A (TRUE) TALE OF HIGGS EVOLUTION IN INFLATION AND THE SURVIVAL OF OUR UNIVERSE

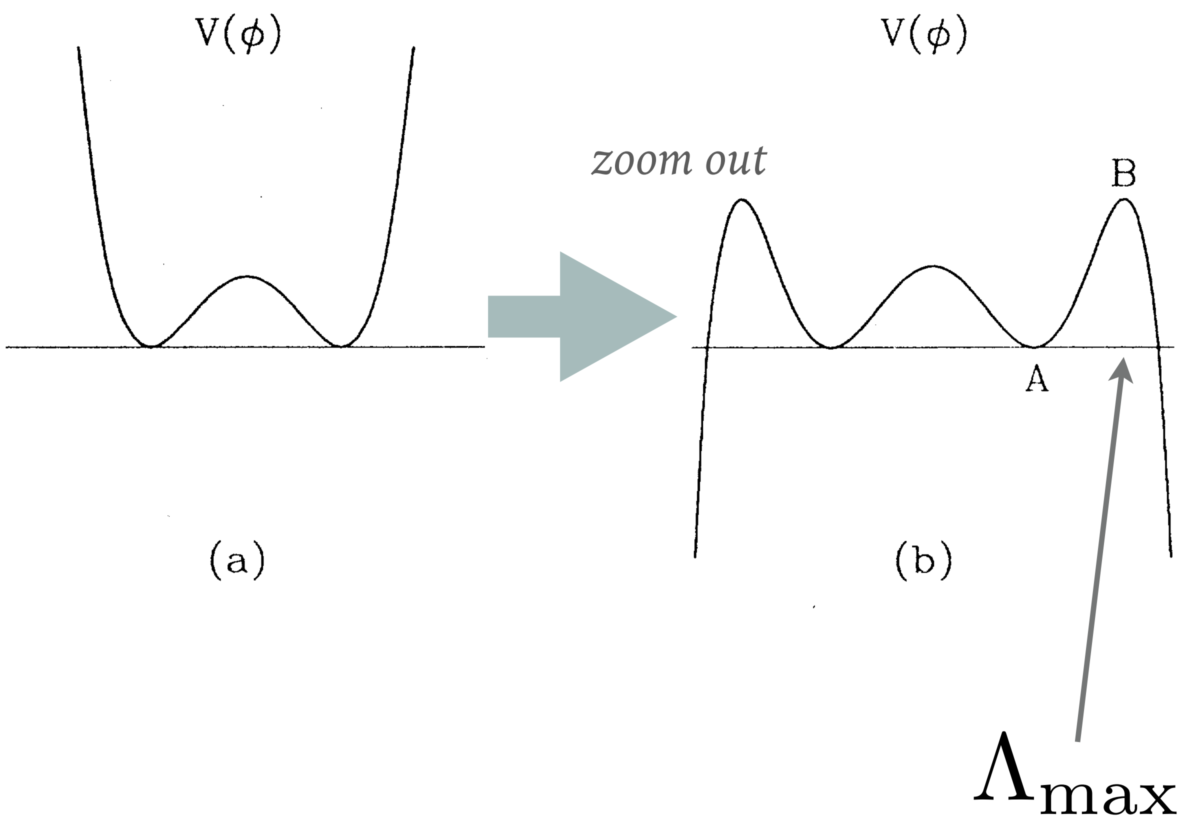
Hook, Kearney, Shakya, KZ 1404.5953
Kearney, Yoo, KZ 1503.05193
East, Kearney, Yoo, KZ 1607.00381

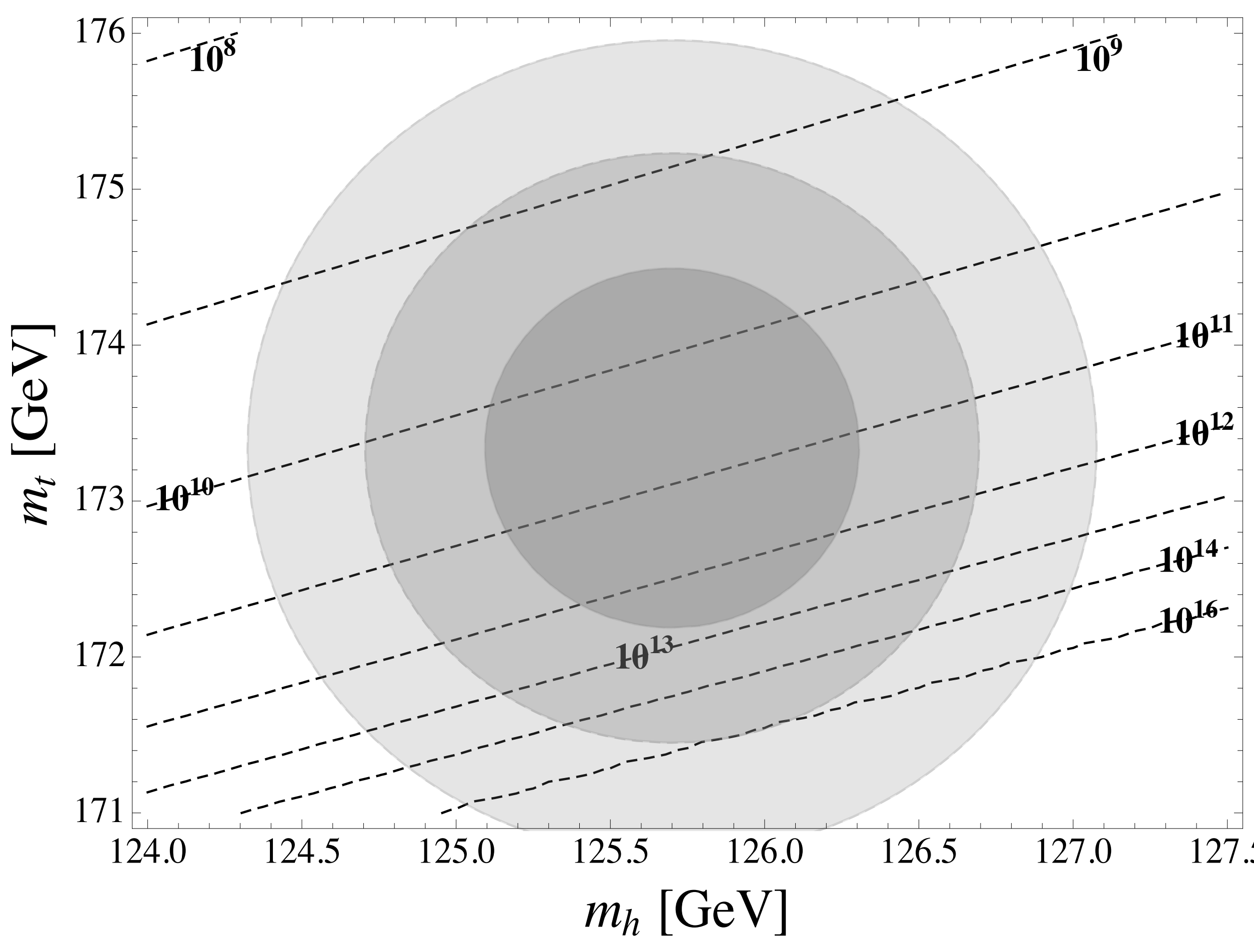
Kathryn Zurek
LBL Berkeley

ONCE UPON A TIME A SM HIGGS WAS DISCOVERED ….

- … and it was found to have a vacuum instability

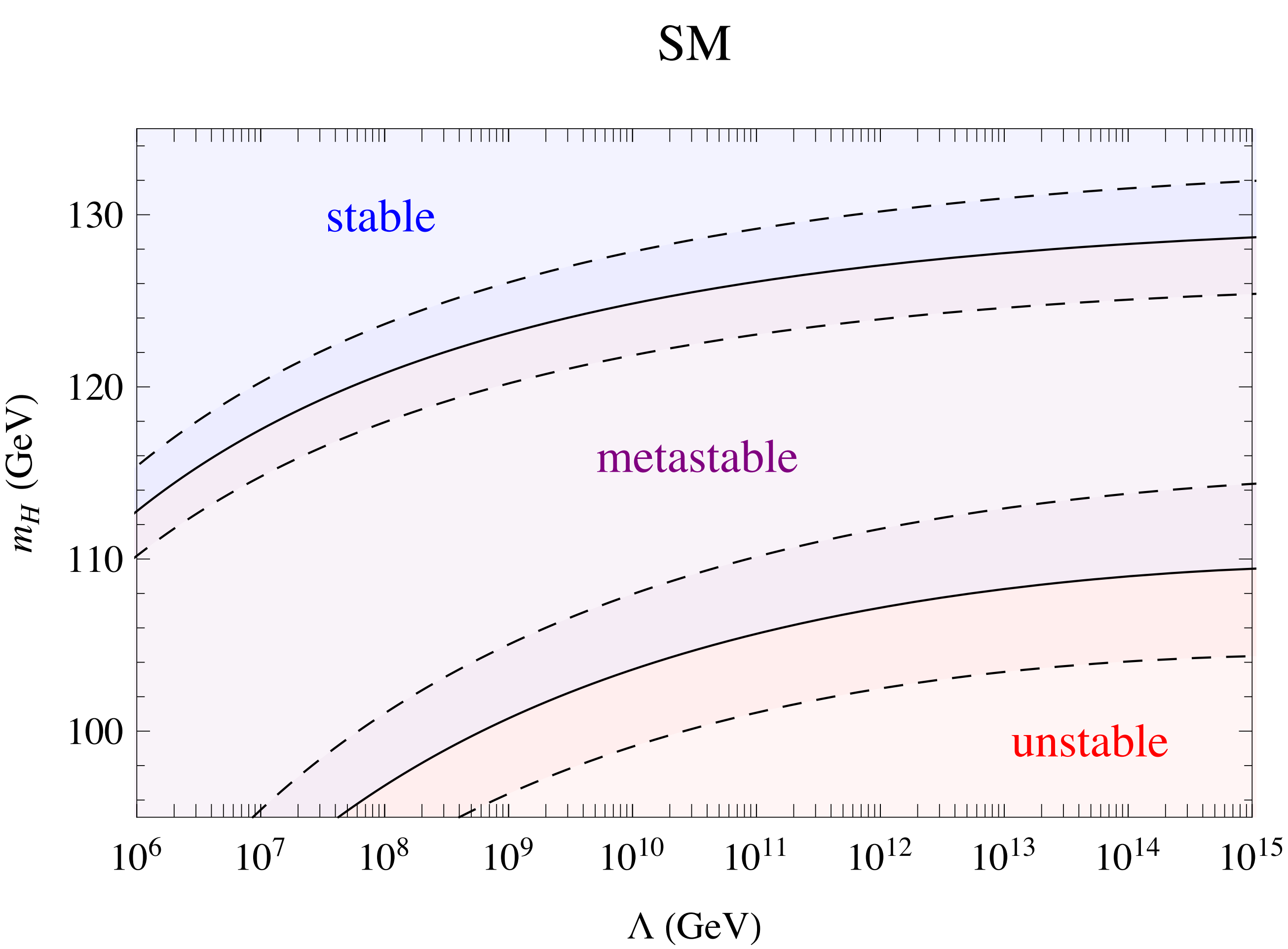
$$V_{\text{eff}}(h) = \frac{\lambda_{\text{eff}}(h)}{4} h^4$$





ONCE UPON A TIME A SM HIGGS WAS DISCOVERED ....

- … that, however, apparently had no Cosmological Implications
- The Universe was deemed to be "Stable Enough"



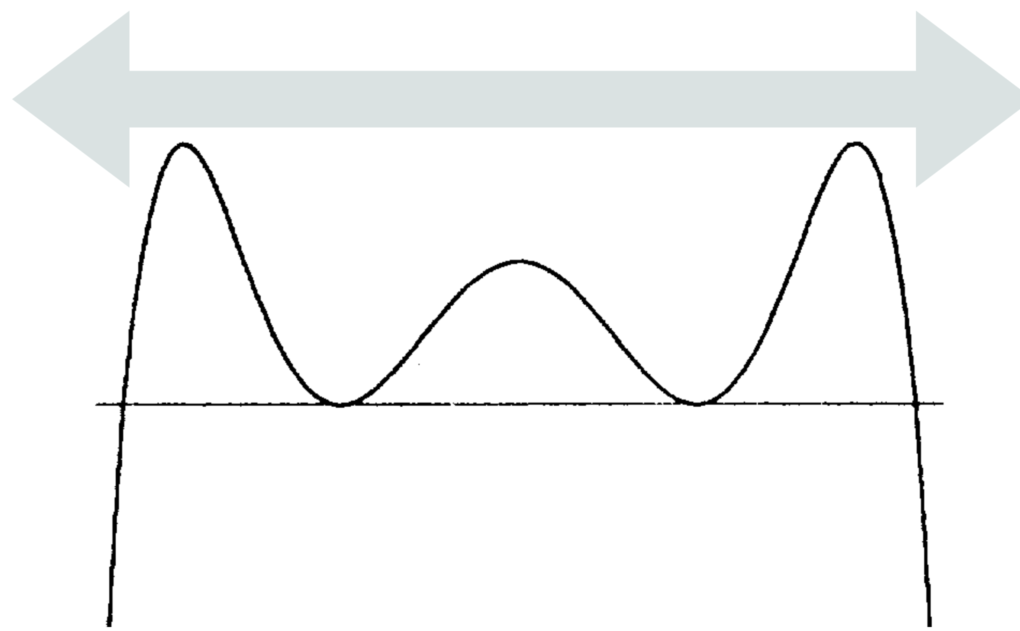
Cheung, Papucci, KZ 2012

Compute Lee-Weinberg bounce

$$p = \max_{h<\Lambda}[V_U h^4 \exp\left(-8\pi^2/3|\lambda(h)|\right)]$$

BUT THE TALE CHANGES IN INFLATION …

- During Inflation Higgs Experiences Same Quantum Fluctuations as the Inflaton

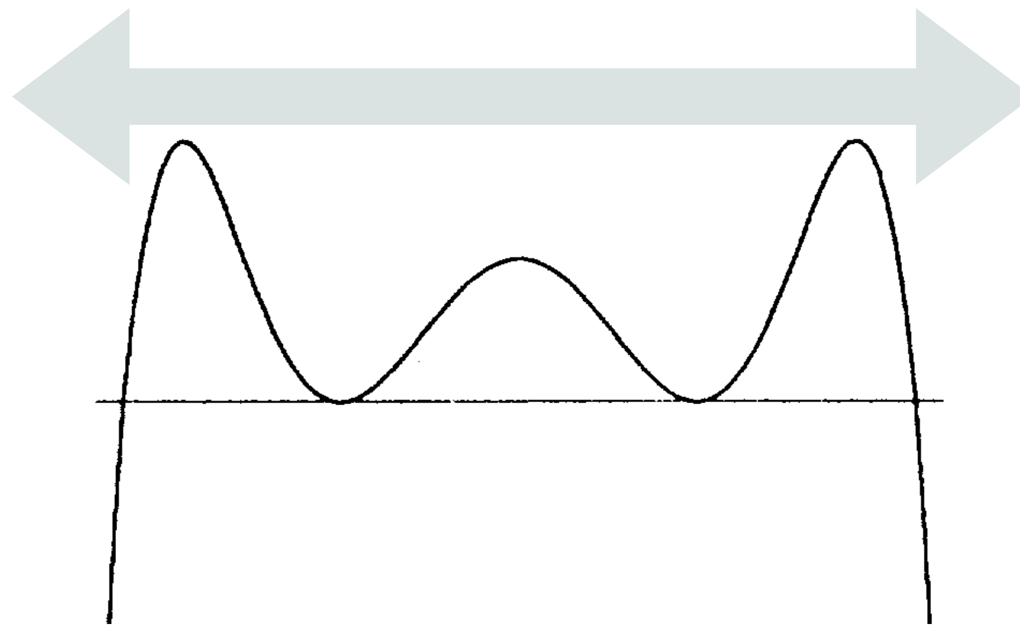


$$\delta h \sim \frac{H}{2\pi}$$

- Implies a potentially dark side for our Universe: a fluctuation can sample unstable part of potential during inflation
- Let us take a close look at this: assume a SM Higgs boson with no gravitational stabilizing corrections

BUT THE TALE CHANGES IN INFLATION …

- During Inflation Higgs Experiences Same Quantum Fluctuations as the Inflaton



$$\delta h \sim \frac{H}{2\pi}$$

- Story is about this “just right” Higgs in an inflating Universe that becomes like ours.
    1. The storyline — how does a fluctuation evolve?
    2. The ending — how does the spacetime react?

A MODEL — STATISTICAL

- How to Describe the Fluctuation Evolution?
- Probabilistic Evolution: Fokker-Planck Equation

$$\frac{\partial P}{\partial t} = \frac{\partial}{\partial \delta h}\left[\frac{V'(\delta h)}{3H}P + \frac{H^3}{8\pi^2}\frac{\partial P}{\partial \delta h}\right]$$

see, e.g., Starobinsky and Yokoyama astro-ph/0407016

- In absence of potential, obtain diffusion

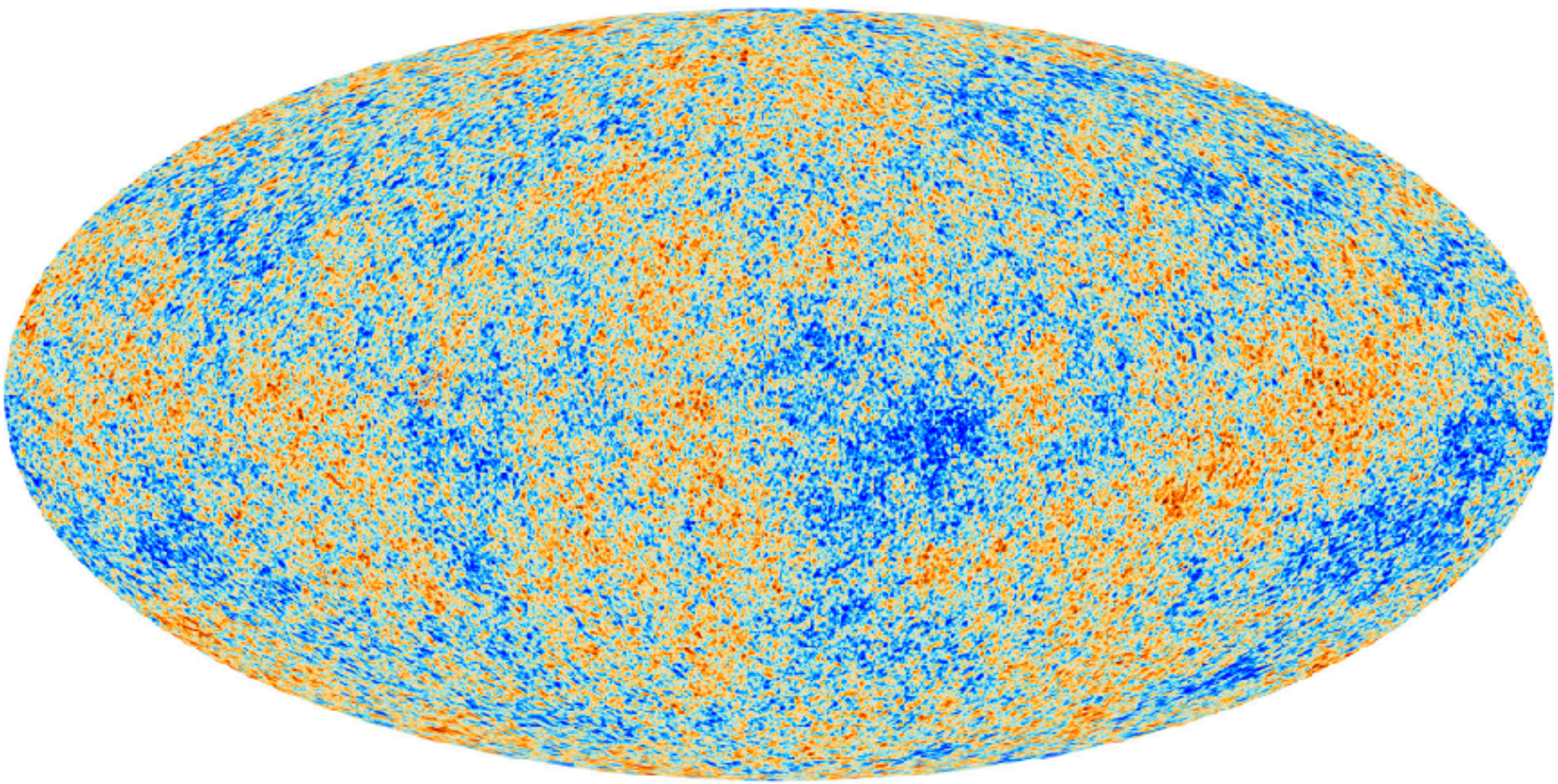
applied to Higgs: Espinosa, Giudice, Riotto '07

$$\langle \delta h^2(t) \rangle = \frac{H^2 \mathcal{N}}{4\pi^2}$$

- Scales like # of e-folds

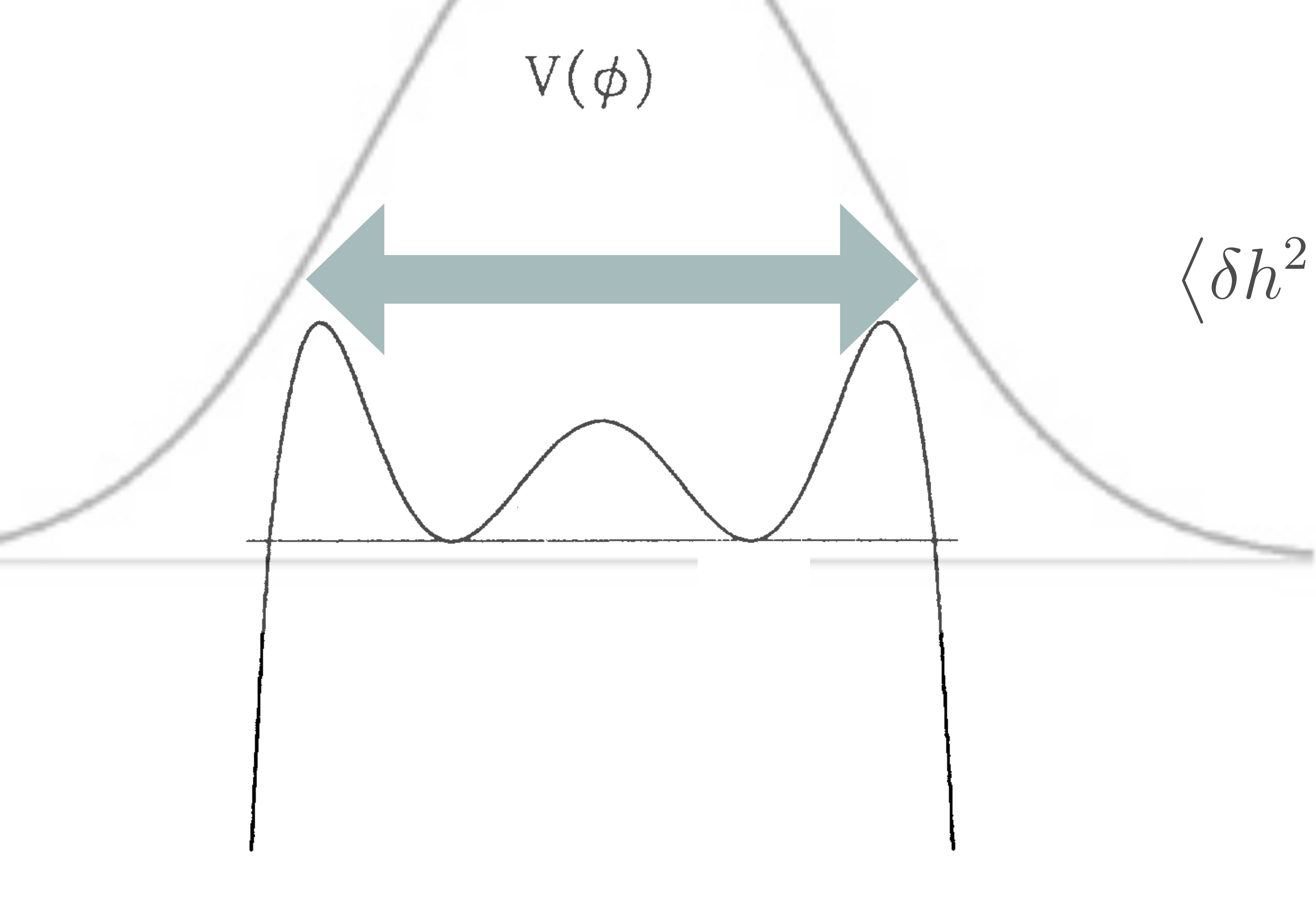
INFLATION & HORIZONS

- Usual story: CMB is sampling of causally separated patches seeded by quantum fluctuations

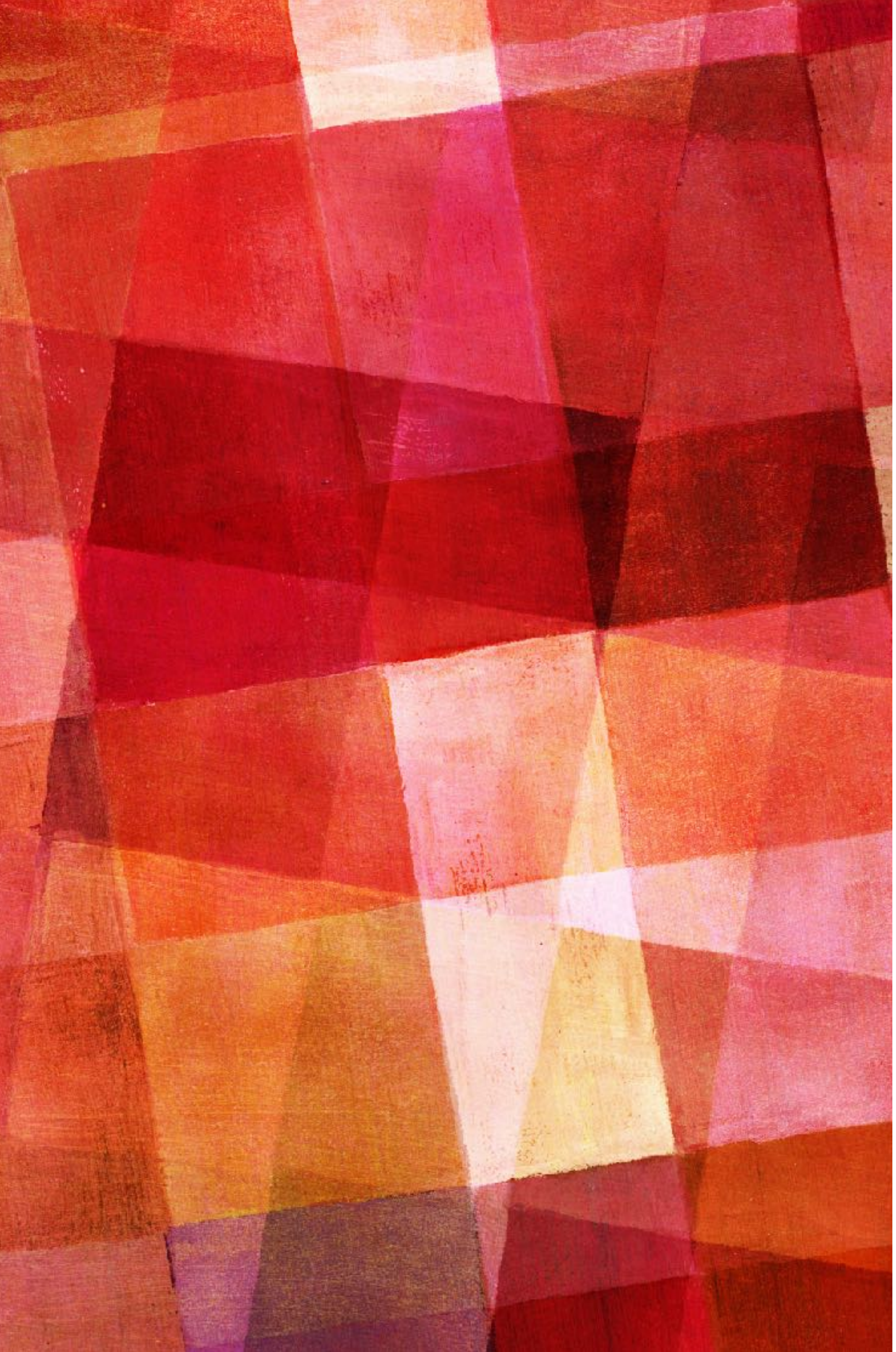


A MODEL — STATISTICAL

- During Inflation Higgs Experiences Same Quantum Fluctuations as the Inflaton

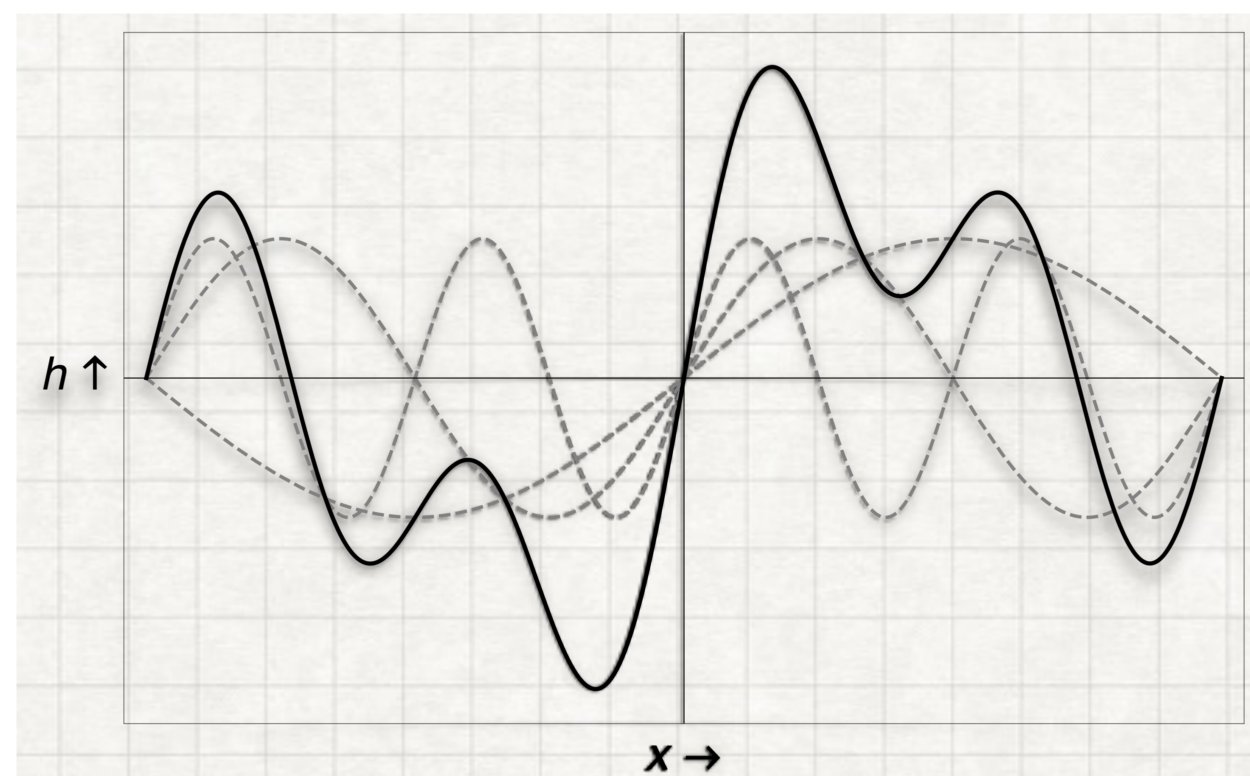


$$\langle \delta h^2(t) \rangle = \frac{H^2 \mathcal{N}}{4\pi^2}$$



INFLATION AND HORIZONS

- Average fluctuation in an inflationary patch (H^{-1}) is sum over super horizon modes
- Higgs undergoes random walk within patch with each subsequent mode crossing



STATISTICAL MODEL FROM HIGGS EOM

$$\ddot{h} + 3H\dot{h} - \left(\frac{\vec{\nabla}}{a}\right)^2 h + V'(h) = 0$$

$\bar{h}(0) = 0,\ \bar{h}(t) = 0$

Hartree-Fock (Gaussian) approximation

$$3H\dot{\delta h}_k(t) + 3\lambda \left\langle \delta h^2(t) \right\rangle \delta h_k(t) = 0$$

$$\left\langle \delta h^2(t) \right\rangle = \int_{k=1/L}^{k=\epsilon a H} \frac{d^3k}{(2\pi)^3} \left|\delta h_k(t)\right|^2$$

See Kearney, Yoo, KZ 1503.05193 for more details

$$\frac{d}{dt}\left\langle \delta h^2(t) \right\rangle = -\frac{2\lambda}{H}\left\langle \delta h^2(t) \right\rangle^2 + \frac{H^3}{4\pi^2}$$

PERPLEXITIES

- When is a statistical treatment appropriate?

  Hook, Kearney, Shakya, KZ 1404.5953

- What is V(h)?

  Kearney, Yoo, KZ 1503.05193

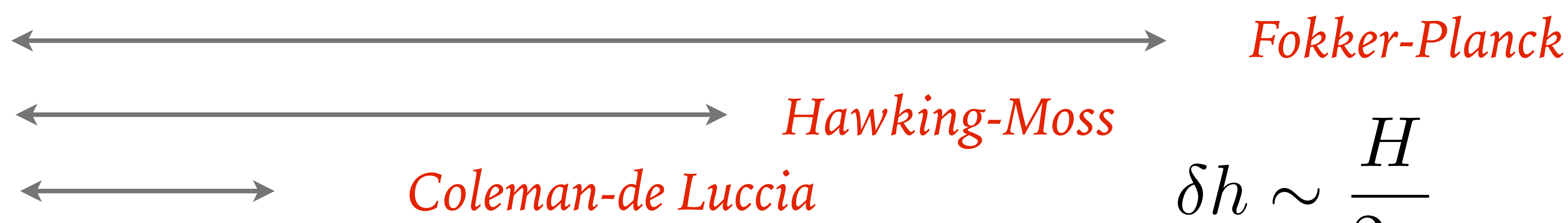
- What is the response of the spacetime to sampling of the unstable potential? When is the sampling problematic for a Universe like ours?

  East, Kearney, Yoo, KZ 1607.00381

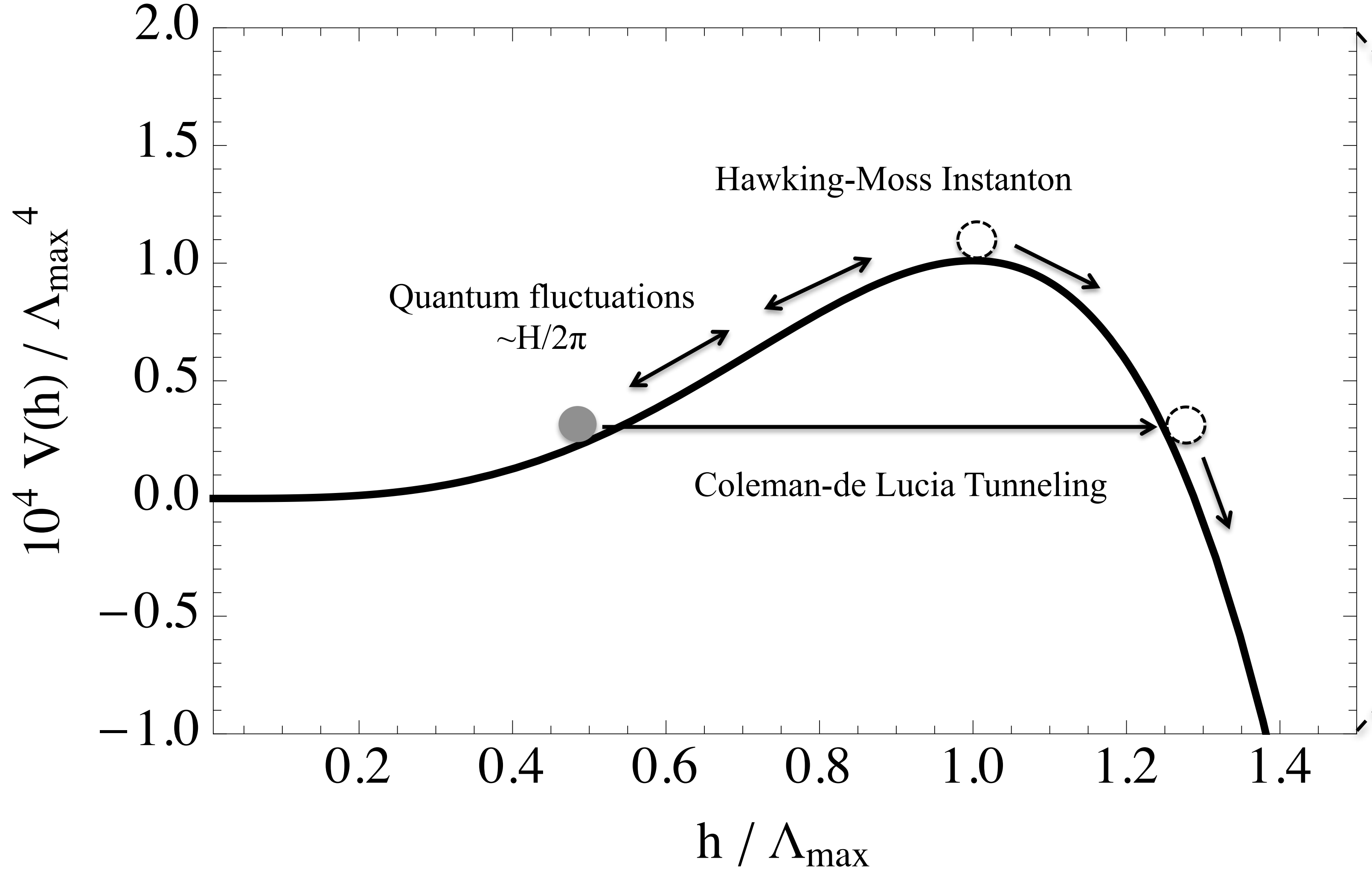
FIELD EXCURSIONS

Hook, Kearney, Shakya, KZ 1404.5953

- When is a Statistical Treatment Appropriate?



$$\delta h \sim \frac{H}{2\pi}$$



FIELD EXCURSIONS

Hook, Kearney, Shakya, KZ 1404.5953

- Smallest Excursions — *Rarest Transitions*

  (Coleman-DeLuccia) — $H^2 \lesssim V''_{\rm eff}(\Lambda_{\rm max})$

- Moderate Excursions — *Rare Transitions*

  (Hawking-Moss) — $V''_{\rm eff}(\Lambda_{\rm max}) \lesssim H^2 \lesssim (V_{\rm eff}(\Lambda_{\rm max}))^{1/2}$

- Big Excursions — *Not Rare Transitions*

  (Fokker-Planck) — $H \gtrsim (V_{\rm eff}(\Lambda_{\rm max}))^{1/4}$

FIELD EXCURSIONS

Hook, Kearney, Shakya, KZ 1404.5953

- Smallest Excursions — *Rarest Transitions*

  (Coleman-DeLuccia) $H^2 \lesssim V''_{\text{eff}}(\Lambda_{\text{max}})$

- Moderate Excursions — *Rare Transitions*

  (Hawking-Moss) $V''_{\text{eff}}(\Lambda_{\text{max}}) \lesssim H^2 \lesssim (V_{\text{eff}}(\Lambda_{\text{max}}))^{1/2}$

- Big Excursions $P_{\text{HM}} \simeq \exp\left[-\frac{8\pi^2 V(\Lambda_{\text{max}})}{3H^4}\right]$ — *Not Rare Transitions*

  (Fokker-Planck) $H \gtrsim (V_{\text{eff}}(\Lambda_{\text{max}}))^{1/4}$

"Single bounce limit of Fokker-Planck solution"

FIELD EXCURSIONS

Kearney, Yoo, KZ 1503.05193

- Large excursions (into unstable part of potential) *do not* signal the end of inflation
- As long as statistical fluctuations dominate over potential, diffusion continues with inflation undisturbed
- Rare fluctuations do not end inflation globally

$$\frac{d}{dt}\left\langle \delta h^2(t)\right\rangle = -\frac{2\lambda}{H}\left\langle \delta h^2(t)\right\rangle^2 + \frac{H^3}{4\pi^2} \quad \Rightarrow \quad \left\langle \delta h^2(t)\right\rangle = \frac{1}{\sqrt{-2\lambda}}\frac{H^2}{2\pi}\tan\left(\sqrt{-2\lambda}\frac{\mathcal{N}}{2\pi}\right)$$

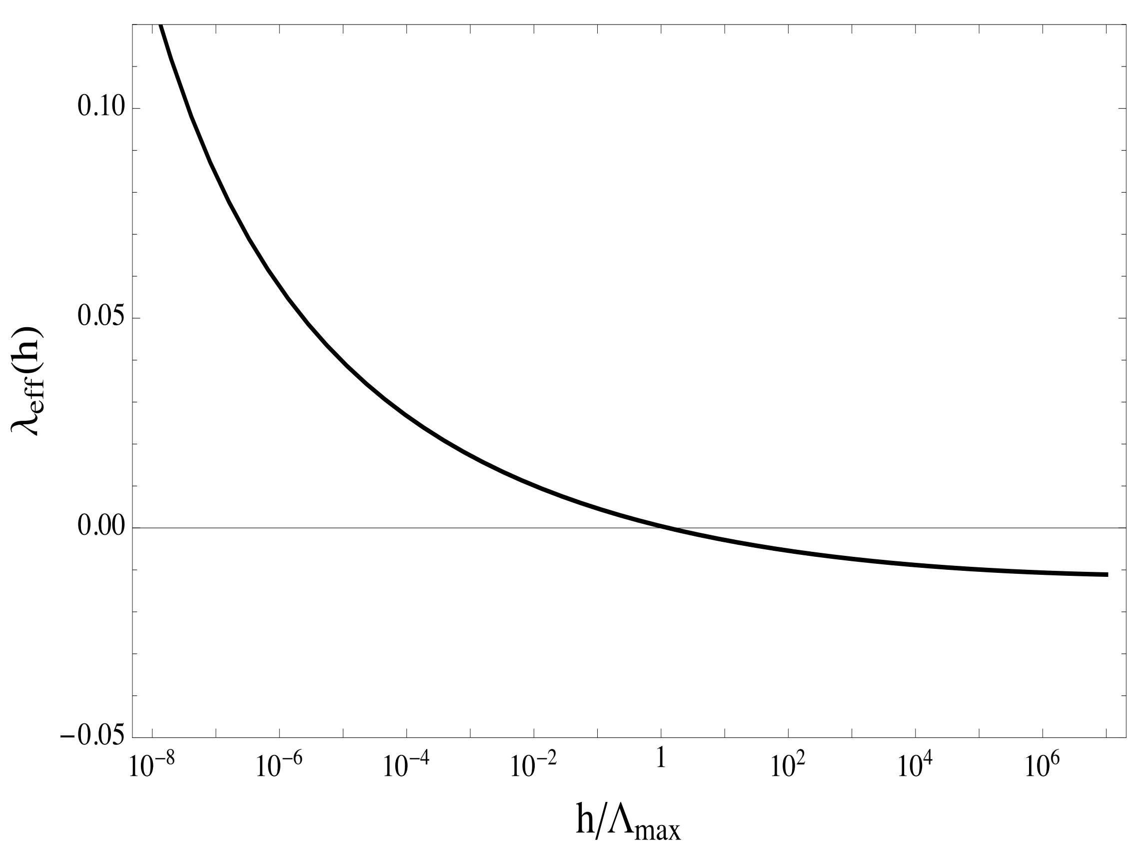
Divergence occurs when

$$\mathcal{N}_{\max} = \frac{\pi^2}{\sqrt{-2\lambda}} \implies 70 \lesssim \mathcal{N}_{\max} \lesssim 100$$

WHAT IS V(H)?

Kearney, Yoo, KZ 1503.05193

- Effective potential evaluated at $\mu = \delta h$ typically employed
- Not immediately clear exactly what this means as effective potential away from extrema is unphysical
- However, in quasi-conformal regime, may not have a big impact on the result



$$\langle \delta h^2(t) \rangle = \frac{1}{\sqrt{-2\lambda}} \frac{H^2}{2\pi} \tan\left(\sqrt{-2\lambda} \frac{\mathcal{N}}{2\pi} \right)$$

WHAT IS V(H)? WILSONIAN EFT

Kearney, Yoo, KZ 1503.05193

- SM fields come in active and passive
  1. Passive modes decay outside the horizon; active grow
     - Fermions & gauge bosons = passive; scalars = active

       (Woodard and collaborators)
  2. Equations describe evolution of super-horizon modes
     - Potential is the RG-improved Higgs potential
  3. Fermions and gauge bosons are active on sub-horizon scales
     - Renormalize coupling as in Minkowski space

WHAT IS V(H)? WILSONIAN EFT

Kearney, Yoo, KZ 1503.05193

- Prescription: run SM down from UV as in Minkowski space, integrating out passive states where the mode functions become suppressed

$$V(h) = \frac{1}{4}\lambda h^4 \qquad \textit{with} \qquad \lambda\left(\mu \simeq \sqrt{H^2 + h^2}\right)$$

- Consistency:
  - $h \ll H$: fermions and gauge bosons decouple at horizon scale, $h \sim H$
  - $h \gg H$: fermions and gauge bosons decouple at "mass threshold," $m_f = y_f h$, $m_V = gh$

Verified by explicit calculation in Herranen et al 1407.3141

CONSISTENCY CHECK: INFRARED LOGS

Kearney, Yoo, KZ 1503.05193

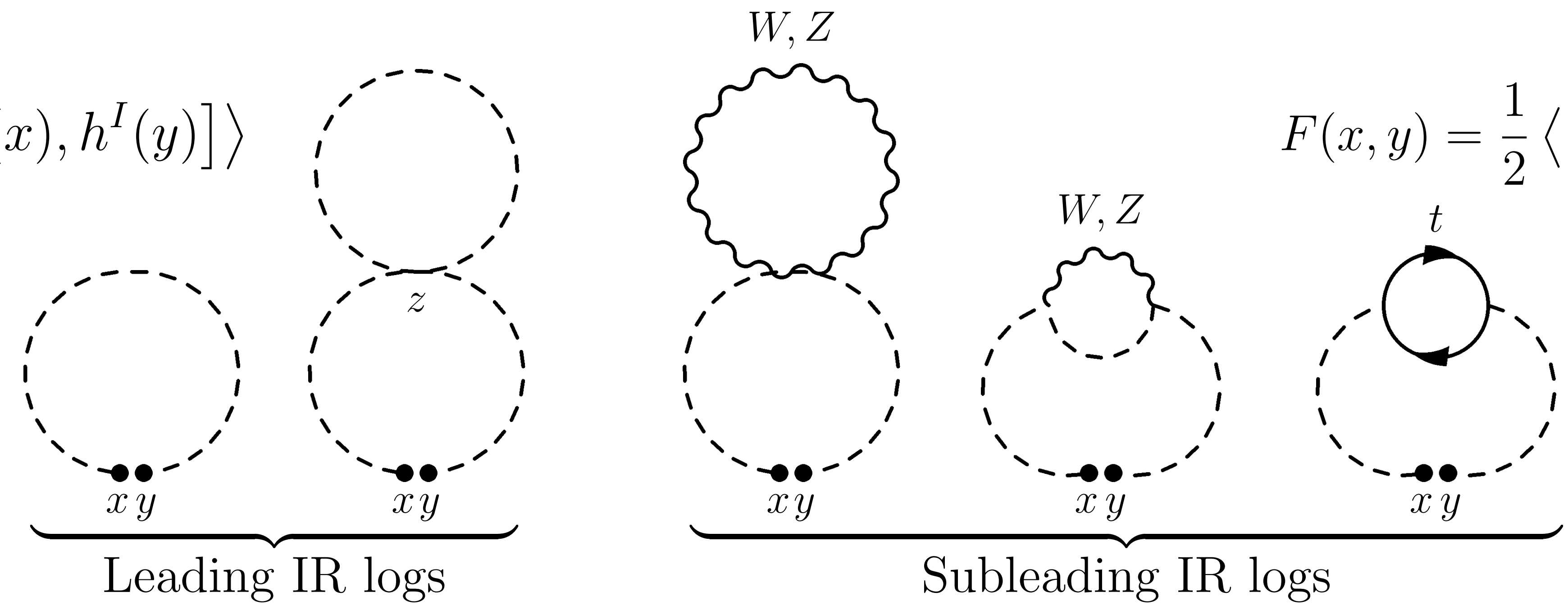
- Calculate divergence of $\langle \delta h^2 \rangle$ utilizing in-in formalism

(e.g. Weinberg, hep-th/0506236)

$$\langle \mathcal{O}(t) \rangle = \sum_n (-i)^n \int_{-\infty}^{t} dt_1 \cdots \int_{-\infty}^{t_{n-1}} dt_n \left\langle \left[\left[\mathcal{O}^I(t), H^I(t_n) \right], \cdots H^I(t_1) \right] \right\rangle$$

$$\rho(x,y) = i \left\langle \left[h^I(x), h^I(y) \right] \right\rangle$$

$$F(x,y) = \frac{1}{2} \left\langle \left\{ h^I(x), h^I(y) \right\} \right\rangle$$



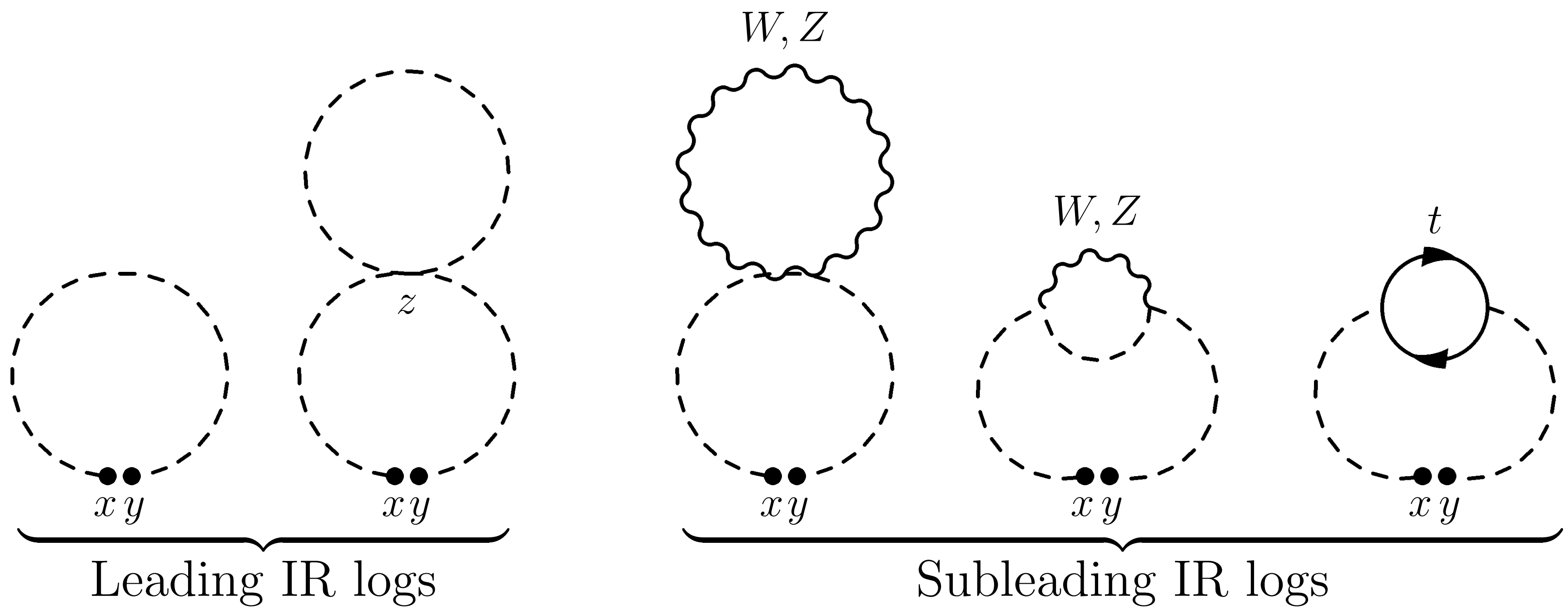
$$\langle h(t,\vec{x}) h(t,\vec{y}) \rangle = F(x,y) - \int^t d^4z\, a^3(t_z) \left[F(x,z)\rho(y,z) + \rho(x,z)F(y,z) \right] \left(3\lambda F(z,z) + \delta m^2 + \delta\xi R(z) \right)$$

CONSISTENCY CHECK: INFRARED LOGS

Kearney, Yoo, KZ 1503.05193

- Calculate divergence of $\langle \delta h^2 \rangle$ utilizing in-in formalism

$$3\lambda F(z,z) = 3\lambda \int_{\Lambda_{IR}}^{a\Lambda} \frac{d^3k}{(2\pi)^3} \left| h_k(t_z) \right|^2 = 3\lambda \left[\frac{\Lambda^2}{8\pi^2} + \frac{H^2}{8\pi^2} \ln \left[\left(\frac{a\Lambda}{\Lambda_{IR}} \right)^2 \right] \right]$$



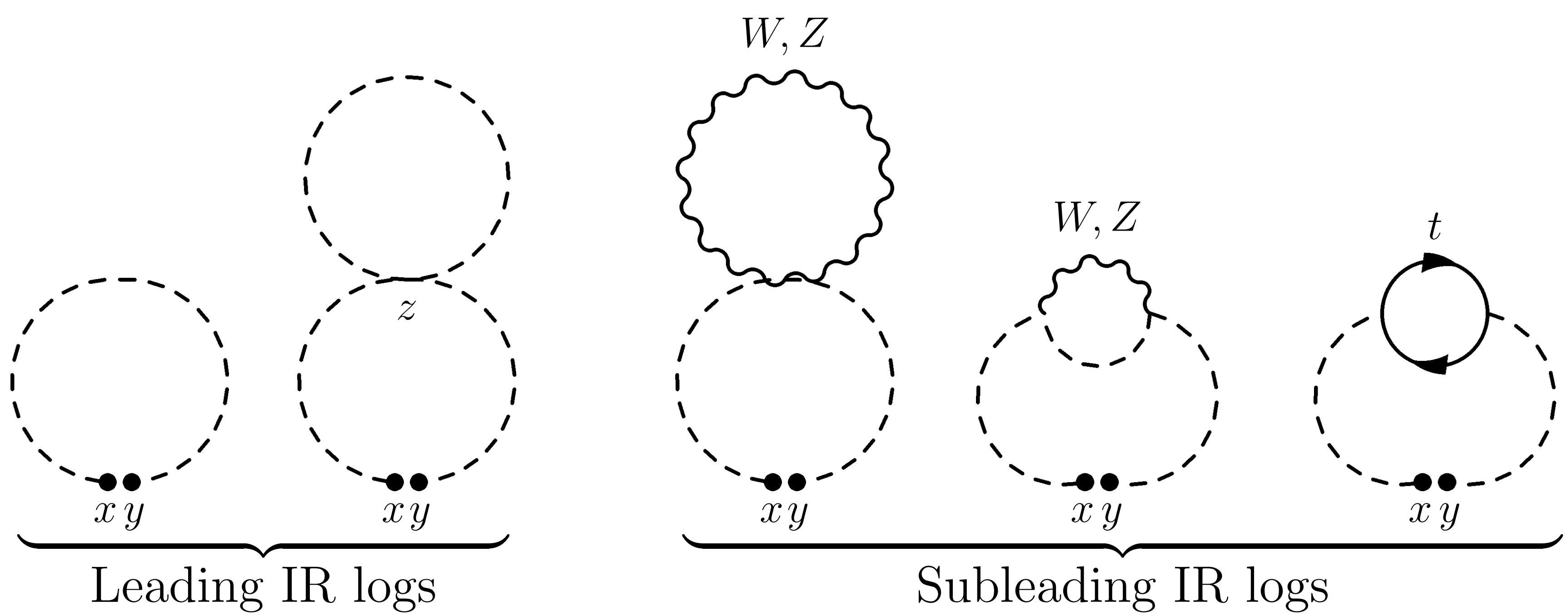
$$\langle h(t,\vec{x}) h(t,\vec{y}) \rangle = F(x,y) - \int^t d^4z\, a^3(t_z) \left[F(x,z)\rho(y,z) + \rho(x,z)F(y,z) \right] \left(3\lambda F(z,z) + \delta m^2 + \delta\xi R(z) \right)$$

CONSISTENCY CHECK: INFRARED LOGS

Kearney, Yoo, KZ 1503.05193

- Calculate divergence of $\langle \delta h^2 \rangle$ utilizing in-in formalism

$$3\lambda F(z,z) + \delta m^2 + \delta\xi R = \frac{3\lambda(\mu)H^2}{8\pi^2}\left(2\mathcal{N} + \ln\frac{\mu^2}{H^2}\right)$$



$$\langle h(t,\vec{x})h(t,\vec{y})\rangle = F(x,y) - \int^t d^4z\, a^3(t_z)\left[F(x,z)\rho(y,z) + \rho(x,z)F(y,z)\right]\left(3\lambda F(z,z) + \delta m^2 + \delta\xi R(z)\right)$$

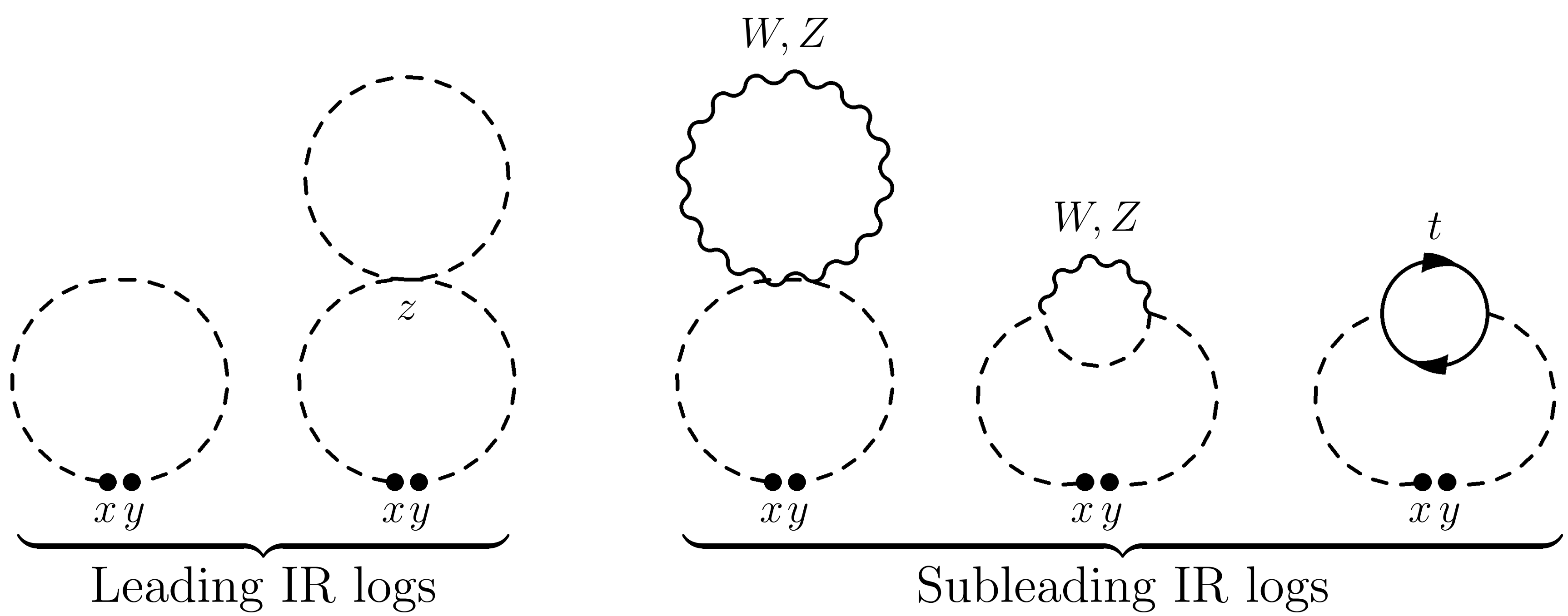
CONSISTENCY CHECK: INFRARED LOGS

Kearney, Yoo, KZ 1503.05193

- Calculate divergence of $\langle \delta h^2 \rangle$ utilizing in-in formalism

$$3\lambda F(z,z) + \delta m^2 + \delta\xi R = \frac{3\lambda(\mu) H^2}{8\pi^2}\left(2\mathcal{N} + \ln\frac{\mu^2}{H^2}\right)$$

$$\frac{H^2}{4\pi^2}\mathcal{N}$$



$$-\frac{\lambda}{24\pi^2}H^2\mathcal{N}^3$$

$$\langle h(t,\vec{x})h(t,\vec{y})\rangle = F(x,y) - \int^t d^4z\, a^3(t_z)\left[F(x,z)\rho(y,z) + \rho(x,z)F(y,z)\right]\left(3\lambda F(z,z) + \delta m^2 + \delta\xi R(z)\right)$$

A CONSISTENT STORY

Kearney, Yoo, KZ 1503.05193

$$\left\langle \delta h^2(t) \right\rangle_{\rm HF} \approx \frac{H^2}{4\pi^2}\mathcal{N} - \frac{\lambda H^2}{24\pi^4}\mathcal{N}^3$$

- Equivalent to expansion of

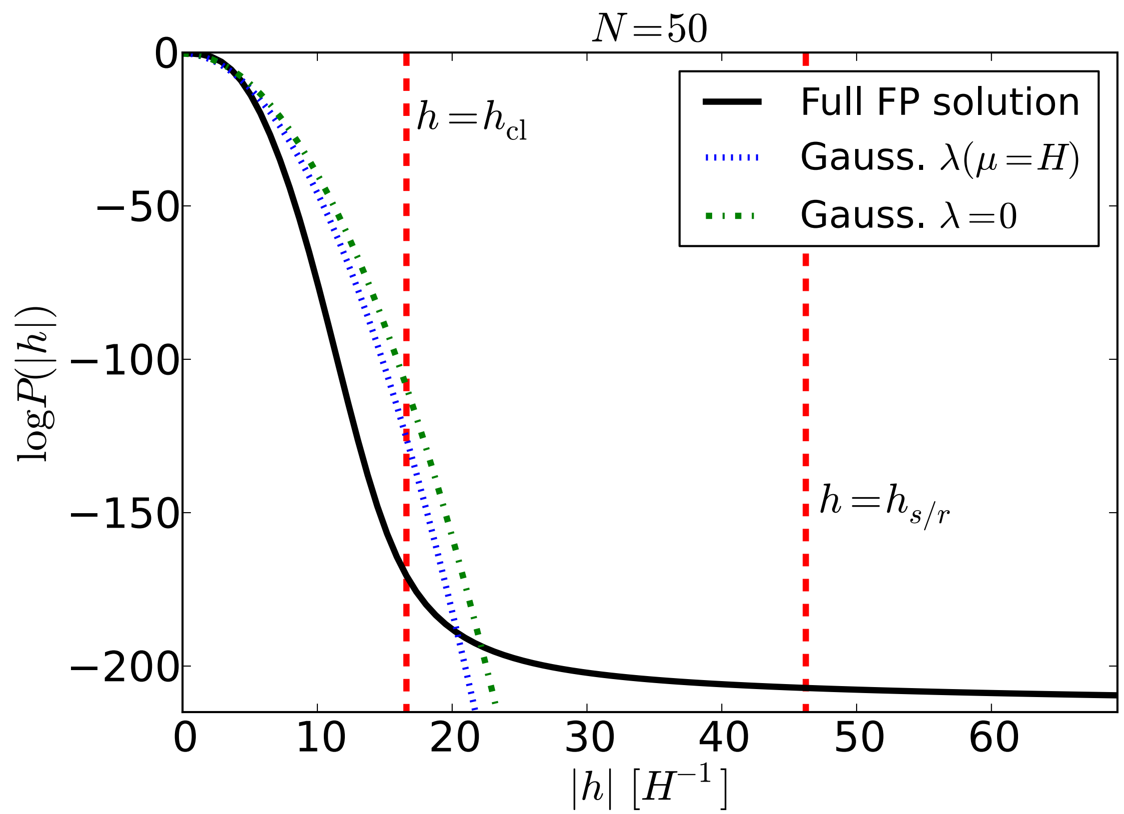
$$\left\langle \delta h^2(t) \right\rangle = \frac{1}{\sqrt{-2\lambda}}\frac{H^2}{2\pi}\tan\left(\sqrt{-2\lambda}\frac{\mathcal{N}}{2\pi}\right)$$

with coupling renormalized at $\mu \simeq H$

IMPLICATION FOR THE END OF THE STORY — OUR UNIVERSE?

- Depends on whether a patch falling to the true vacuum expands outwards, or not
- If patch expands outwards, then a single patch in the past light cone is disastrous; so need e^{3N_0} good patches

Tails are important!

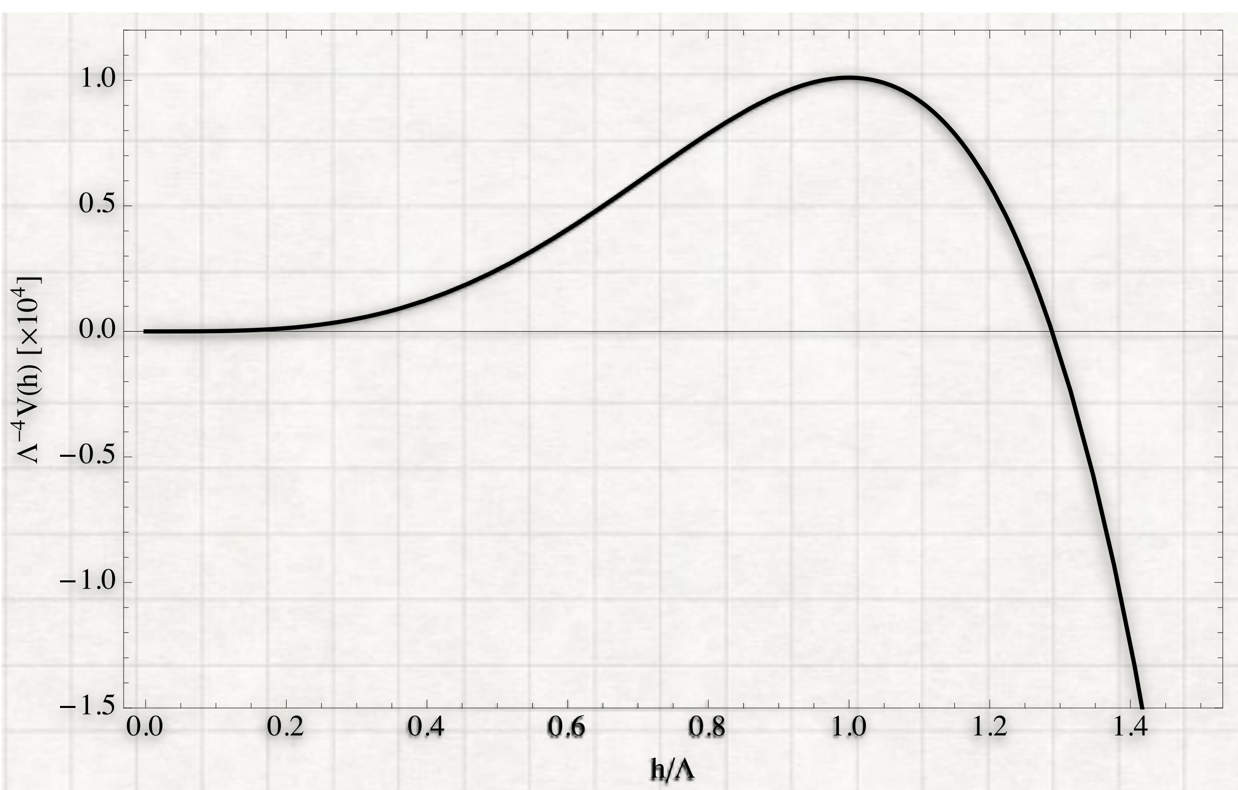


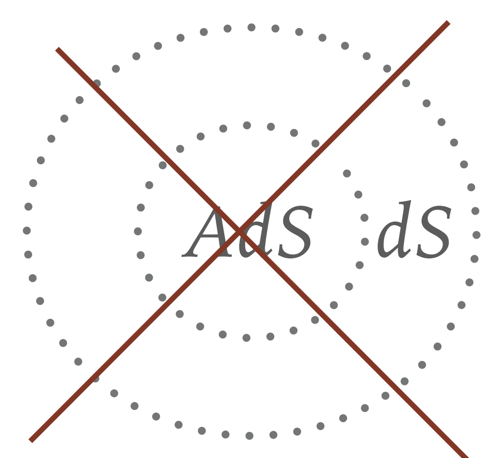
DOES A BAD PATCH EXPAND?

East, Kearney, Yoo, KZ 1607.00381

- Normal intuition is that a true vacuum expands outwards
- Here, though, the true vacuum is crunching; also potential energy from potential is liberated as the field enters the true vacuum

No thin wall bubble nucleation





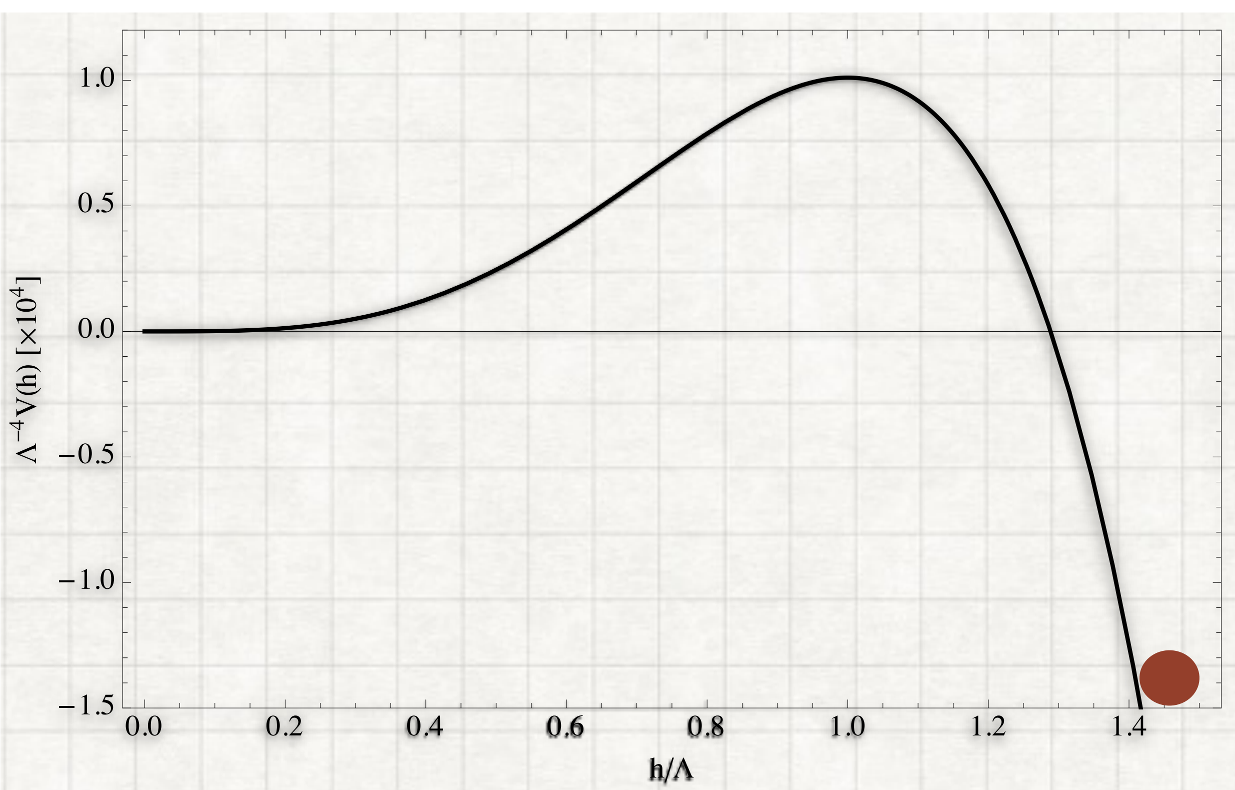
treated with thin wall: Espinosa et al. 1505.04825

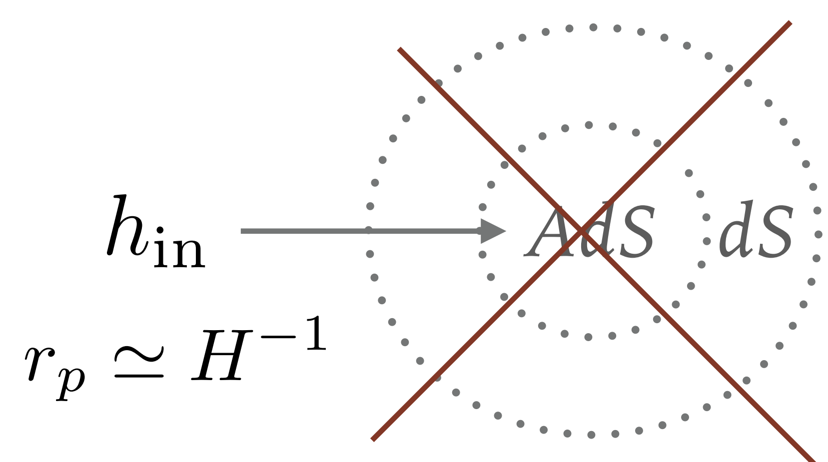
DOES A BAD PATCH EXPAND?

East, Kearney, Yoo, KZ 1607.00381

- Normal intuition is that a true vacuum expands outwards
- Here, though, the true vacuum is crunching; also potential energy from potential is liberated as the field enters the true vacuum

No thin wall bubble nucleation

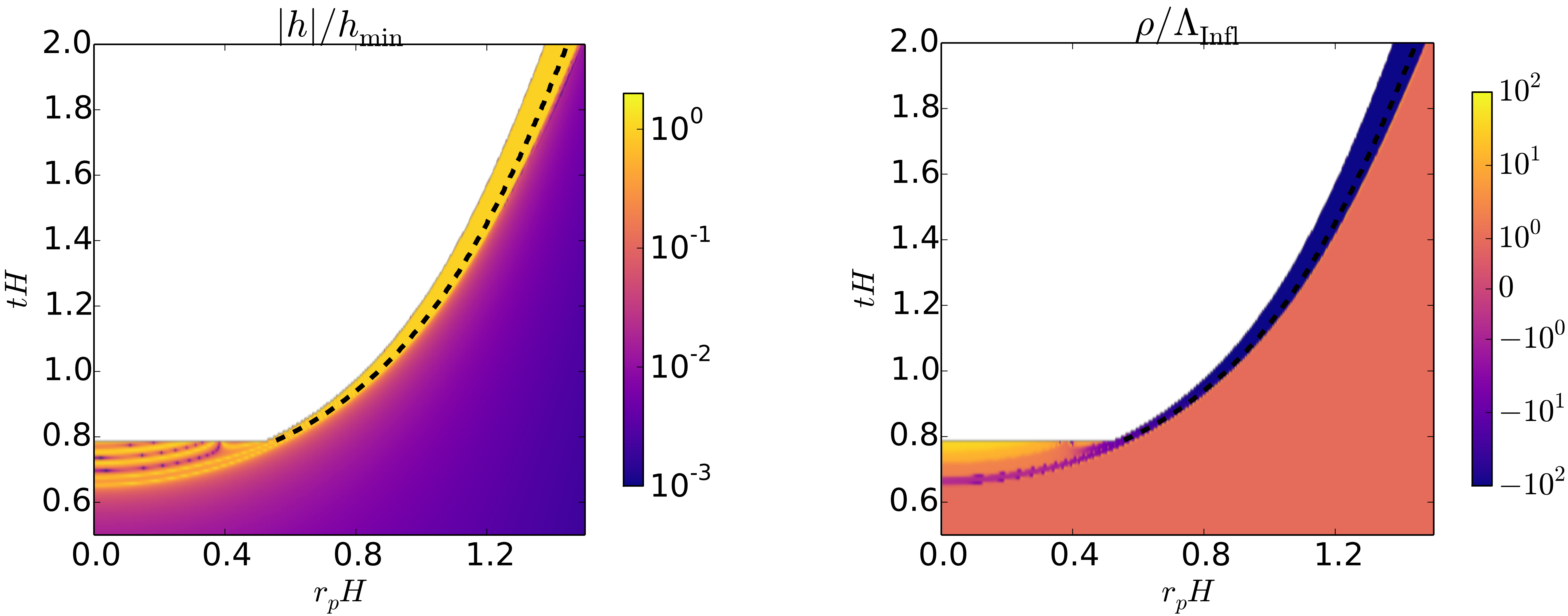




$h_{\rm in}$ *defined by point where classical motion dominates over diffusion*

DOES A BAD PATCH EXPAND?

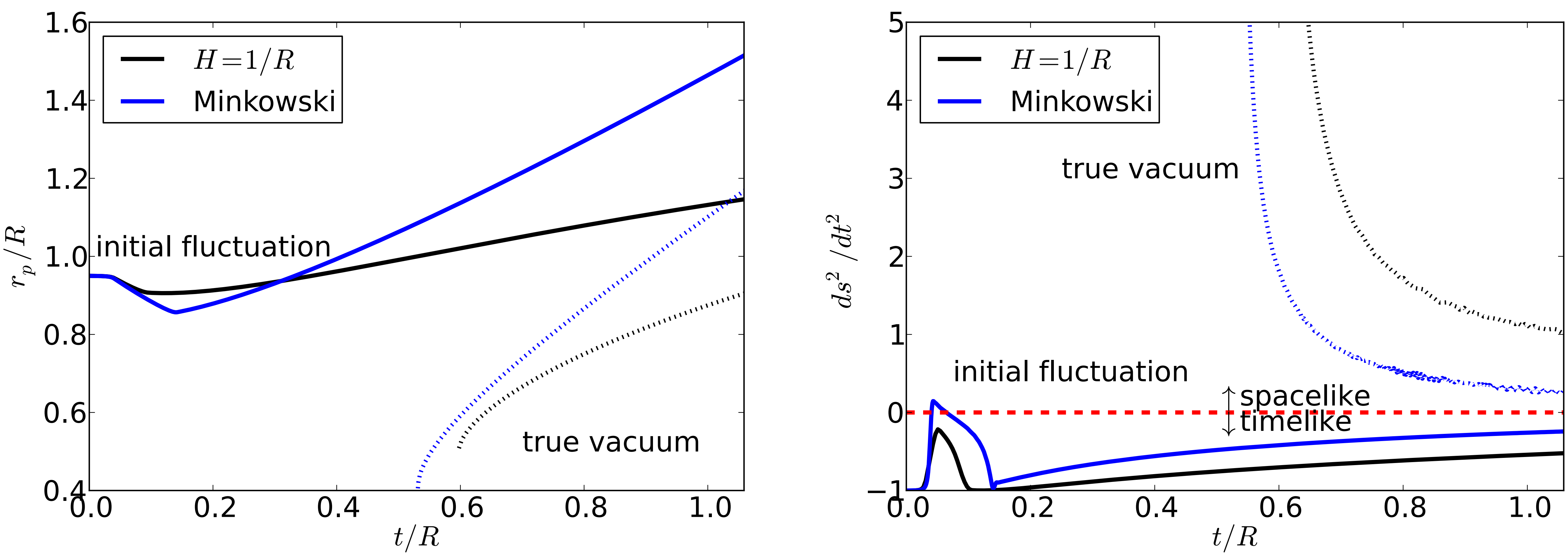
East, Kearney, Yoo, KZ 1607.00381



DOES A BAD PATCH EXPAND?

East, Kearney, Yoo, KZ 1607.00381

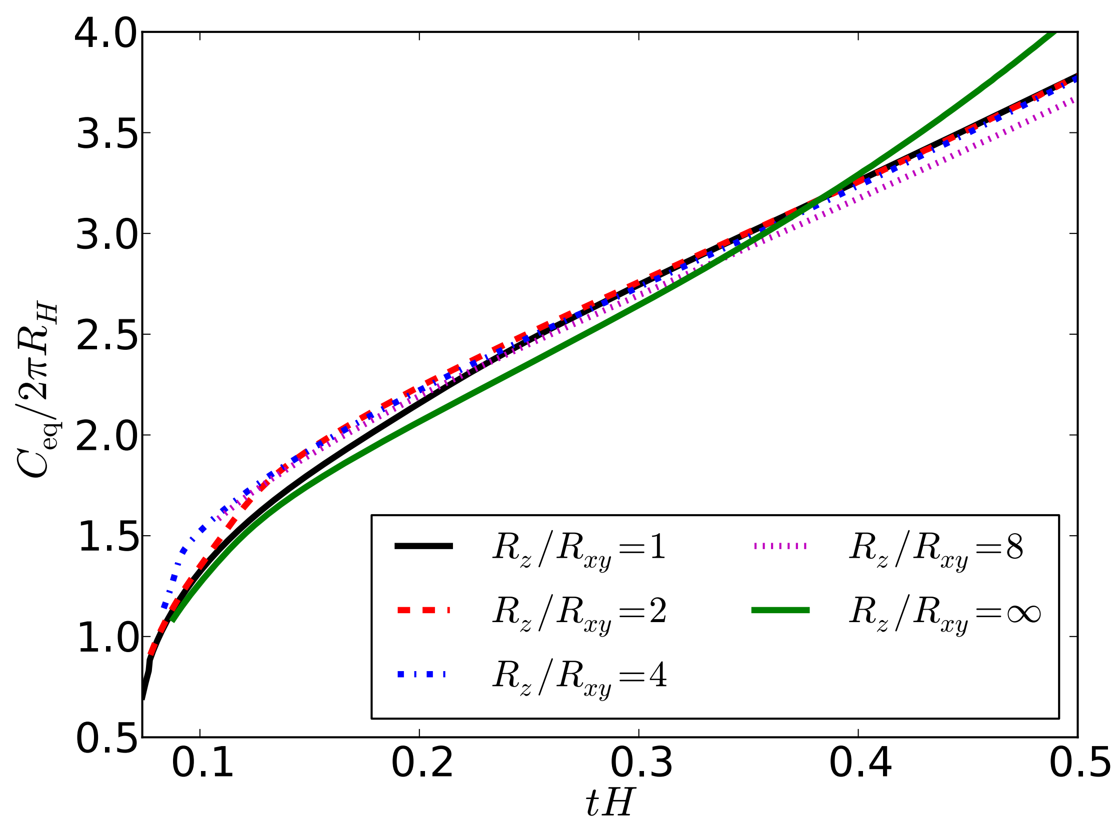
➤ True vacuum patch formation is initially spacelike

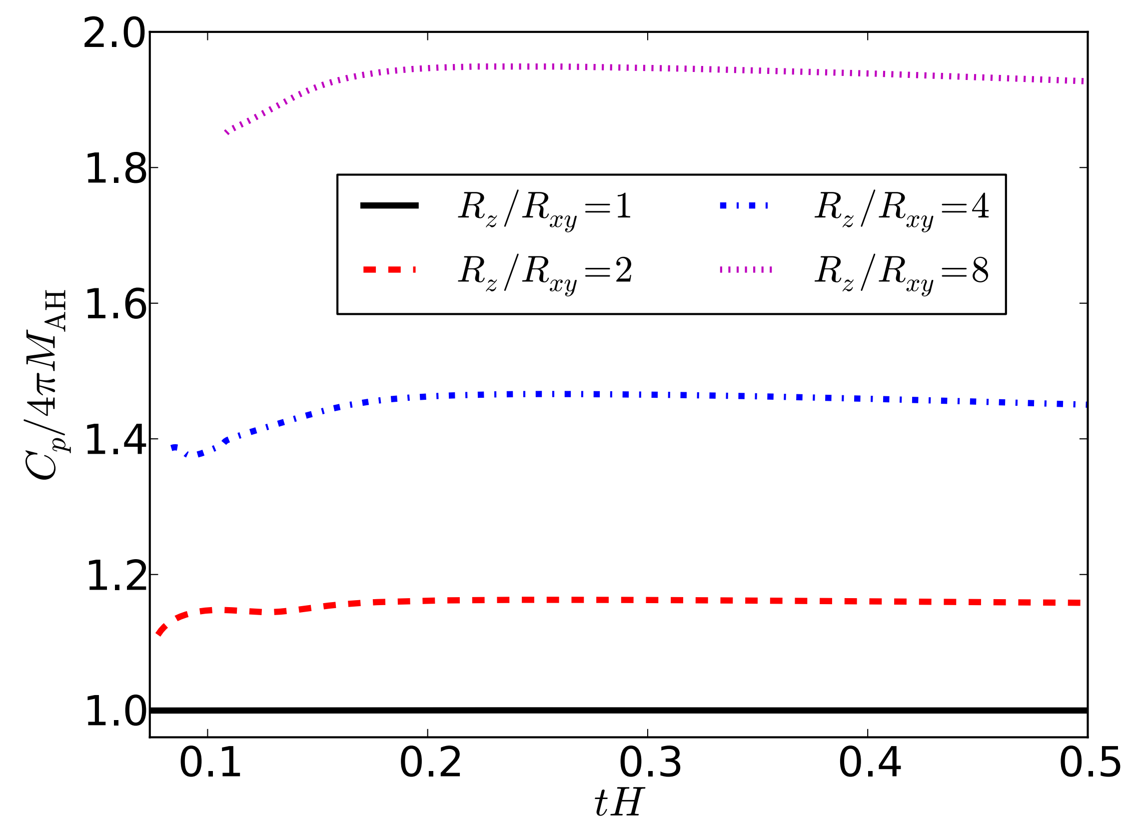


NON-SPHERICAL CONFIGURATIONS

East, Kearney, Yoo, KZ 1607.00381

- Black Holes and the Hoop Conjecture
- Region of mass M will create black hole iff a "hoop" of circumf. $4\pi M$ can be passed over region in every direction
- Manifestly violated here

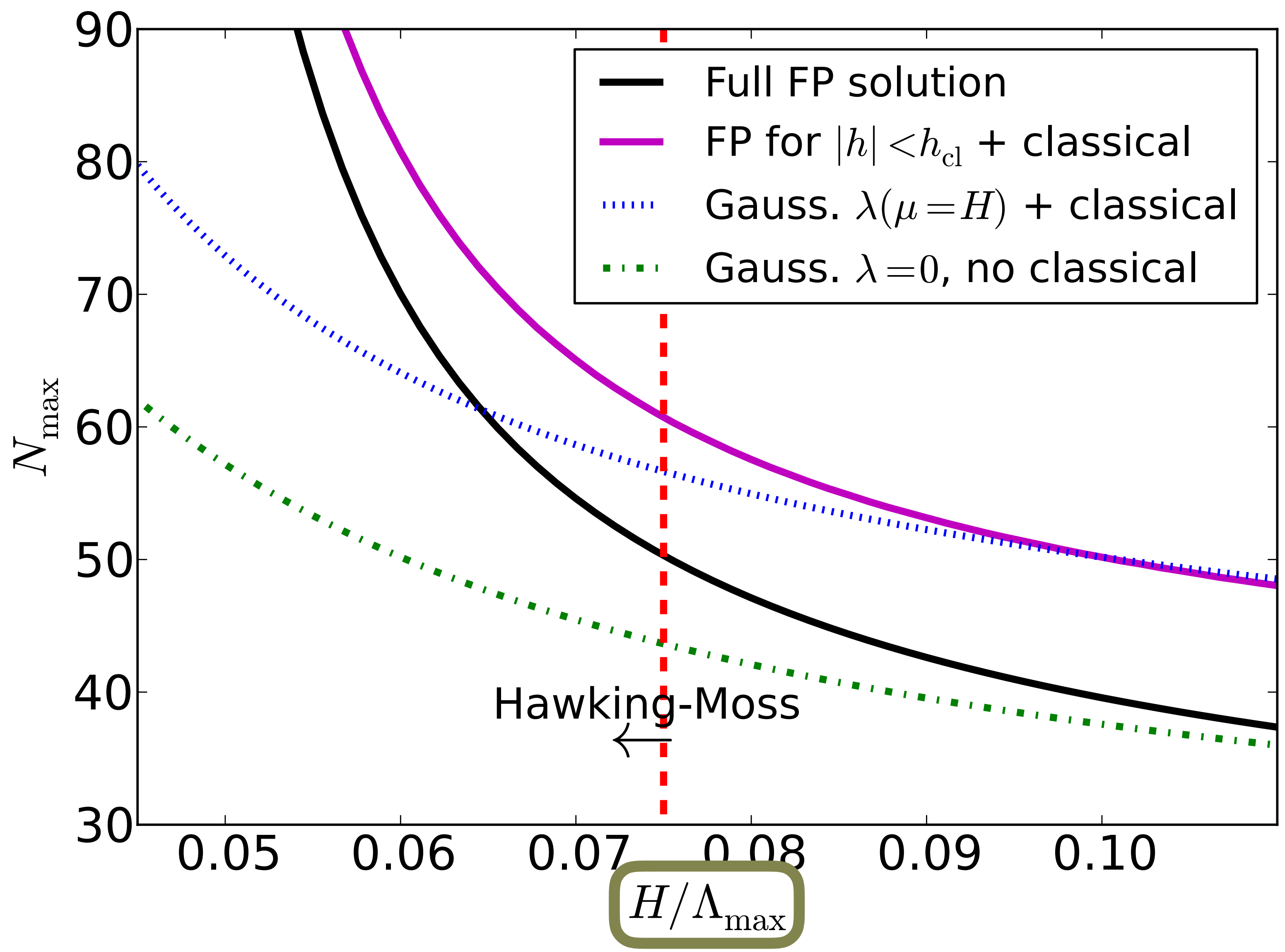




A TECHNICAL SIDE STORY

East, Kearney, Yoo, KZ 1607.00381

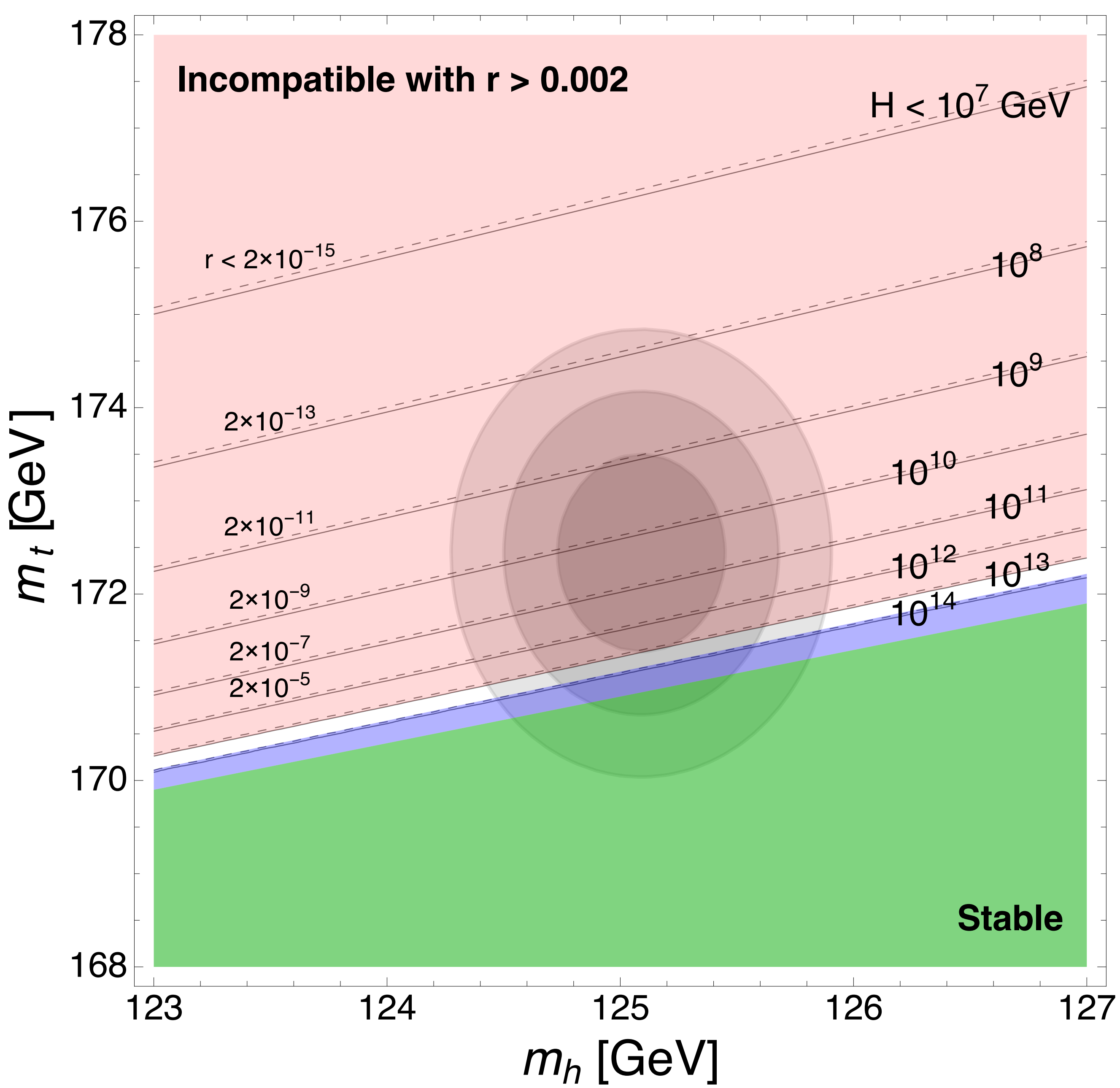
Need to resolve tails of distribution to obtain correct constraint



IMPLICATIONS FOR OUR UNIVERSE

East, Kearney, Yoo, KZ 1607.00381

If one observes primordial B-modes either
(a) the Higgs has stabilizing corrections, or
(b) the top mass must come down



WHAT HAVE WE LEARNED?

- The presence of a SM Higgs boson vacuum instability provides an ideal laboratory for considering novel aspects of scalar field evolution in inflation
  - single vs multi-bounce (HM vs FP)
  - interacting field in an inflating background; scale and gauge dependence
  - evolution of AdS bubble with thick wall
  - test of Hoop conjecture
  - *observation of primordial B-modes may tell us something about the nature of the Higgs potential at large field values*