Gravity dual of FFLO states

James Alsup

Computer Science, Engineering and Physics
University of Michigan-Flint

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work in collaboration with E. Papantonopoulos, G. Siopsis
Outline

1. Introduction
   - BCS
   - AdS/CFT
   - FFLO

2. AdS/CFT Model
   - Bulk Geometry
   - Instability
   - Gravity Dual
Superconductivity

BCS theory of superconductivity uses a spontaneously broken gauge symmetry

\[ \frac{Re \sigma_s}{Re \sigma_n} \]

2nd order phase transition at \( T_C \)

- conductance contains delta function and is gapped
- Meissner Effect

[picture from Gubser group]
AdS/CFT

some features found via gravitational duals

- allows for calculation of transport properties
  - $\sigma$, gap, Meissner [Hartnoll, Herzog, Horowitz, …]
  - Nernst effect [Hartnoll, Kovtun, Muller, Sachdev]

Differences

- global vs local symmetry breaking
- large $N$ limit?
some features found via gravitational duals

- allows for calculation of transport properties
- $\sigma$, gap, Meissner [Hartnoll, Herzog, Horowitz, ...]
- Nernst effect [Hartnoll, Kovtun, Muller, Sachdev]

### Similarities

- finite chemical potential, charge density
- DC superconductivity
- gap, $\omega_g \sim 8T_C$

### Conductivity

$\frac{Re \sigma_s}{Re \sigma_n}$

[Graph showing conductivity with $T<T_c$ and $\delta(\omega)$]
AdS/CFT

some features found via gravitational duals
▶ allows for calculation of transport properties
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Similarities
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  - DC superconductivity
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Gap Measurement [Gomes, et al.]
Condensed Matter Motivation

FFLO states [Fulde, Ferrell, and Larkin, Ovchinnikov]

- near the superconducting/normal transition line

\[
F_G = a|\psi|^2 + \gamma|\vec{\nabla}\psi|^2 + \frac{b}{2}|\psi|^4 + \frac{\eta}{2}|\vec{\nabla}^2\psi|^2
\]

- \(a, b, \gamma, \eta\) are functions of \(B, T\)

Inhomogeneous Ground State

- finite electron pair momentum
- translation/rotation symmetries broken
- no direct experimental evidence

\[
\psi \sim e^{iqx}
\]

Build a gravitational dual.
Hairy black hole

**Approach**

- **top-down:** stringy solutions \([Denef, Hartnoll, Gubser, \ldots]\)
- **bottom-up:** pick a supergravity solution, assume it works (for the time being) and analyze

we use a gravity with \(\Lambda_{\text{AdS}}\), scalar field \(\phi\) of mass \(m\) and charge \((q, 0)\) coupled to \(U(1)\) vector potential \(A_\mu + U(1)\) with \(A_\mu\):

\[
S = \int d^4x \sqrt{-g} \left[ \frac{R + 6/L^2}{16\pi G} - \frac{1}{4} F_{AB} F^{AB} - \frac{1}{4} \mathcal{F}_{AB} \mathcal{F}^{AB} + S_\phi \right]
\]
Scalar

\[ S_\phi = \int d^4 x \sqrt{-g} \left[ (g^{AB} - \xi G^{AB}) (D_A \phi)^* D_B \phi - m^2 |\phi|^2 \right] \]

with

\[ D_\mu = \partial_\mu - iqA_\mu \]

**Coupling**

- scalar coupled with Einstein tensor
  - used cosmology with vanishing \( \Lambda \)
  - entry/exit quasi-de Sitter
  - scalar-tensor theory with second order \( \Psi \) eqn. [Sushkov]

includes terms in the Lagrangian

\[ |F^{AB} \partial_B \phi|^2, \quad |\mathcal{F}^{AB} \partial_B \phi|^2 \]

\( \Rightarrow \) similar to Landau-Ginzburg gradient term
Hairy black hole

Asymptotics
as \( z \to 0 \)

\[ h \to 1 \quad \Rightarrow \quad \psi \sim \psi^\pm z^{\Delta_\pm}, \quad A_t \sim \mu - \rho z, \quad A_t \sim \delta \mu - \delta \rho z \]

\[ \Delta_\pm = \frac{3}{2} \pm \sqrt{\frac{9}{4} + m^2} \]

- chemical potentials, \( \mu, \delta \mu \) and charge density, \( \rho, \delta \rho \)
- \( \langle O_{\Delta \pm} \rangle = \sqrt{2} \psi^\pm \)

Horizon
as \( z \to 1 \)

\[ A_t \to 0, \quad h \to 0 \]
**Mechanism for Instability - I** \[ \text{Gubser} \]

- scalar develops effective mass

\[
\mathcal{L}_\psi = |\partial_r \psi|^2 - \left( m^2 + g^{tt} q^2 \Phi^2 \right) |\psi|^2
\]

\[
\rightarrow m_{\text{eff}}^2 = m^2 + g^{tt} q^2 \Phi^2 = m^2 - 2q^2
\]

- Breitenlohner-Freedman bound near \( z \sim 1 \)

\[
m_{\text{eff}}^2 < m_{BF,3+1}^2 = -9/4
\]

**Mechanism for Instability - II** \[ \text{Denef, Hartnoll} \]

Extremal black holes near horizon exhibit \( AdS_2 \times \mathbb{R}^2 \)

- effective mass can be below 2D \( m_{BF}^2 \)

\[
m_{\text{eff}}^2 = m^2 - 2q^2 < 6m_{BF,1+1}^2 = -3/2
\]
Hairy black hole

instability for $q^2 > \frac{3+2m^2}{4} \rightarrow$ including $q^2 = 0$

violations of geometric perspective [J.A., G. Siopsis, J. Therrien]
Hairy black hole

Solution with $\phi = 0$

$$
\begin{align*}
\text{ds}^2 &= \frac{1}{z^2} \left[ -h(z) dt^2 + d\vec{x}^2 + \frac{dz^2}{h(z)} \right], \\
h(z) &= 1 - \left(1 + \frac{\mu^2 + \delta \mu^2}{4} \right) z^3 + \frac{\mu^2 + \delta \mu^2}{4} z^4 \\
A_t &= \mu (1 - z), \quad A_t = \delta \mu (1 - z)
\end{align*}
$$

In general, equations of motion must be solved numerically. Use scaling symmetries: $z_H = 1$, with $z \in [0, 1]$

$\rightarrow$ AdS boundary at $z = 1$

Temperature:

$$
T = -\frac{h'(1)}{4\pi} = \frac{3}{4\pi} \left[ 1 - \frac{\mu^2 + \delta \mu^2}{12} \right]
$$

$\triangleright$ look at only scale invariant quantities: $\frac{\delta \mu}{\mu^0}$
Scalar

Scalar field equation

\[ \partial_z^2 \phi + \left( \frac{h'}{h} - \frac{2}{z} \right) \partial_z \phi + \frac{1}{h} \nabla^2 \phi - \frac{1}{h} \left[ \frac{m^2}{z^2} - q^2 \frac{A_t^2}{h} \right] \phi = 0 \]

interested in transition temperature \( T_c, \phi \gtrsim 0 \)

- introduce \( x \)-dependence
  \[ \phi \sim e^{i \sqrt{\tau} x} \psi(z) \]
- fix \( \frac{\tau}{(q \mu)^2}, \delta \mu \)
- \( \mu \rightarrow \) eigenvalue in scalar equation
  \[ \frac{T_c}{\mu_c} = \frac{3}{4\pi \mu_c} \left[ 1 - \frac{\mu_c^2 + \delta \mu^2}{12} \right] \]
Homogeneous Solution, $\xi = 0$

**Critical Temperature**

\[ \Delta = 2.5, \quad q = 10 \]

\[ \frac{\tau}{(q\mu)^2} = 0, .05, .10, .15, .25, \text{ and } .35. \]

$\Rightarrow$ $T_c$ of the homogeneous higher for all allowed $\frac{\delta \mu}{\mu}$

\[ \beta = \frac{\delta \mu}{\mu}, \quad \beta_{max} = \frac{q^2}{q^2_{min}} - 1, \quad q^2_{min} = \frac{3 + 2m^2 + 2\tau}{4} \]

J.A. L. Papantonopoulos, G. Siopsis
Inhomogeneous Solution, $\xi \neq 0$

- turn on Einstein tensor and scalar coupling

**Critical Temperature**

$$\Delta = 2.5, \ q = 10, \ \xi = .2, \ \frac{\tau}{(q\mu)^2} = 0, .15, \text{ and } .35$$

J.A. L. Papantonopoulos, G. Siopsis
Inhomogeneous Solution, $\xi \neq 0$

Critical Temperature

$\Delta = 2.5$, $q = 10$
$\xi = .2$
$\frac{\tau}{(q\mu)^2} = 0, .15, \text{ and } .35$

$T_c$ lines cross for large $\beta$

J.A. L. Papantonopoulos, G. Siopsis
Inhomogeneous Solution, $\xi \neq 0$

\[ \beta = \frac{\delta \mu}{\mu}, \quad \beta_{\text{max}} = \frac{q^2}{q_{\text{min}}^2} - 1, \quad q_{\text{min}}^2 = \frac{3 + 2m^2 + 2\tau(1 - 6\xi)}{4} \]

For finite $\tau$, $\xi > 1/6$, near extremality

- minimum charge decrease
- maximum value of $\beta$ increases

- inhomogeneous solutions possess higher critical temperature than homogenous solution
- in CFT, dominant terms possess modulated order parameter

\[ \langle \mathcal{O} \rangle \sim e^{i\sqrt{\tau}x} \]
Summary

- gravity provides tools to strongly coupled superconductors
- can build dual theory to condensed matter’s FFLO states
  - AdS + scalar + $U(1)^2$
  - couple Einstein tensor and scalar

Inhomogenous states dominate CFT at low temperature $\Rightarrow$ FFLO states

work to be done