Gravity dual of FFLO states

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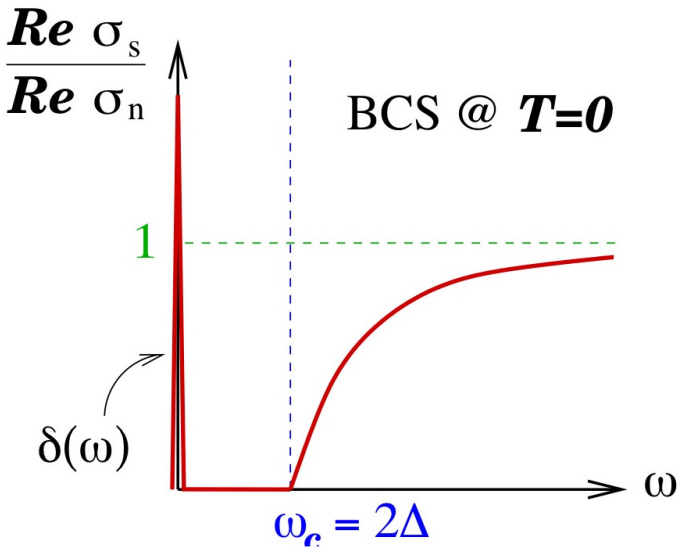
work in collaboration with E. Papantonopoulos, G. Siopsis

Outline

Superconductivity

BCS theory of superconductivity uses a spontaneously broken gauge symmetry



[picture from Gubser group]

- 2nd order phase transition at T_C
- conductance contains delta function and is gapped
- Meissner Effect

AdS/CFT

some features found via gravitational duals

- allows for calculation of transport properties
  - σ, gap, Meissner *[Hartnoll, Herzog, Horowitz, . . .]*
  - Nernst effect *[Hartnoll, Kovtun, Muller, Sachdev]*

Differences

- global vs local symmetry breaking
- large N limit?

AdS/CFT

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- ▶ allows for calculation of transport properties
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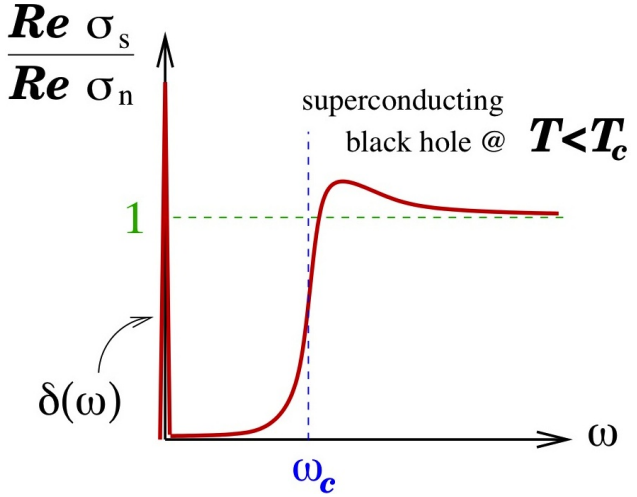
  - Nernst effect *[Hartnoll, Kovtun, Muller, Sachdev]*

Similarities

- finite chemical potential, charge density
- DC superconductivity
- gap, $\omega_g \sim 8T_C$

Conductivity *[Gubser group]*



AdS/CFT

some features found via gravitational duals

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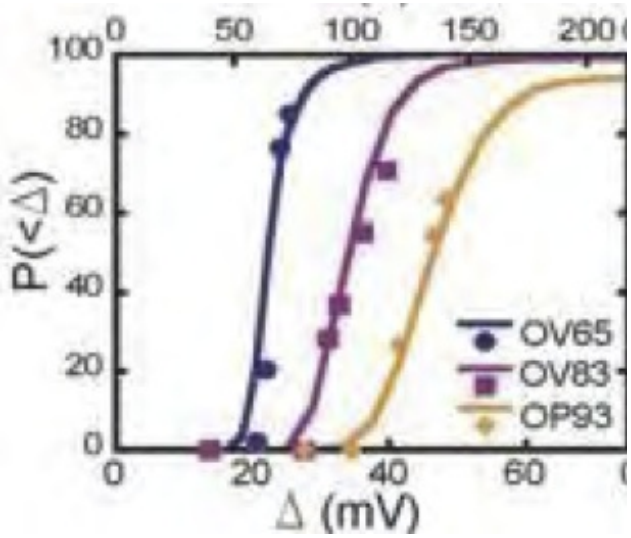
Gap Measurement *[Gomes, et al.]*



Condensed Matter Motivation

FFLO states *[Fulde, Ferrell, and Larkin, Ovchinnikov]*

▶ near the superconducting/normal transition line

Ginzburg-Landau functional

$$F_G = a|\psi|^2 + \gamma|\vec{\nabla}\psi|^2 + \frac{b}{2}|\psi|^4 + \frac{\eta}{2}|\vec{\nabla}^2\psi|^2$$

▶ a, b, γ, η are functions of B, T

Inhomogeneous Ground State

- finite electron pair momentum
- translation/rotation symmetries broken
- no direct experimental evidence

$$\psi \sim e^{iqx}$$

Build a gravitational dual.

Hairy black hole

Approach

- top-down: stringy solutions *[Denef, Hartnoll, Gubser, . . .]*
- bottom-up: pick a supergravity solution, assume it works (for the time being) and analyze

we use a gravity with Λ_{AdS}, scalar field ϕ of mass m and charge $(q, 0)$ coupled to $U(1)$ vector potential A_μ + $U(1)$ with $\mathcal{A}_\mu$

$$S = \int d^4x\sqrt{-g}\left[\frac{R + 6/L^2}{16\pi G} - \frac{1}{4}F_{AB}F^{AB} - \frac{1}{4}\mathcal{F}_{AB}\mathcal{F}^{AB} + S_\phi\right]$$

Scalar

$$S_\phi = \int d^4x\sqrt{-g}\left[\left(g^{AB} - \xi G^{AB}\right)(D_A\phi)^* D_B\phi - m^2|\phi|^2\right]$$

with

$$D_\mu = \partial_\mu - iqA_\mu$$

Coupling

- scalar coupled with Einstein tensor
  - used cosmology with vanishing Λ
    - entry/exit quasi-de Sitter
    - scalar-tensor theory with second order Ψ eqn. *[Sushkov]*

includes terms in the Lagrangian

$$|F^{AB}\partial_B\phi|^2 \quad , \quad |\mathcal{F}^{AB}\partial_B\phi|^2$$

$\Rightarrow$ similar to Landau-Ginzburg gradient term

Hairy black hole

Asymptotics

as $z \to 0$

$$h \to 1 \quad \Rightarrow \quad \Psi \sim \Psi^{\pm} z^{\Delta_{\pm}} \ , \quad A_t \sim \mu - \rho z \ , \quad \mathcal{A}_t \sim \delta\mu - \delta\rho z$$

$$\Delta_{\pm} = \frac{3}{2} \pm \sqrt{\frac{9}{4} + m^2}$$

- chemical potentials, $\mu, \delta\mu$ and charge density, $\rho, \delta\rho$
- $\langle \mathcal{O}_{\Delta_{\pm}} \rangle = \sqrt{2}\Psi^{\pm}$

Horizon

as $z \to 1$

$$A_t \to 0 \ , \quad h \to 0$$

Hairy black hole

Mechanism for Instability - I *[Gubser]*

- scalar develops effective mass

$$\mathcal{L}_\Psi = |\partial_r \Psi|^2 - \left(m^2 + g^{tt} q^2 \Phi^2\right) |\Psi|^2$$
$$\rightarrow m_{eff}^2 = m^2 + g^{tt} q^2 \Phi^2 = m^2 - 2q^2$$

- Breitenlohner-Freedman bound near $z \sim 1$

$$m_{eff}^2 < m_{BF,3+1}^2 = -9/4$$

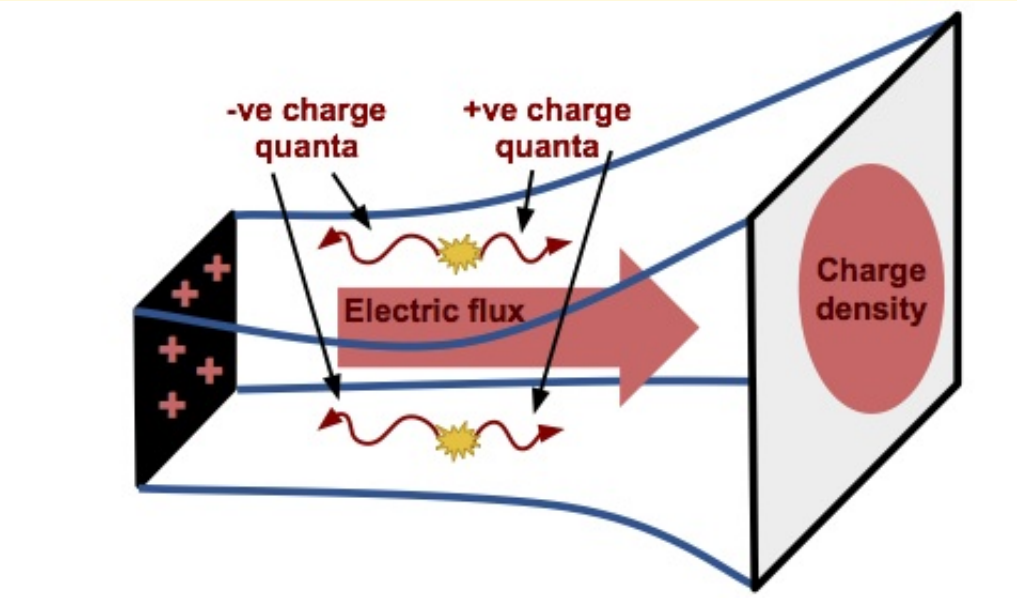
Mechanism for Instability - II *[Denef, Hartnoll]*

Extremal black holes near horizon exhibit $AdS_2 \times \mathbb{R}^2$

- effective mass can be below 2D m_{BF}^2

$$m_{eff}^2 = m^2 - 2q^2 < 6 m_{BF,1+1}^2 = -3/2$$

Hairy black hole



picture from Hartnoll

instability for $q^2 > \frac{3+2m^2}{4}$ $\rightarrow$ including $q^2 = 0$

violations of geometric perspective *[J.A., G. Siopsis, J. Therrien]*

Hairy black hole

Solution with $\phi = 0$

$$\begin{aligned} ds^2 &= \frac{1}{z^2}\left[-h(z)dt^2 + d\vec{x}^2 + \frac{dz^2}{h(z)}\right], \\ h(z) &= 1 - \left(1 + \frac{\mu^2 + \delta\mu^2}{4}\right) z^3 + \frac{\mu^2 + \delta\mu^2}{4} z^4 \\ A_t &= \mu(1-z),\ \mathcal{A}_t = \delta\mu(1-z) \end{aligned}$$

In general, equations of motion must be solved numerically.
use scaling symmetries: $z_H = 1$, with $z \in [0, 1]$
$\longrightarrow$ AdS boundary at $z = 1$
Temperature:

$$T = -\frac{h'(1)}{4\pi} = \frac{3}{4\pi}\left[1 - \frac{\mu^2 + \delta\mu^2}{12}\right]$$

- look at only scale invariant quantities: $\frac{\delta\mu}{\mu}$...

Scalar

Scalar field equation

$\longrightarrow$ conventional case $\xi = 0$

$$\partial_z^2\phi + \left[\frac{h'}{h} - \frac{2}{z}\right]\partial_z\phi + \frac{1}{h}\nabla_2^2\phi - \frac{1}{h}\left[\frac{m^2}{z^2} - q^2\frac{A_t^2}{h}\right]\phi = 0$$

interested in transition temperature T_c, $\phi \gtrsim 0$

- introduce x-dependence

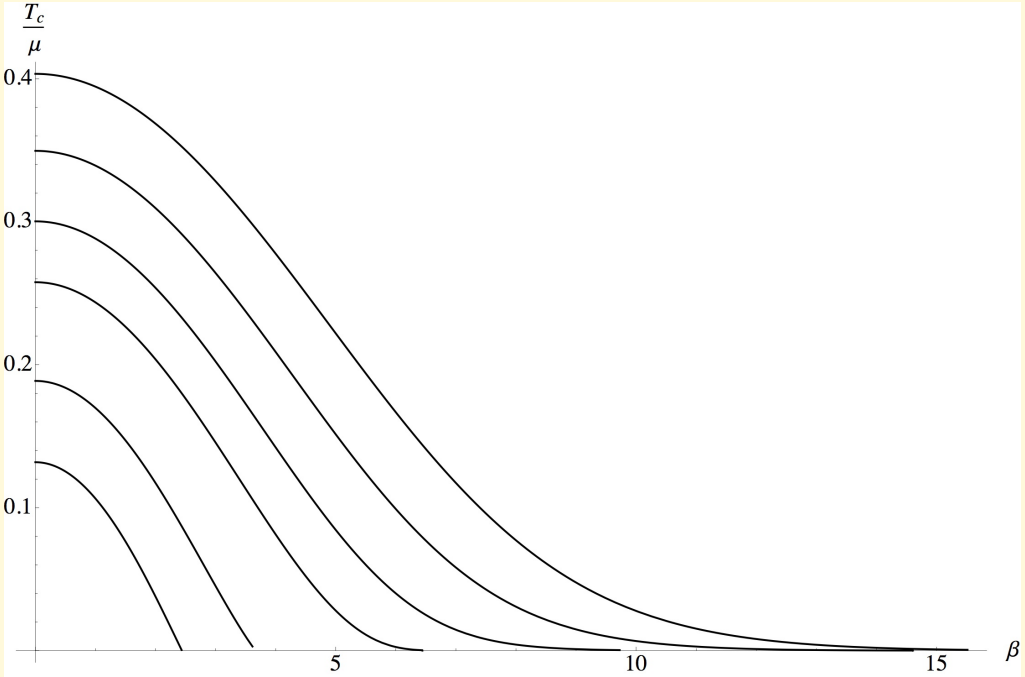
$$\phi \sim e^{i\sqrt{\tau}x}\psi(z)$$

- fix $\frac{\tau}{(q\mu)^2}$, $\delta\mu$
- $\mu \rightarrow$ eigenvalue in scalar equation

$$\frac{T_c}{\mu_c} = \frac{3}{4\pi\mu_c}\left[1 - \frac{\mu_c^2 + \delta\mu^2}{12}\right]$$

Homogeneous Solution, $\xi = 0$

Critical Temperature



$\Delta = 2.5, \; q = 10$

$\frac{\tau}{(q\mu)^2} =$

0, .05, .10, .15, .25, *and* .35.

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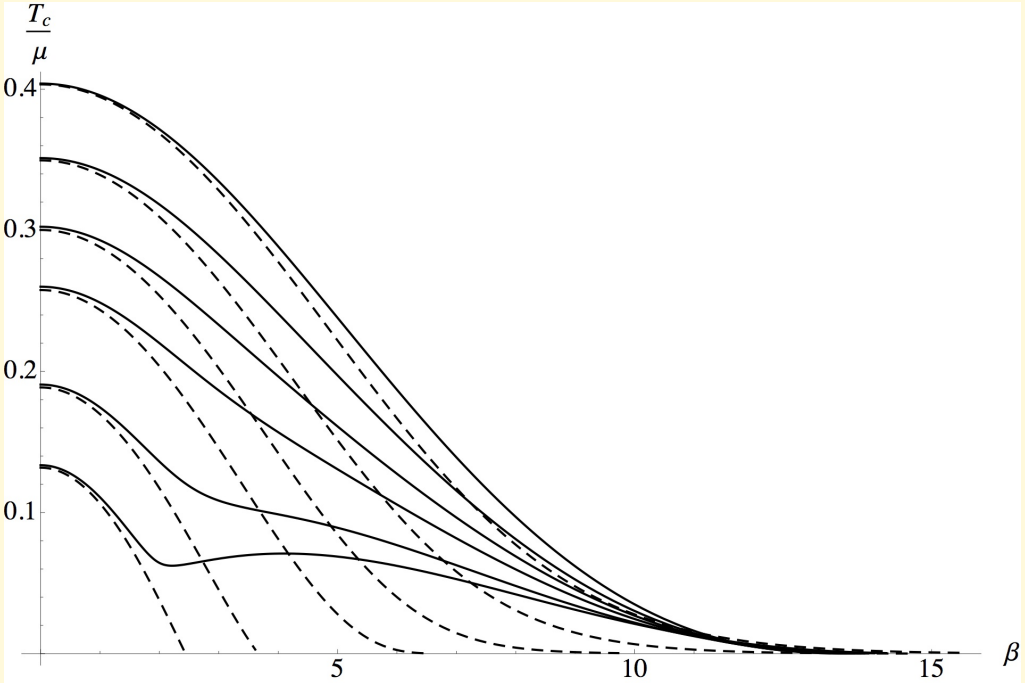
$\Rightarrow$ T_c of the homogeneous higher for all allowed $\frac{\delta\mu}{\mu}$

$$\beta = \frac{\delta\mu}{\mu}\,, \quad \beta_{max} = \frac{q^2}{q^2_{min}} - 1\,, \quad q^2_{\min} = \frac{3 + 2m^2 + 2\tau}{4}$$

Inhomogeneous Solution, $\xi \neq 0$

- turn on Einstein tensor and scalar coupling

Critical Temperature



$\Delta = 2.5, \; q = 10$
$\xi = .2$

$\frac{\tau}{(q\mu)^2} = 0, .15, and .35$

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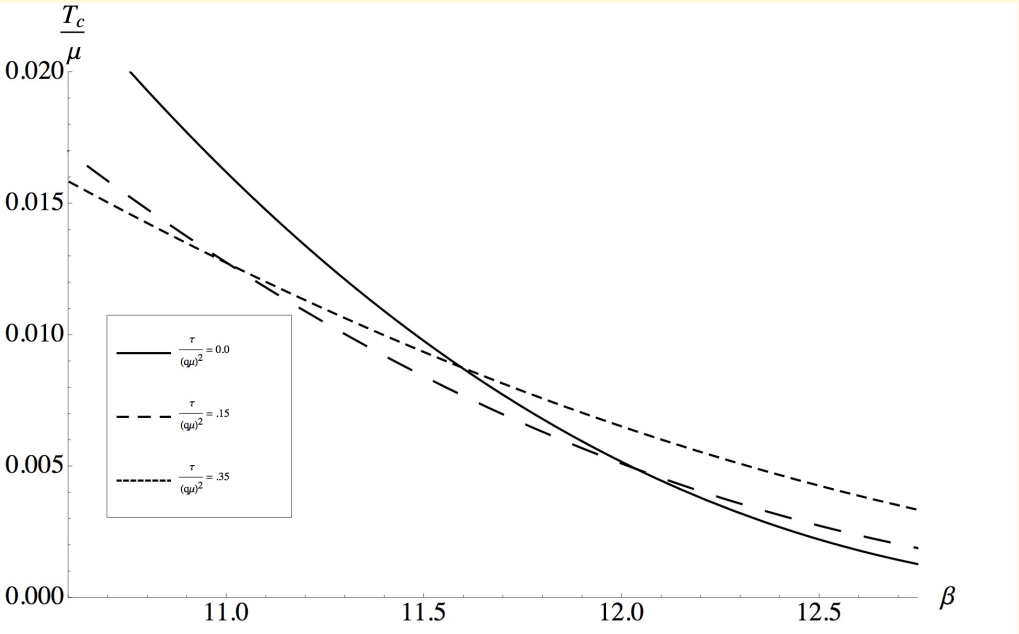
Inhomogeneous Solution, $\xi \neq 0$

Critical Temperature



$\Delta = 2.5, \; q = 10$
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$\Rightarrow$ T_c lines cross for large β

Inhomogeneous Solution, $\xi \neq 0$

$$\beta = \frac{\delta\mu}{\mu}\ , \quad \beta_{max} = \frac{q^2}{q^2_{min}} - 1\ , \quad q^2_{\min} = \frac{3 + 2m^2 + 2\tau(1 - 6\xi)}{4}$$

For finite τ, $\xi > 1/6$, near extremality

- minimum charge decrease
- maximum value of β increases

▶ inhomogeneous solutions possess higher critical temperature than homogenous solution
▶ in CFT, dominant terms possess modulated order parameter

$$\langle \mathcal{O} \rangle \sim e^{i\sqrt{\tau}x}$$

Summary

- gravity provides tools to strongly coupled superconductors
- can build dual theory to condensed matter's FFLO states
  - AdS + scalar + $U(1)^2$
  - couple Einstein tensor and scalar

Inhomogenous states dominate CFT at low temperature $\Rightarrow$ FFLO states

work to be done