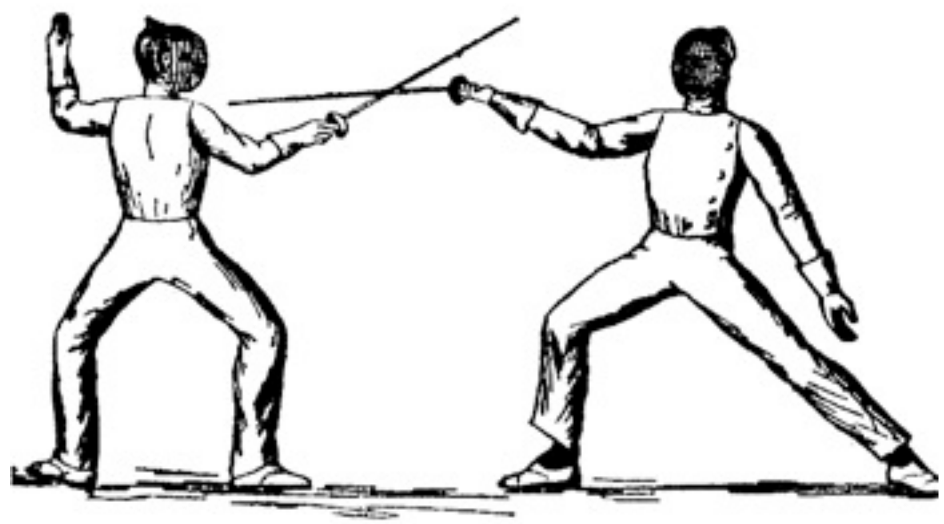
Higuchi VS Vainshtein



ArXiv: 1206.3852+ soon to appear.

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Outline

- The Higuchi bound
- The Vainshtein mechanism
- Higuchi vs Vainshtein
- dRGT setup
- FRW on FRW
- Despair not: a quicker route to the resolution

The Higuchi bound is a condition that stems from requiring stability from the classical theory of linear Massive Gravity

$$\mathcal{L} = \mathcal{L}_{EH} + \mathcal{L}_m = \sum p^T \dot{q} - \left[\frac{1}{2} p^T \cdot P \cdot p + \frac{1}{2} q^T \cdot Q \cdot q + p^T \cdot P\bar{Q} \cdot q\right] \tag{1}$$

Roughly speaking: stability <==> Q , P positive definite (Higuchi + gradient instability)

(2)

Essential literature:

A. Higuchi
Nucl.Phys. B282 (1987) 397

S. Deser, A. Waldron
Phys.Lett. B508 (2001) 347-353
hep-th/0103255

L.Grisa, L.Sorbo
Phys.Lett. B686 (2010) 273-278
arXiv:0905.3391

Let's take a look

example: Fierz-Pauli

$$S = S_{EH} - \frac{m^2}{4}\int d^4x\sqrt{-\bar{g}^{(4)}}h_{\mu\nu}h_{\rho\sigma}\left[f^{\mu\rho}f^{\nu\sigma} - f^{\mu\nu}f^{\rho\sigma}\right]$$

where:

$$f^{\mu\nu} = \bar{g}^{\mu\nu}_{EH}$$

- usual tensor decomposition $\quad T_{ij} = T^{Tt}_{ij} + 2\partial_{(i}T^t_{j)} + \frac{1}{2}\left(\delta_{ij} - \hat{\partial}_{ij}\right)T^t + \hat{\partial}_{ij}T^l$

We are looking at the scalar here, the helicity 0 mode

- use ADM formalism
- solve constraint equations, solve for $\quad p^t, h^t$
- canonical transformation: $\quad p^l \to p_0 + h^t\left(m^2 - 2H^2\right)/4H \;\; ; \;\; h^l \to q_0 + h^t/2$

$$I_0 = p_0 \dot{q}_0 - \left[\frac{1}{2}\left[\frac{3\nu^2 m^2}{12H^2}\right] p_0^2 + \frac{1}{2}\left[\frac{12H^2}{\nu^2 m^2}\right] q_0 \left(-\nabla^2 + m^2 - \frac{9H^2}{4}\right) q_0\right]$$

$$\nu^2 = m^2 - 2H^2$$

Immediately then, stability dictates:

$$\nu^2 > 0$$

in this setup, the Higuchi bound reads:

$$\boxed{m^2 > 2H^2}$$

Vainshtein radius

* underlying assumption:
$f_{\mu\nu} \neq \bar{g}_{\mu\nu}$

a quick, heuristic derivation:

$$R_{\mu\nu} + m^2 h_{\mu\nu} \sim T_{\mu\nu}$$

$$h_{\mu\nu} \sim 1 \qquad\qquad R \sim m^2$$

$$R \sim \nabla^2\phi \; ; \qquad \phi \sim \frac{GM}{r} \Rightarrow R \sim \frac{GM}{r^3} \sim m^2$$

therefore

$$r < r_V \longleftarrow \boxed{r_V = \left(\frac{M}{M_P^2 m^2}\right)^{1/3}} \longrightarrow r > r_V$$

C. Deffayet, G. Dvali, G. Gabadadze, A. Vainshtein hep-th/0106001
Phys.Rev. D65 (2002) 044026

G. Chkareuli, D. Pirtskhalava
airXiv 11.05.1783

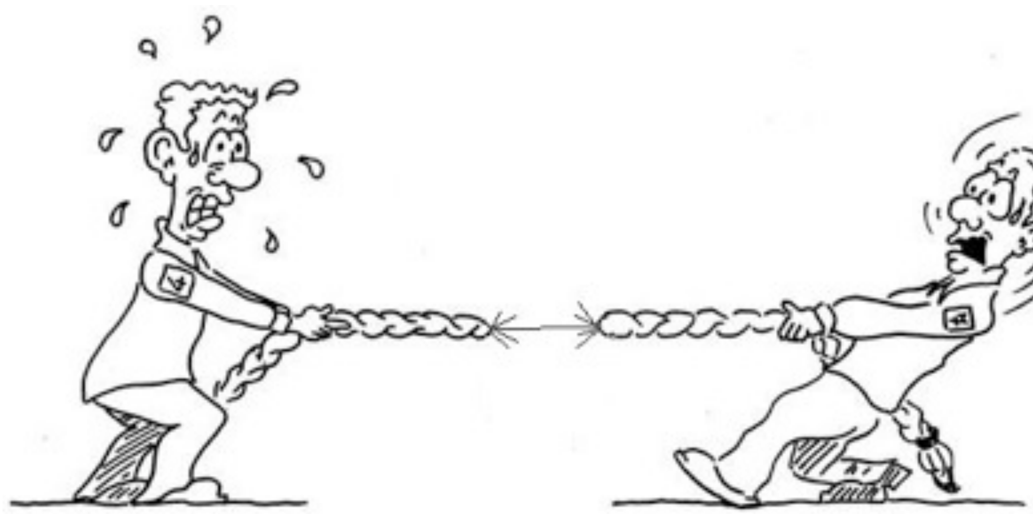
Inside the Vainshtein radius lies the region where one recovers GR

schematically:

$$3H^2 = \Lambda + 3m^2 \times \Theta(1)$$

one must require $\boxed{m^2 < H^2}$

Combining Higuchi and Vainshtein then:



want our theory to be stable

$$m^2 > 2H^2$$

GR works all around us

$$m^2 < H^2$$

Clearly, there's a problem...

In deriving the Higuchi bound, a number of assumptions were made:

- Shall we add matter content? Of course:

$$S = S_{EH} + S_{m^2} - \int d^4x \sqrt{-g^{(4)}} \left[\frac{1}{2} g^{\mu\nu} \partial_\mu \Phi \partial_\nu \Phi + V(\Phi) \right]$$

[Grisa and Sorbo, 2010]

$$m^2 > 2(H^2 + \dot{H})$$

but FP gravity, i.e. ghosts!

- Shall we use a different reference metric "f"? Yes, no reason not to.

$$f_{\mu\nu} \neq \bar{g}_{\mu\nu}$$

Plan: Study a ghost-free theory of massive gravity with matter content

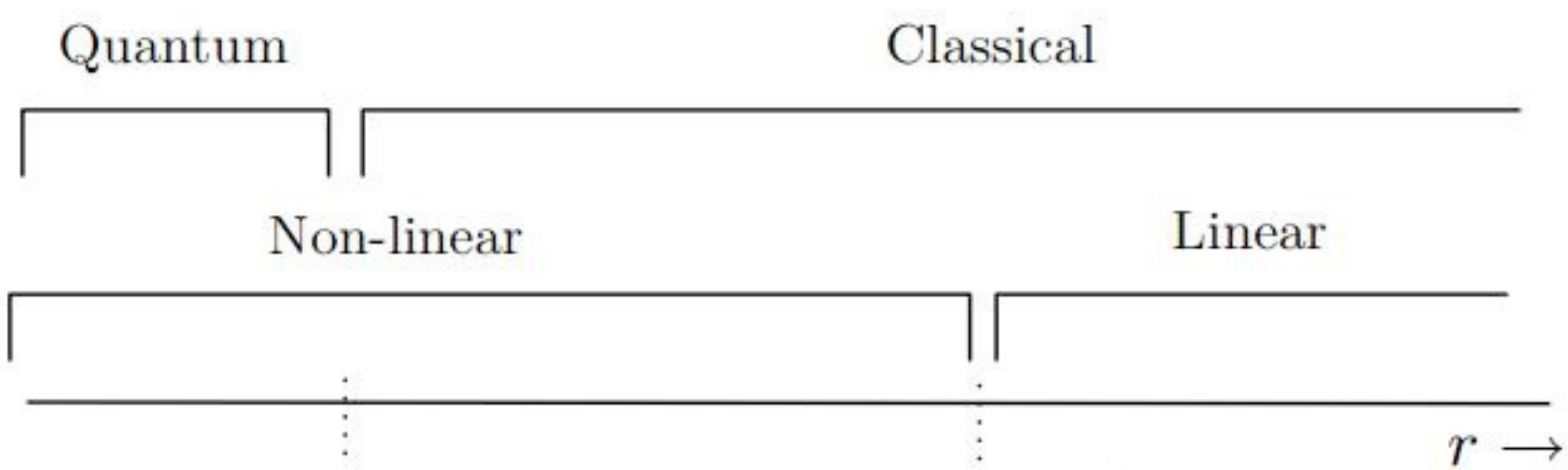
dRGT: Ghost-free m.g. theory at fully non-linear level

De Rham, Gabadadze, Tolley
Hassan, Rosen

* No Boulware-Deser Ghost, at all orders

* Screening mechanism in the non linear regime that restores continuity with G.R. as m approaches 0

* High enough cutoff so that the theory different regimes can be described



$$S = S_{EH} + 2m^2 \int d^4x \sqrt{-g}\Big[\varepsilon_2(\delta - \sqrt{g^{-1}f}) + \alpha_3\, \varepsilon_3(\delta - \sqrt{g^{-1}f}) + \alpha_4\, \varepsilon_4(\delta - \sqrt{g^{-1}f})\Big]$$

Our set up

* dRGT theory of massive gravity

$$S_{m^2} = 2m^2 \int d^4x \sqrt{-g} \Big[\varepsilon_2(\delta - \sqrt{g^{-1}f}) + \alpha_3\, \varepsilon_3(..) + \alpha_4\, \varepsilon_4(..) \Big]$$

with

$$\varepsilon_2(X) = \frac{1}{2}\left(Tr^2[X] - Tr[X^2]\right);$$
$$\varepsilon_3(X) = \frac{1}{6}\left(Tr^3[X] - 3Tr[X^2]Tr[X] + 2Tr[X^3]\right)$$
$$\varepsilon_4(X) = \frac{1}{24}\left(Tr^4[X] - 6Tr[X^2]Tr^2[X] + 3Tr^2[X^2] + 8Tr[X^3]Tr[X] - 6Tr[X^4]\right)$$

** The reference metric "f" and "$g_\circ$" need not be the same, parametrize this as:

$$f_{\mu\nu} = (1+z)\bar{g}_{\mu\nu}{}^{*}$$

* *in dS*

Higuchi bound:

$$m^2(1-z-2z^2)(m^2(1-z-2z^2)-2H^2)>0$$

in other words, the Higuchi bound has the generic form

$$\tilde{m}^2(\tilde{m}^2-2H^2)>0$$

$\tilde{m}$ *is the dressed mass, we ask* $\tilde{m}^2>0$ *to avoid instabilities in the vector sector.*

Two branches of solutions:

$$0<H<\frac{3}{2}H_0 \quad ;$$
$$m^2>\frac{2HH_0^2}{3H_0-2H} \; .$$

includes the $H_0 = H$ *branch*

* $1+z=H/H_0$

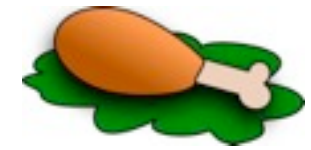
$$H>\frac{3}{2}H_0\,;$$
$$m^2<-\frac{2HH_0^2}{2H-3H_0} \; .$$

new branch

apparently, for H>>H$_0$*,* $|m^2|/H_0^2>1$

this is a much weaker Higuchi bound, but Vainshtein will require the opposite inequality to hold, a.k.a. : *back to square 1.*

Add matter:



$$L = L_{EH} + L_{dRGT} + \int d^3x \sqrt{-{}^{(4)}g}\left(-\frac{1}{2}g^{\mu\nu}\partial_\mu\Phi\partial_\nu\Phi + V(\Phi)\right)$$

Background:

$$H^2 = m^2(z - z^2) + \frac{\bar{\pi}^2}{12} + \frac{V}{6};$$

$$\dot{H} = -\frac{\bar{\pi}^2}{4} - \frac{m^2}{2}\left(1 - z - 2z^2 - M + 2Mz\right);$$

$$\dot{\bar{\pi}} + 3H\bar{\pi} + V_1 = 0\ ; \quad V_1 = \frac{dV(\phi)}{d\phi};$$

$$\dot{z} = H\left(M - 1 - z\right); \quad 1 + z = \frac{b}{a} = \frac{H}{H_0}$$

$$\bar{\pi} \equiv \dot{\phi}_0; \quad H_b \equiv \frac{\dot{b}}{Mb}.$$

$^*f_{\mu\nu} = \mathrm{diag}\left[-\mathrm{M}^2(\mathrm{t}), (1 + \mathrm{z}(\mathrm{t}))^2\, \mathrm{a}(\mathrm{t})^2\right]; \quad \alpha_3 = 0 = \alpha_4.$

$\dot{H}$ *drops out of the Higuchi inequality ! The bound is independent from the equation of state for matter:*

$$\tilde{m}^2(H) = m^2 \frac{H}{H_0}\left((3+3\alpha_3+\alpha_4) - 2(1+2\alpha_3+\alpha_4)\frac{H}{H_0} + (\alpha_3+\alpha_4)\frac{H^2}{H_0^2}\right) \geq 2H^2.$$

interesting feature, but the problem remains.

$$\alpha_3 = -1 = -\alpha_4$$

Could the freedom on the alpha's pay off?

It doesn't .Time evolution does not help either.

$$\frac{\mathrm{poly}_1^{(k)}(z)}{\mathrm{poly}_2^{(k)}(z)} \gg 1$$

structure also makes it hard.

In this setup there is no regime which is simultaneously observationally acceptable and ghost-free.

A quicker method and a resolution of the H-V tension

Use the properties of the mini superspace action:

$$ds^2 = -\dot{\phi}^2 dt^2 + b(\phi^2) d\vec{x}^2 \qquad\qquad B_n \sim m^2 M_{Pl}^2 \times (1, \alpha_3, \alpha_4)$$

$$S = \int dt N a^3 \Big[-3M_{Pl}^2 \left(\frac{\dot{a}^2}{N^2 a^2} - \frac{k}{a^2} \right) - \frac{\dot{\phi}}{N} \sum_{n=0}^{3} A_n \left(\frac{b(\phi)}{a} \right)^n - \sum_{n=0}^{3} B_n \left(\frac{b(\phi)}{a} \right)^n - \rho(a) \Big]$$

$$\boxed{A_n (3-n) = B_{n+1}(n+1)}$$

field redefinition:

$$\phi = 1/H_0 \, ln(a) + \chi/H_0; \quad M = N a^3; \quad \psi = a^3/3$$

fluctuations + diagonalize

$$S_{(2)} = \int dt \left(-\delta M \Big[-\frac{6M_{Pl}^2\dot{\psi}}{M^2}\delta\dot{\psi} \Big] - \frac{3M_{Pl}^2\delta\dot{\psi}^2}{M} - \frac{1}{2}\nu\delta\chi^2 \right)$$

gravity sector has decoupled from helicity zero mode

 !

we read off the Higuchi bound

Proceeding analogously for bigravity, when f too is dynamical:

$$\boxed{m^2_{\text{dressed}}(H)\left(H^2 + \frac{H_0^2 M_P^2}{M_f^2}\right) \geq 2H^4}$$

new !

What to do now?

<u>Fully bi-metric theories</u> ✓

<u>Inhomogeneities</u> in the ϕ's ?

~work in progress...

Reasons to be hopeful: see
Gabadadze et al., "massive cosmologies".