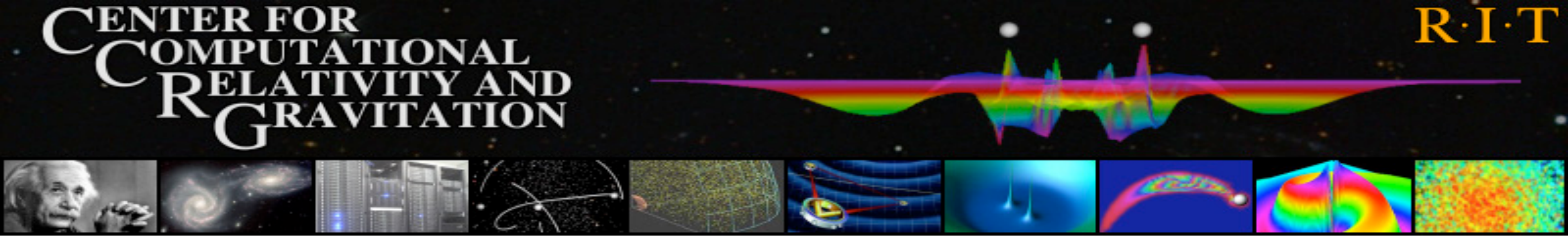

Exploring the Outer Limits of Numerical Relativity

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Overview/Introduction

- Since 2005 breakthroughs we can evolve BBHs
- Small mass ratios, q=m1/m2=1/100 (RIT)
- High energy collisions, γ =2.9 (Sperhake et al)
- Highly spinning black holes, S/M^2=0.97 (Lovelace et al)
- But how far can we evolve BBHs?
  - Can NR reproduce the Newtonian/low post-Newtonian regimes?
  - Very good NR/PN waveform matching requires 10 times more cycles than currently produced (lindblom, Damour, et al)
  - Matter around BBHs requires hundred of cycles to reach quasitationary regimes (Illinois, RIT)
- We study now a prototypical case of BBHs at a separation R=100M

0^{th} Generation Runs

TABLE I: The grid structure for the three numerical simulations presented here. The coarsest run had $h = h_0 = 4M$, while the two higher resolution runs had $h = h_1 = 4M/1.2$ and $h = h_2 = 4M/1.44$, respectively.

	radius	resolution	CFL
0	400	h	0.019
1	220	$h/2$	0.038
2	110	$h/4$	0.076
3	55	$h/8$	0.152
4	25	$h/16$	0.152
5	10	$h/32$	0.304
6	5	$h/64$	0.304
7	2	$h/128$	0.304
8	0.65	$h/256$	0.304

TABLE II: Initial data parameters for the full numerical simulations. The punctures are located at $\pm(x, 0, 0)$, with momenta $\pm(p_x, p_y, 0)$, spin $(0, 0, 0)$, and puncture mass m_p. m_H is the measured horizon masses, while T_i is the period of the first orbit for the three resolution runs (0 for coarsest,1 for medium, 2 for finest). $m_{\rm PN}$ and $T_{\rm PN}$ are the corresponding post-Newtonian values

$x = 50$	$T_{\rm PN} = 6365$
$p_x = -7.97939 \times 10^{-7}$	$T_0 = 6406$
$p_y = 2.55526 \times 10^{-2}$	$T_1 = 6420$
$m_p = 0.49920645$	$T_2 = 6422$
$m_H = 0.500615$	$m_{\rm PN} = 0.500616$

Setup adapted from runs optimized for BBHs with initial separations ~ 10M

A. Post-Newtonian Analysis

To the second post-Newtonian order [35], we have for the orbital frequency, Ω of quasicircular orbits

$$\Omega^2 = \frac{M}{r^3}\left[1-(3-\eta)\frac{M}{r}+\left(6+\frac{41}{4}\eta+\eta^2\right)\frac{M^2}{r^2}\right]+\cdots \tag{2}$$

and the radial decay of this orbit

$$\dot{r} = -\frac{64M^3}{5r^3}\eta\left[1-\frac{(1751+588\eta)}{336}\frac{M}{r}-4\pi\left(\frac{M}{r}\right)^{3/2}\right]+\cdots \tag{3}$$

The orbital period, T is given by

$$T = \frac{2\pi}{\Omega} \approx 6368M, \tag{4}$$

with $\Omega \approx 0.001$ for our orbital case at $r \approx 100M$ and for equal mass binary with $\eta = 1/4$.

We can thus estimate the radial decay after one orbit as

$$\Delta r = \dot{r}\,T \approx -0.02M. \tag{5}$$

Note that we have neglected here the effects of eccentricity present in the full numerical run.

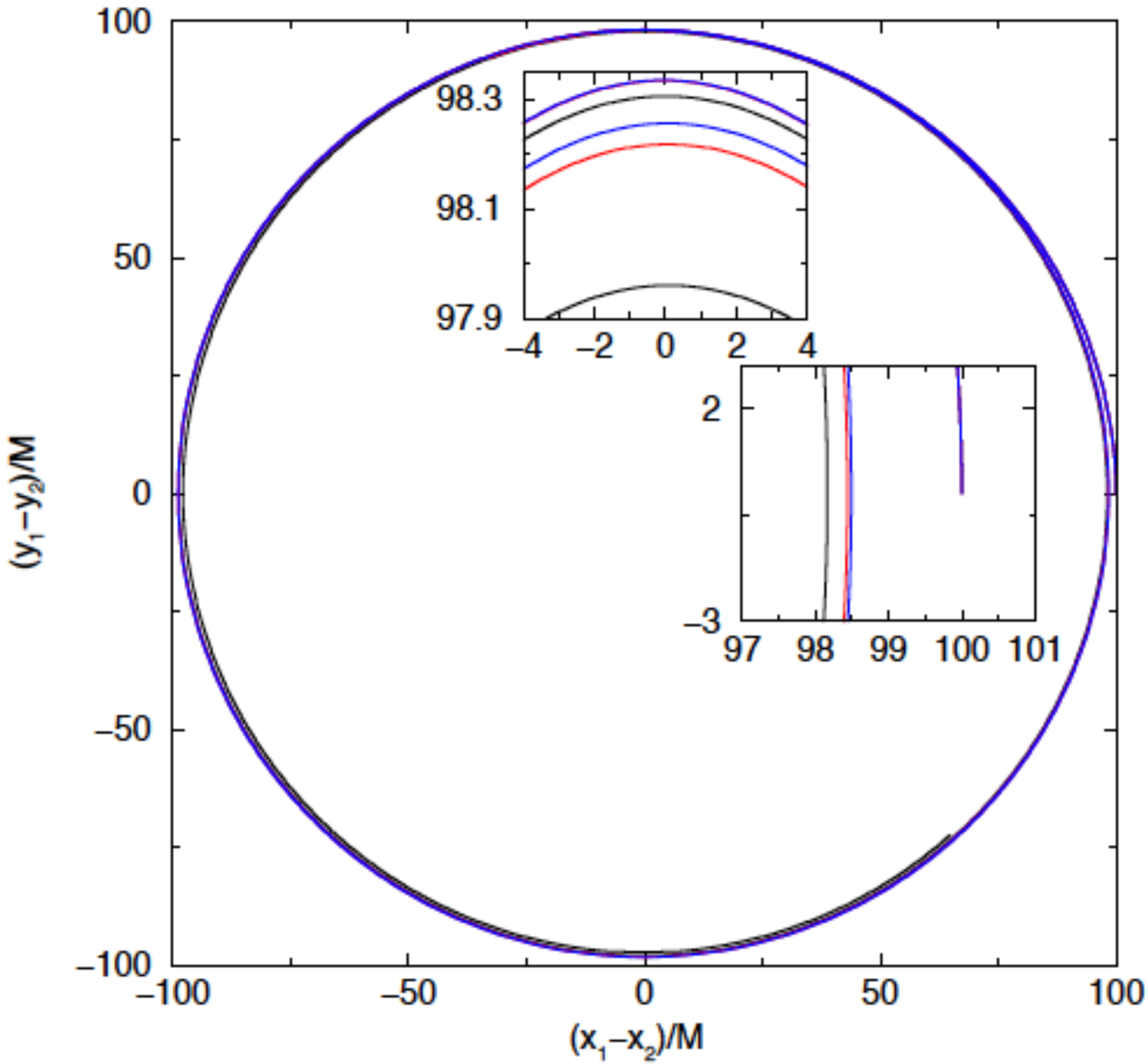


FIG. 1: xy plane projections of the relative trajectories for our configuration at different resolutions.

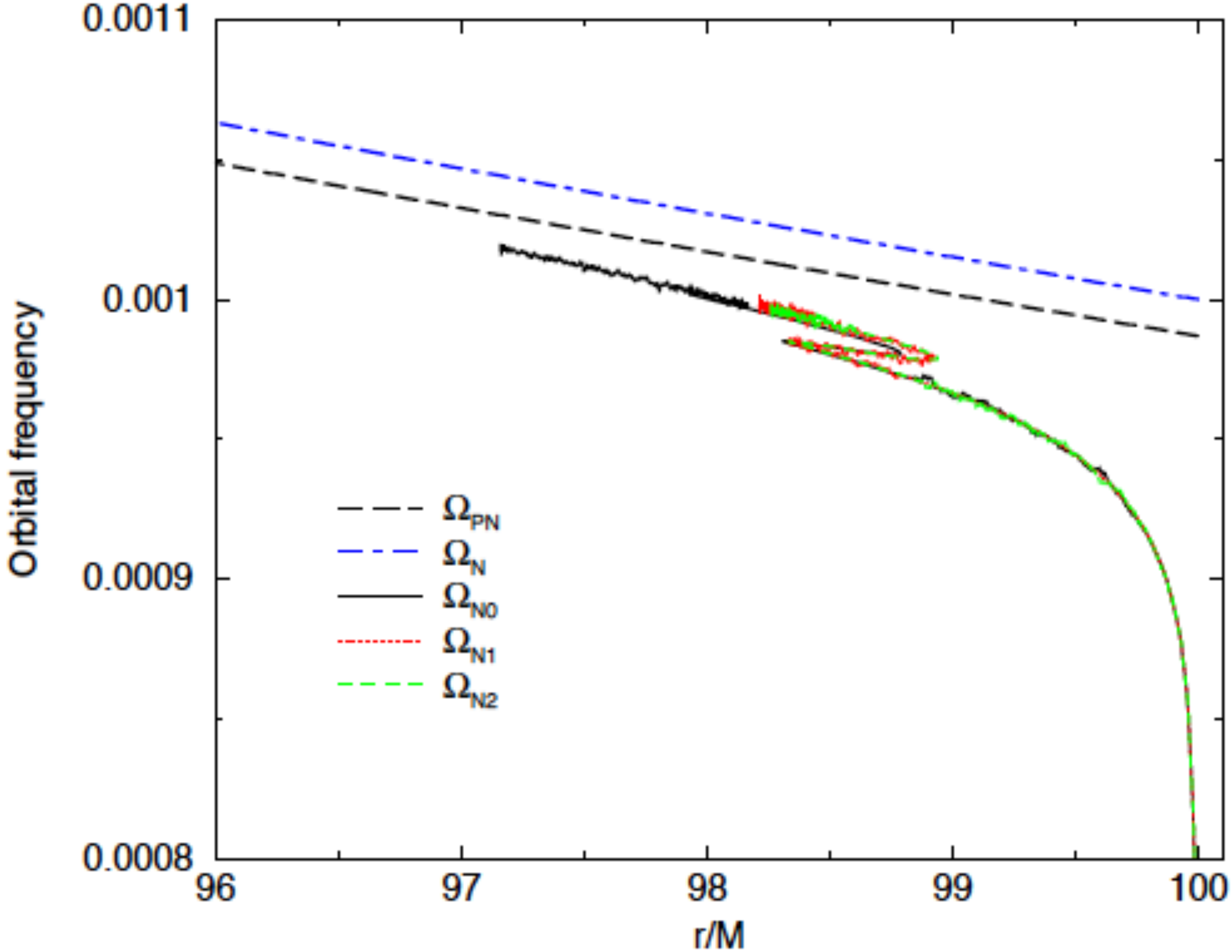


FIG. 2: The orbital frequency $d\phi_{\mathrm{orbit}}/dt$ versus orbital separation r, as well as the Newtonian and 3.5 PN predictions. The mapping between r and ω is not unique because r is not a monotonic function of t.

Results 0^{th} Generation Runs

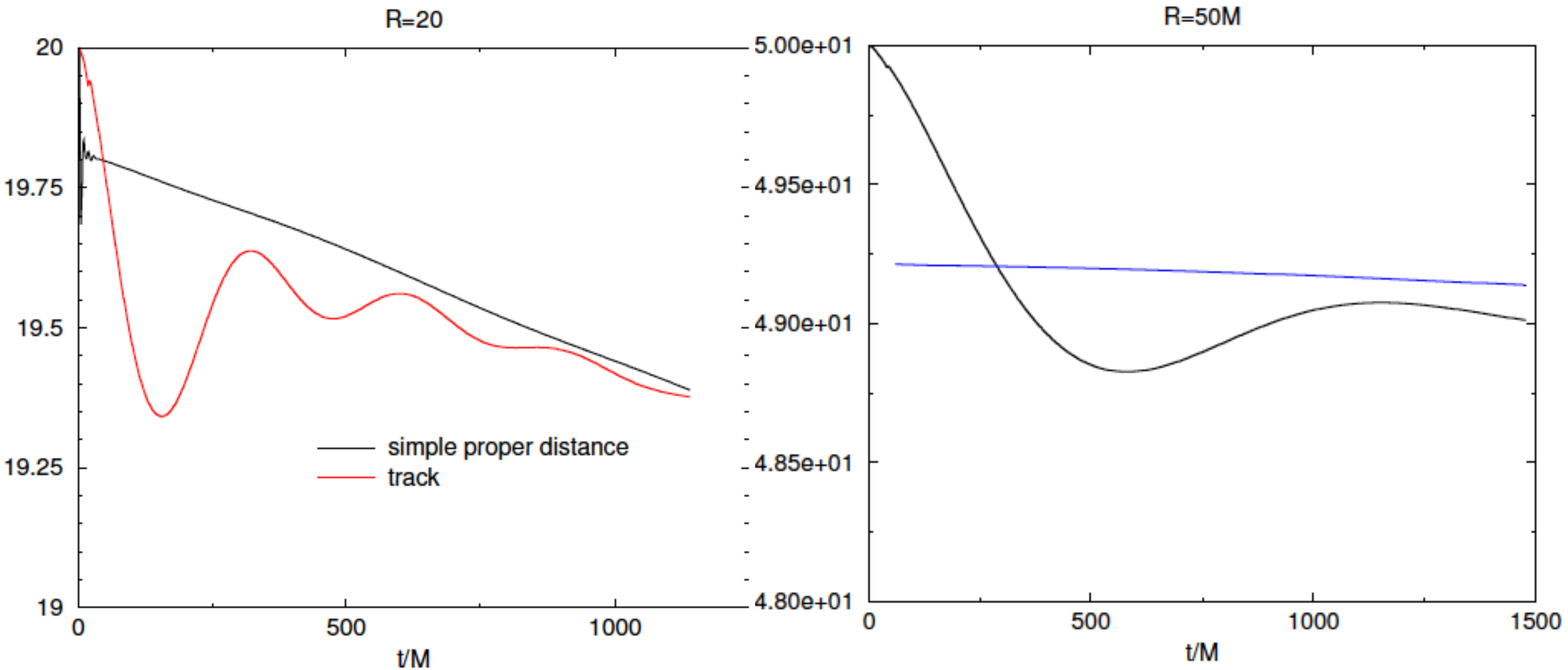
- PN predicts a $\Delta r=-0.02M$
- we observe at high resolution (8^{th} order convergent) NR $\Delta r=-0.08M$ (4 times larger)
- But over a radius of 100M it is 0.06% accurate
- PN predicts $M\Omega=0.00103$ at R=97M
- NR evolution after two orbits at lowest resolution produces $M\Omega=0.00102$. T = 6100M vs. 6160M

Issues with 0th Generation Runs

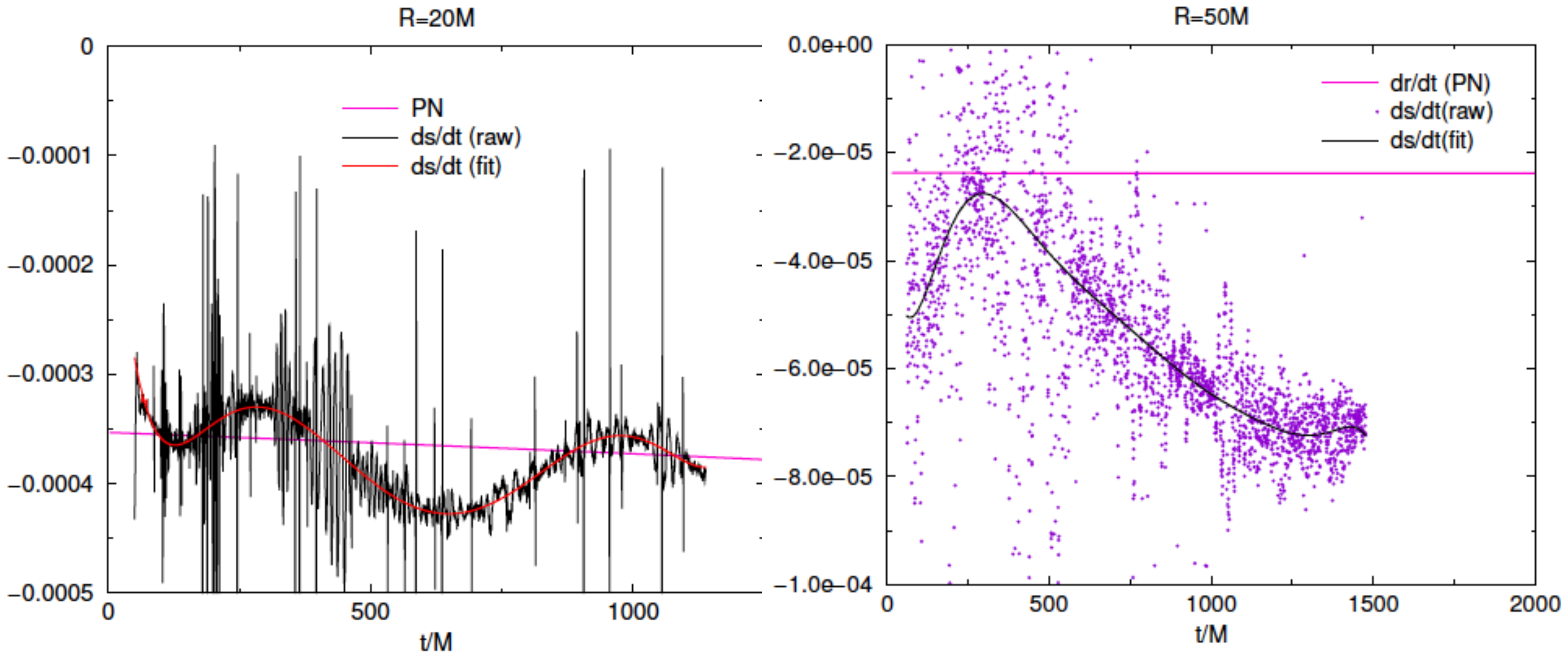
- The initial pulse of spurious radiation still needs ~ ½ orbit to relax.
- Initial gauge takes a similar time to catch up with track
- Boundary conditions at a few hundred M can affect evolutions early and since radiation levels are much smaller BC can be dynamically more important than for smaller orbits
- Waveform extraction needs to be made at two wavelengths away ~ several thousand M and make sure BC does not contaminate them.
- New scale emerges at R=100M to be resolved accordingly by AMR set up
- NR evolution take several months per orbit

BBH Tracks: The Next Generation

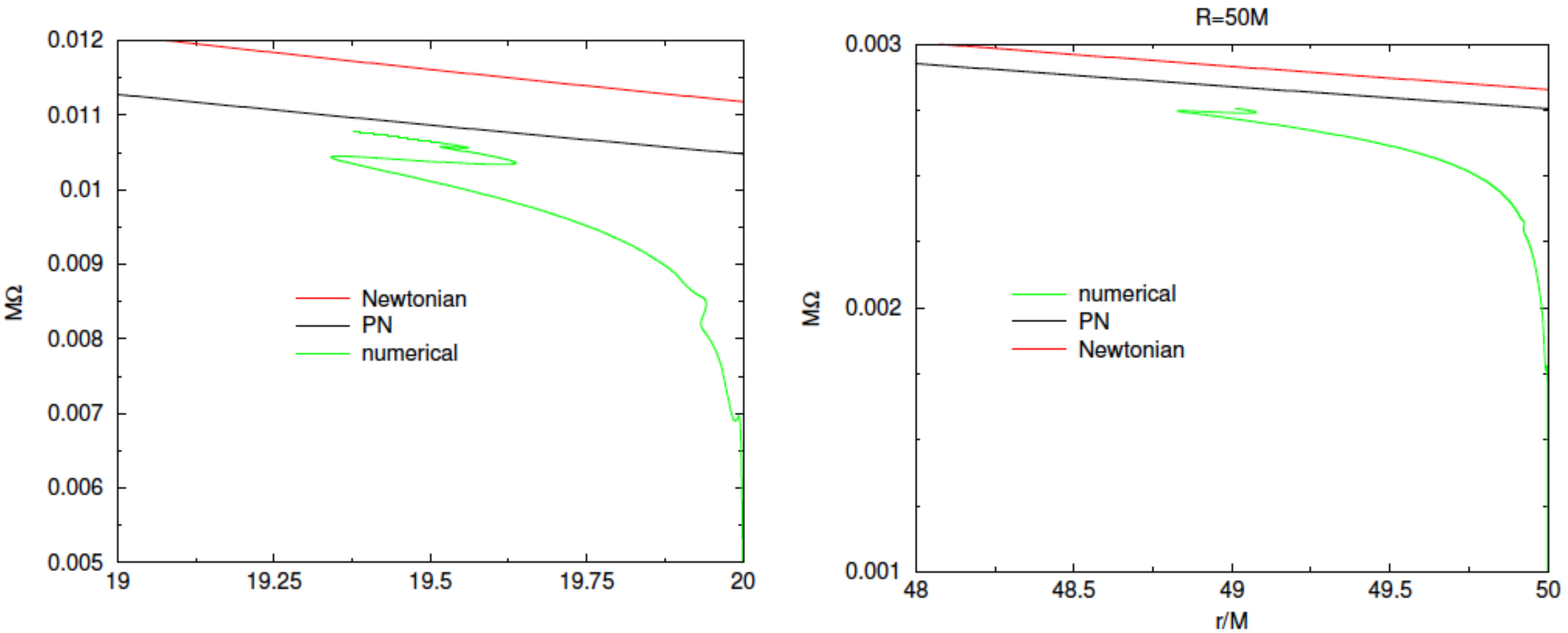
- Develop initial data with less radiation content
- Better initial gauges (shift)
- Use proper distances as measure of separation (Ω already gauge invariant)
- Use more accurate BC or Multipatch schemes or Pseudospectral to push boundaries to 10000M or more
- Extract waveforms in the above setup or CCE
- Code optimization, AMR or pseudospectral can speed up things. IMEX promising.
- GPU or other radical hardware acceleration needed.



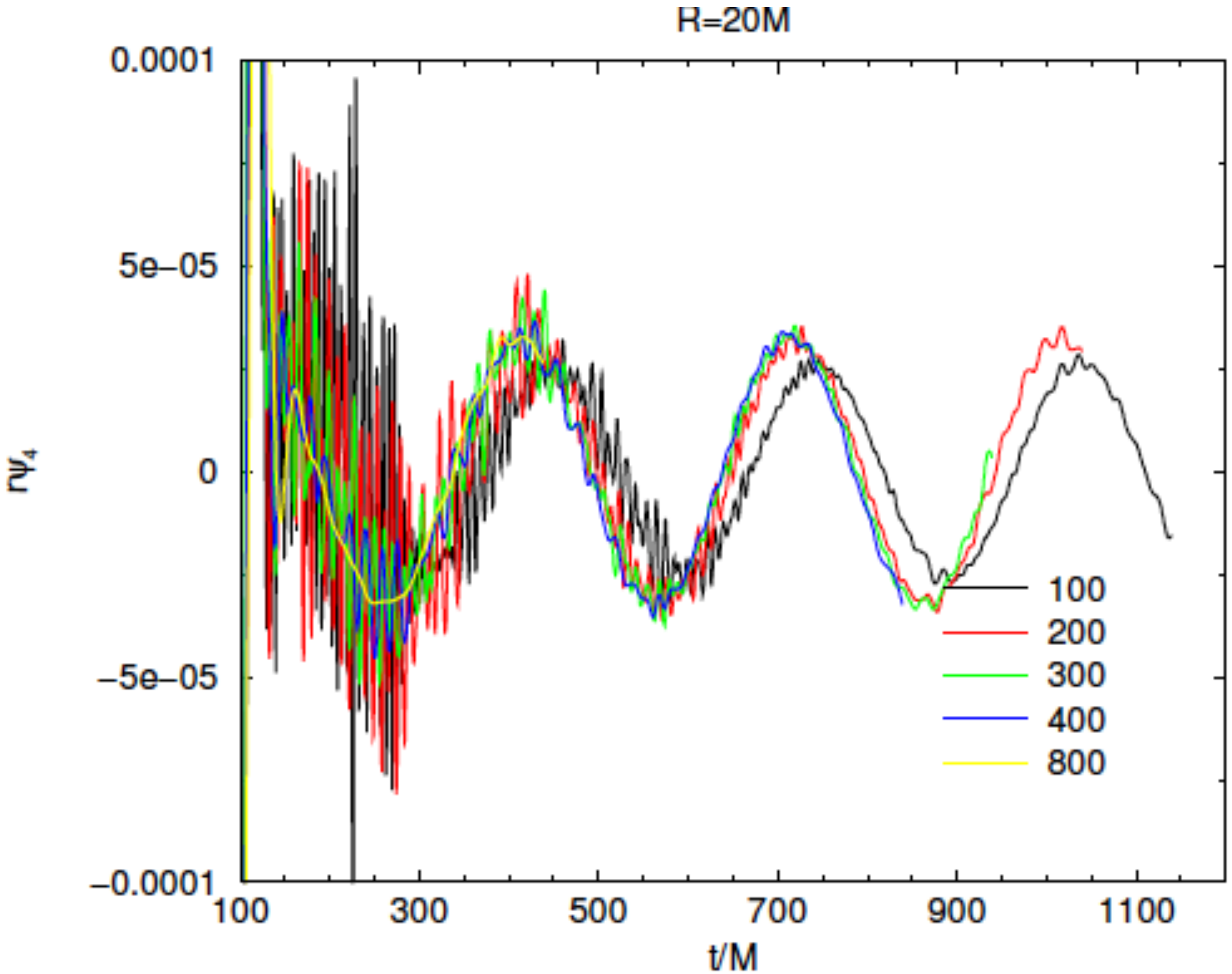
Coordinate vs. simple proper distance decay



NR proper distance decay vs post-Newtonian predictions



NR orbital frequency evolution vs. Newtonian and post-Newtonian predictions



NR direct waveform extraction at different radii.
Note the small amplitude and 2.$\lambda_{wavelength}$=562M

Discussion

- Comparison with PN through proper distance, Orbital frequency and waveforms
- For R=100M, 3.5 PN evolutions predicts 2064 orbits to merger and t = 8.2 million M
- NR took nearly 6 month per orbit.
- This would give 200,000 days to evolve to merger:
  Over 500 years.