Gravitational waves from BH-NS binaries: Effective Fisher matrices and parameter estimation using higher-harmonics

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Outline

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   - Precessing results

3. Conclusions
Motivation

- At leading order, the GW signal from a binary oscillates at twice the orbital frequency. Other harmonics of the orbital frequency enter at higher post-Newtonian (PN) order.

- Are higher harmonics/amplitude corrections (ACs) important for parameter estimation, particularly for BH-NS systems?

- This can be addressed with Bayesian inference techniques like MCMC or nested sampling runs, but these are expensive.

- We would like to get some insights with simpler, cheaper analytic methods.

- The Fisher matrix is a standard approach, but can be perilous.
Fisher matrix assumptions

- The Fisher matrix formalism is widely used because it is much cheaper than a full Bayesian analysis.

- However, it makes several assumptions that are rather optimistic for ground-based GW data analysis:
  - High SNR
  - Stationary, Gaussian noise
  - Signal depends linearly on all of its parameters

- What could possibly go wrong?
The Fisher matrix

Those assumptions are meant to justify approximating the likelihood (of an observed signal given some proposed waveform parameters) as a multivariate Gaussian in some neighborhood of the true parameters

\[ \mathcal{L} \simeq e^{-\Gamma_{ij} \delta \lambda^i \delta \lambda^j / 2} \]

where \[ \Gamma_{ij} = \left\langle \frac{\partial h}{\partial \lambda_i} \left| \frac{\partial h}{\partial \lambda_j} \right. \right\rangle \]

is the Fisher matrix.

The covariance matrix, the inverse of the Fisher matrix, will estimate the expected errors and correlations of the parameter measurements

\[ \Sigma = \Gamma^{-1} \]
\[ \sigma_i = \sqrt{\Sigma_{ii}} \]
\[ c_{ij} = \frac{\Sigma_{ij}}{\sqrt{\Sigma_{ii} \Sigma_{jj}}} \]
The likelihood that $h(\lambda')$ will produce a signal $s = h(\lambda) + n$ is:

$$\mathcal{L}(s|\lambda') = \exp \left[ -\frac{1}{2} \langle s - h(\lambda')|s - h(\lambda') \rangle \right]$$

Expanding $s$ and neglecting the noise contribution to the likelihood, we get:

$$\mathcal{L} = \exp \left[ -\frac{1}{2} \left( \langle h(\lambda)|h(\lambda) \rangle + \langle h(\lambda')|h(\lambda') \rangle - 2\text{Re}\langle h(\lambda)|h(\lambda') \rangle \right) \right]$$

Assuming $\rho^2 = \langle h(\lambda)|h(\lambda) \rangle = \langle h(\lambda')|h(\lambda') \rangle$,

$$\ln \mathcal{L} = -\rho^2 \left( 1 - \frac{\text{Re}\langle h(\lambda)|h(\lambda') \rangle}{\sqrt{\langle h(\lambda)|h(\lambda) \rangle \langle h(\lambda')|h(\lambda') \rangle}} \right)$$
Ambiguity function

- The ambiguity function, the normalized overlap between different points in the parameter space, is

\[ P(\lambda, \lambda') = \frac{\text{Re}\langle h(\lambda) | h(\lambda') \rangle}{\sqrt{\langle h(\lambda) | h(\lambda) \rangle \langle h(\lambda') | h(\lambda') \rangle}} \]

- Computing the ambiguity function allows us to map out the log likelihood, and it can be related to the Fisher matrix

\[ \ln \mathcal{L} = -\rho^2 (1 - P(\lambda, \lambda')) \approx -\Gamma_{ij} \delta \lambda^i \delta \lambda^j / 2 \]

- The (normalized) *effective* Fisher matrix is obtained by fitting the ambiguity function with a multivariate quadratic

\[ P_{\text{fit}}(\delta \lambda) = 1 - \hat{\Gamma}_{ij}^{\text{eff}} \delta \lambda^i \delta \lambda^j / 2 \]

- The range over which you fit is determined by the expected SNR of your signal: \( 1 - P \leq 1 / \rho^2 \)
Advantages of the Effective Fisher matrix

- Plotting the ambiguity function will tell you if the Fisher matrix approximation is at all reasonable.

- Fitting to a scale set by the SNR makes it robust against fine-scale structure that is unobservable at that SNR.
Fiducial Binary

- We study a \((10 + 1.4) M_\odot \) BH-NS binary \((M_c = 2.99, \eta = 0.1077)\) in initial LIGO
- Use precessing PN waveforms (SpinTaylorT4) with 0PN or 1.5PN amplitude
- Non-spinning/Spin-aligned: \(L\) has an inclination of \(\pi/4\) to line of sight \(N\); BH has spin 0 or 1 along \(L\); NS is non-spinning
- Precessing: \(J\) is inclined to line of sight by \(\theta_{NJ} = \pi/4\); precession cone has an opening angle \(\beta_{JL} = \pi/4\); \(L\) is initially either along \(N\) or perpendicular to it
Non-spinning results

\[ \Delta \eta / \eta \begin{array}{l} 3.83 \% \ \ ... \\ 3.38 \% \end{array} \]

\[ \Delta \nu_L N \begin{array}{l} 0.30 \end{array} \]

\[ \Delta \phi_{ref} \begin{array}{l} 0.585 \end{array} \]
Spin-aligned results

- Non-spinning: ACs do not improve measurement of intrinsic parameters

- Spin-aligned: ACs give a modest improvement to intrinsic parameter errors

- In both cases, extrinsic parameters are unmeasurable without amp. cor., and are measurable (but with large errors) with ACs

<table>
<thead>
<tr>
<th>amp. order</th>
<th>0PN</th>
<th>1.5PN</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta M_c/M_c$</td>
<td>1.08%</td>
<td>.795%</td>
</tr>
<tr>
<td>$\Delta \eta/\eta$</td>
<td>18.7%</td>
<td>12.7%</td>
</tr>
<tr>
<td>$\Delta \chi$</td>
<td>0.149</td>
<td>0.108</td>
</tr>
<tr>
<td>$\Delta \iota_{LN}$</td>
<td>—</td>
<td>0.282</td>
</tr>
<tr>
<td>$\Delta \phi_{ref}$</td>
<td>—</td>
<td>0.821</td>
</tr>
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The errors are smaller for precessing binaries than for spin-aligned.

The gains from ACs become less pronounced, however.

The divide between intrinsic and extrinsic parameters becomes blurred, as angles describing the precession cone correlate to both.

<table>
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<th>amp. order</th>
<th>0PN</th>
<th>1.5PN</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta M_c/M_c$</td>
<td>.208%</td>
<td>.195%</td>
</tr>
<tr>
<td>$\Delta \eta/\eta$</td>
<td>6.14%</td>
<td>4.91%</td>
</tr>
<tr>
<td>$\Delta \chi$</td>
<td>0.0495</td>
<td>0.0421</td>
</tr>
<tr>
<td>$\Delta \beta_{JL}$</td>
<td>0.0241</td>
<td>0.0210</td>
</tr>
<tr>
<td>$\Delta \theta_{JN}$</td>
<td>0.117</td>
<td>0.113</td>
</tr>
<tr>
<td>$\Delta \alpha_{JL}$</td>
<td>0.187</td>
<td>0.191</td>
</tr>
<tr>
<td>$\Delta \phi_{ref}$</td>
<td>—</td>
<td>—</td>
</tr>
</tbody>
</table>
Caveats

- We see non-quadratic behavior in the precessing case for some orientations (but not others)
- We use a complex inner product s.t. \( \text{Re} \langle h|h'\rangle = (h_+|h'_+)+(h_\times|h_\times) \) corresponding to an idealized network.
- The results should lie between those of a single-IFO and a real multi-detector network
- We cautiously believe the trends seen here and are working to confirm with MCMC
Conclusions

- The effective Fisher matrix allows one to see whether the Fisher matrix formalism is likely to be valid (and for which parameters)
- It is also robust against small-scale structure that can be problematic for the standard Fisher matrix
- Higher harmonics will likely have little effect on the measurement of intrinsic parameters, but they can improve the measurement of extrinsic parameters
- For non-precessing systems, there is a clear separation of intrinsic and extrinsic parameters, but this becomes blurred for precessing
- For more details, see arXiv:1209.4494
- Full MCMC runs underway to get more precise results and confirm the trends seen here