Nonsingular big-bounce cosmology from spin and torsion

Nikodem J. Popławski

Department of Physics, Indiana University, Bloomington, IN

22nd Midwest Relativity Meeting
University of Chicago, Chicago, IL

29 September 2012
Problems of standard cosmology

- **Big-bang singularity** – can be solved by LQG
  But LQG has not been shown to reproduce GR in classical limit

- Flatness and horizon problems – solved by inflation
  consistent with cosmological perturbations observed in CMB
  But:
  - Scalar field with a specific (slow-roll) potential needed
    fine-tuning problem not resolved
  - What physical field causes inflation?
  - What ends inflation?

- Dark energy
- Dark matter
- Matter-antimatter asymmetry

Existing alternatives to GR:
- Use exotic fields
- Are more complicated
- Do not address all problems
  (usually 1, sometimes 2)
Einstein-Cartan-Sciama-Kibble theory

Spacetime with gravitational torsion

This talk:
Big-bang singularity, inflation problems all naturally solved by torsion
**Affine connection**

- Vectors & tensors – under coordinate transformations behave like differentials and gradients & their products.

- Differentiation of vectors in curved spacetime requires subtracting two vectors at two infinitesimally separated points with different transformation properties.

- **Parallel transport** brings one vector to the origin of the other, so that their difference would make sense.

\[ \delta A^k = - \Gamma^k_{li} A^l dx^i \]
Curvature and torsion

Calculus in curved spacetime requires geometrical structure: affine connection

Covariant derivative of a vector

\[ B^k_{;i} = B^k_{,i} + \Gamma^k_{li} B^l \]

Two tensors constructed from affine connection:

- **Curvature tensor**

\[ R^i_{mjk} = \partial_j \Gamma^i_{mk} - \partial_k \Gamma^i_{mj} + \Gamma^i_{lj} \Gamma^l_{mk} - \Gamma^i_{lk} \Gamma^l_{mj} \]

- **Torsion tensor** – antisymmetric part of connection

\[ S^k_{ij} = \Gamma^k_{[ij]} \]

Contortion tensor

\[ C^i_{jk} = S^i_{jk} + S^i_{jk} + S^i_{kj} \]

É. Cartan (1921)
Theories of spacetime

*Special Relativity* – flat spacetime (no affine connection)
Dynamical variables: matter fields

*General Relativity* – (curvature, no torsion)
Dynamical variables: matter fields + metric tensor \( g_{ik} \)
\[ S^k_{ij} = 0 \]
Connection restricted to be symmetric – ad hoc
(equivalence principle)

*ECSK gravity* (simplest theory with curvature & torsion)
Dynamical variables: matter fields + metric + **torsion**
ECSK gravity

Riemann-Cartan spacetime – metricity
\[ g_{ik;j} = 0 \]
\[ \Gamma^k_{ij} = \{^k_{ij}\} + C^k_{ij} \]

Christoffel symbols of metric

Matter Lagrangian density

Total Lagrangian density like in GR:
\[ -\frac{1}{2\kappa} R\sqrt{-g} + \mathcal{L}_m \]

Two tensors describing matter:

- Energy-momentum tensor
  \[ T_{ik} = 2(\delta \mathcal{L}_m / \delta g^{ik}) / \sqrt{-g} \]

- Spin tensor
  \[ s_{ijk} = 2(\delta \mathcal{L}_m / \delta C_{ijk}) / \sqrt{-g} \]

D. W. Sciama, Rev. Mod. Phys. 36, 463 (1964)
ECSK gravity

Curvature tensor = Riemann tensor + tensor quadratic in torsion + total derivative

Stationarity of action under $\delta g^{ik} \rightarrow$ Einstein equations

$$G_{ik} = \kappa (T_{ik} + U_{ik})$$

$$U_{ik} = \frac{1}{\kappa} \left( C_{ij}^j C_{kl}^l - C_{ij}^l C_{kl}^j - \frac{1}{2} g_{ik} (C_{jm}^m C_{ml}^l - C_{mj}^m C_{lj}^m) \right)$$

Stationarity of action under $\delta C_{ijk} \rightarrow$ Cartan equations

$$S_{ik}^j - S_i \delta_{k}^j + S_k \delta_{i}^j = -\frac{1}{2} \kappa S_{ik}^j$$

$S_i = S_{ik}^k$

- Torsion is proportional to spin density
- Contributions to energy-momentum from spin are quadratic
Dirac spinors with torsion

Simplest case: minimal coupling

Dirac Lagrangian density (natural units)

\[ \mathcal{L}_m = \frac{i}{2} \sqrt{-g} (\bar{\psi} \gamma^i \psi^i - \bar{\psi} \gamma^i \gamma^j \psi_j) - m \sqrt{-g} \bar{\psi} \psi \]

Dirac equation

\[ i \gamma^k \psi_{;k} = m \psi \]

\[ \bar{\psi}_{;k} = \bar{\psi}_{;k} - \frac{1}{4} C_{ijk} \gamma^{[i} \gamma^{j]} \bar{\psi} \]

Covariant derivative of a spinor

GR covariant derivative of a spinor

Dirac spinors with torsion

Spin tensor is completely antisymmetric

\[ s_{ijk} = -e^{ijkl}s_l \]  \[ s^i = \frac{1}{2} \bar{\psi} \gamma^i \gamma^5 \psi \]

Torsion and contortion tensors are also antisymmetric

\[ C_{ijk} = S_{ijk} = \frac{1}{2} \kappa e_{ijkl}s^l \]

LHS of Einstein equations

\[ T_{ik} + U_{ik} = \frac{i}{2} (\bar{\psi} \delta^j_{(i} \gamma^k) \psi_{;j} - \bar{\psi}_{;j} \delta^j_{(i} \gamma^k) \psi) + \frac{3}{4} \kappa s^l s_l g_{ik} \]

\[ \langle s^2 \rangle = \frac{3}{4} n^2 \]

Fermion number density

\[ -\frac{3}{4} \kappa s^2 g_{ik} \]

comoving frame
ECSK gravity

Torsion significant when $U_{ik} \sim T_{ik}$ (at Cartan density)

$$\rho_C = \frac{m_n^2 c^4}{G \hbar^2}$$

For fermionic matter $\rho_C > 10^{45}$ kg m$^{-3}$ $\gg$ nuclear density

Other existing fields do not generate torsion

- Gravitational effects of torsion are negligible even for neutron stars (ECSK passes all tests of GR)
- Torsion vanishes in vacuum $\rightarrow$ ECSK reduces to GR
- Torsion is significant in very early Universe and black holes

Imposing symmetric connection is unnecessary
ECSK has less assumptions than GR
Cosmology with torsion

Spin corrections to energy-momentum act like a perfect fluid

\[ \ddot{\epsilon} = -\ddot{p} = -\alpha n^2 \]

\[ \alpha = \frac{9}{16} \kappa \]

Friedman equations for a homogeneous and isotropic Universe:

\[ \dot{a}^2 + k = \frac{1}{3} \kappa (\epsilon - \alpha n^2) a^2 \]

\[ a^3 d\epsilon - 2\alpha a^3 n dn + (\epsilon + p) d(a^3) = 0 \]

Statistical physics in early Universe (neglect \( k \))

\[ \epsilon(T) = \frac{\pi^2}{30} g_\star(T) T^4 \]

\[ p(T) = \frac{\epsilon(T)}{3} \]

\[ n(T) = \frac{\zeta(3)}{\pi^2} g_n(T) T^3 \]

\[ h_\star \]

\[ h_n \]
Cosmology with torsion

Scale factor vs. temperature

\[ \frac{dT}{T} - \frac{3\alpha h_n^2}{2h_*} TdT + \frac{da}{a} = 0 \]

Solution

\[ a = a_r T_r \exp\left(\frac{3\alpha h_n^2}{4h_*} T^2\right) \]

Singularity avoided

\[ T_{cr} = \left(\frac{2h_*}{3\alpha h_n^2}\right)^{1/2} \]
Cosmology with torsion

Temperature vs. time

\[ \dot{T}^2 \left( \frac{1}{T^2} - \frac{3\alpha h_n^2}{2h_*} \right)^2 = \frac{\kappa}{3} (h_* T^2 - \alpha h_n^2 T^4) \]

\[ |\beta| = \sqrt{\frac{\kappa h_*}{3} \sqrt{\frac{\beta^2 - \frac{2}{3} \beta_{cr}^2}{\beta^2 - \beta_{cr}^2}}} \]

\[ \beta = T^{-1} \quad \rightarrow \quad T \leq T_{cr} \]

Can be integrated parametrically

\[ \beta = \sqrt{\frac{2}{3} \beta_{cr} \cosh \eta} \]

\[ \eta_{cr} = \text{arcosh} \sqrt{\frac{3}{2}} \]

\[ \frac{t}{t_0} = \frac{1}{6} \sinh(2\eta) - \frac{2}{3} \eta + \frac{\sqrt{3}}{6} - \frac{2}{3} \eta_{cr}, \quad \eta \leq -\eta_{cr}, \quad t \leq 0 \]

\[ \frac{t}{t_0} = \frac{1}{6} \sinh(2\eta) - \frac{2}{3} \eta - \frac{\sqrt{3}}{6} + \frac{2}{3} \eta_{cr}, \quad \eta \geq \eta_{cr}, \quad t \geq 0 \]
Cosmology with torsion

Temperature vs. time

\[ t_0 = \beta_{cr}^2 \sqrt{\frac{3}{kh_*}} \]

Time scale of torsion era

Cusp-like bounce
Nonsingular big bounce instead of big bang

Scale factor vs. time

Big bounce

NP, Phys. Rev. D 85, 107502 (2012)
Nonsingular big bounce instead of big bang

Scale factor vs. time

![Graph showing scale factor vs. time with a slope discontinuity at t = t0]

Big bounce
Nonsingular big bounce

Scale factor vs. time

Mass generation

Big bounce
Nonsingular big bounce

Singularity theorems?

Spinor-torsion coupling enhances strong energy condition

$$\ddot{\epsilon} + 3\ddot{p} = 2\alpha n^2 > 0$$

Expansion scalar (decreasing with time) in Raychaudhuri equation

$$\theta = \frac{3\dot{a}}{a}$$

is discontinuous at the bounce, preventing it from decreasing to $-\infty$ (reaching a singularity)
Torsion as alternative to inflation

For a closed Universe ($k = 1$):

Velocity of the antipode relative to the origin

$$v_{\text{ant}}(T) = \pi \dot{a}(T)$$

At the bounce

$$|\dot{a}(T_{\text{cr}})| = \left(\frac{32e}{243}\right)^{1/2} \frac{h_{\star}}{h_n} a_r T_r$$

Density parameter

$$\Omega(T) = 1 + \frac{1}{\dot{a}^2(T)}$$

Current values (WMAP)

$$\Omega = 1.002$$

$$a_0 = 2.9 \times 10^{27} \text{ m}$$

NP, Phys. Rev. D 85, 107502 (2012)
Torsion as alternative to inflation

Big bounce:

\[ T_{cr} \approx 0.78 m_p \]
\[ a_{cr} \approx 5.9 \times 10^{-4} \text{ m} \]

Horizon problem solved

\[ v_{\text{ant}}(T_{cr}) \approx 8.9 \times 10^{34} \]

Flatness problem solved

\[ \Omega(T_{cr}) \approx 1 + 1.3 \times 10^{-70} \]

Cosmological perturbations – in progress

\[ N \sim v_{\text{ant}}^3 \]

Minimum scale factor

Number of causally disconnected volumes

No free parameters
Summary

Torsion in the ECSK theory of gravity:

• Averts the big-bang singularity, replacing it by a nonsingular, cusp-like big bounce
• Solves the flatness and horizon problems without inflation

No free parameters