A First Class Formulation of Massive Gravity - *or Massive Gravity as a Gauge Theory*

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Based on work to appear

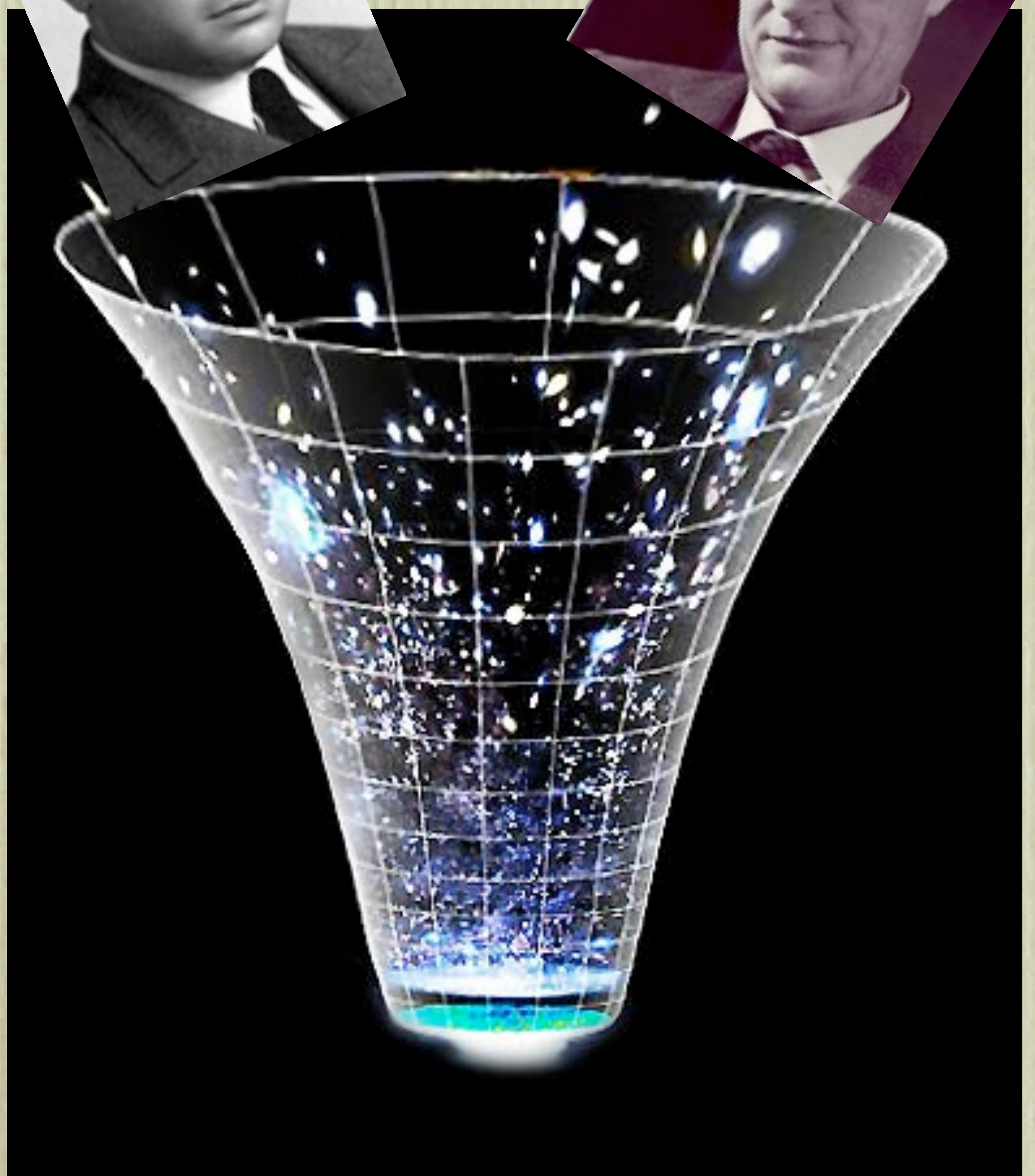
Why Massive Gravity?

Massive Gravity Theories are a remarkably a constrained modification of general relativity at large distance scales - graviton is assumed to acquire a mass

In present talk I shall only be concerned with models where this occurs without breaking Lorentz or de Sitter symmetries

They are interesting in that as in GR, there are a finite number of consistent allowed terms in the Lagrangian that do not give rise to ghosts

By Massive Gravity we mean a nonlinear completion of Fierz-Pauli coupled to matter



Markus Fierz and Wolfgang Pauli, 1939

$$\Box h_{\mu\nu} + \cdots = m^2(h_{\mu\nu} - 1\,\eta_{\mu\nu} h)$$

$$5 = 2s + 1$$

Fierz-Pauli mass term

guarantees 5 rather than 6 propagating degrees of freedom

Massless spin-two in Minkowski makes sense!

Why Massive Gravity?

Adding a mass to gravity weakens the strength of gravity at large (cosmological) distances

$$V_{Yukawa} \sim \frac{e^{-mr}}{r}$$

But thats not all!

Self-acceleration?

Screening mechanism

Degravitation mechanism?

Why Massive Gravity?

Self-acceleration?

Gravitons can condense to form a condensate whose energy density **sources** self-acceleration

$$\rho_{\mathrm{matter}} \sim 0 \qquad\qquad H \sim m \neq 0$$

Analogous to well-known mechanism in Dvali-Gabadadze-Porrati model (DGP), however here it seems possible to remove the DGP ghost??

Deffayet 2000

Koyama 2005
Charmousis 2006

Why Massive Gravity?

Gravitons can condense to form a condensate whose energy density **compensates** the cosmological constant

Screening mechanism - The Cosmological Constant can be LARGE with the cosmic acceleration SMALL

In a Massive Theory - the c.c. is a `redundant' operator

Why Massive Gravity?

mass term

$$G_{\mu\nu} + m^2 \frac{\partial L_M}{\partial g_{\mu\nu}} = -\Lambda g_{\mu\nu}$$

Graviton condensate:

Spacetime is Minkowski in presence of an arbitrary large Λ

$$g_{\mu\nu} = \left(1 + f\left(\frac{\Lambda}{m^2}\right)\right)\eta_{\mu\nu} \qquad G_{\mu\nu} = 0 \qquad m^2 \frac{\partial L_M}{\partial g_{\mu\nu}} = -\Lambda g_{\mu\nu}$$

Equivalent Statement: The cosmological constant can be reabsorbed into a **redefinition** of the metric and coupling constants - and is hence a **redundant** operator

Why Massive Gravity?

Screening $\longrightarrow$ Degravitation

One strong motivation for considering Massive Gravity is as a toy model of higher dimensional gravity models (eg Cascading Gravity) that potentially exhibit degravitation

de Rham et al 2007

Degravitation = Dynamical Evolution to a Screened Solution from generic initial conditions

Dvali, Hofmann, Khoury 2007

so far it is safe to say that this idea has not YET been fully realized

Why Massive Gravity?

Departure from GR is governed by essentially a single parameter - Graviton Mass

Vainshtein Screening mechanism ensures recovery of GR in limit $m \to 0$

This ensures massive gravity can be easily made to be consistent with most tests of GR (effectively placing an upper bound on m) without spoiling its role as an IR modification

Why Massive Gravity?

Massive Gravity is a natural Infrared Completion of
Galileon Theories

Galileon: Nicolis, Rattazzi
Trincherini 2010

Decoupling limit of Massive Gravity on Minkowski is a Galileon Theory

de Rham and Gabadadze 2010

Decoupling limit of Massive Gravity on de Sitter is a Galileon Theory (with slightly different coefficients)

de Rham and Renaux-Petel 2012

The allowed Galileon Interactions are in direct correspondence with the allowed MG interactions

Why Massive Gravity?

Massive Gravity models share many nice features in common with extra dimensional models such as DGP and Cascading Gravity .....

e.g. Vainshtein mechanism, Galileon limit, self-acceleration, possible screening

.... however without the difficulty of having to solve fundamentally higher dimensional equations

Ghost-free Massive Gravity

$$\mathcal{L} = M_{\rm Pl}^2 \sqrt{-\,^{(4)}g}\left(\,^{(4)}R + 2m^2\mathcal{U}(g,f)\right) + \mathcal{L}_M$$

$$\mathcal{K}^\mu_\nu(g,f) = \delta^\mu_\nu - \sqrt{g^{\mu\alpha} f_{\alpha\nu}} \qquad \mathcal{U}(g,H) = \mathcal{U}_2 + \alpha_3 \mathcal{U}_3 + \alpha_4 \mathcal{U}_4$$

$$\begin{aligned}
\mathcal{U}_2 &= \left([\mathcal{K}]^2 - [\mathcal{K}^2]\right),\\
\mathcal{U}_3 &= \left([\mathcal{K}]^3 - 3[\mathcal{K}][\mathcal{K}^2] + 2[\mathcal{K}^3]\right),\\
\mathcal{U}_4 &= \left([\mathcal{K}]^4 - 6[\mathcal{K}^2][\mathcal{K}]^2 + 8[\mathcal{K}^3][\mathcal{K}] + 3[\mathcal{K}^2]^2 - 6[\mathcal{K}^4]\right)
\end{aligned}$$

de Rham, Gabadadze, Tolley, PRL, 106, 231101 (2011)

Proven fully ghost free in ADM formalism: Hassan and Rosen 2011

Result reconfirmed in Stueckelberg decomposition:
de Rham, Gabadadze, Tolley 2011
Hassan, Schmidt-May, von Strauss 2012
Kluson 2012

Result reconfirmed in helicity decomposition:
de Rham, Gabadadze, Tolley 2011

Now several other proofs: Mehrdad Mirbaryi 2011, AJT to appear

dRGT model: allowed mass terms

de Rham, Gabadadze, Tolley 2011

Build out of unique combination

$$K^{\mu}{}_{\nu} = \delta_{\mu\nu} - \sqrt{g^{\mu\alpha} f_{\alpha\nu}}$$

$$U(g,f) = \sum_i \beta_i U_i(K)$$

Mass terms are *characteristic polynomials*

$$det(\delta^{\mu}{}_{\nu} + \lambda K^{\mu}{}_{\nu}) = \sum_{n=0}^{n=d} \lambda^n U_n(K)$$

Finite number of allowed interactions in any dimension

Interactions protected by a Nonrenormalization theorem

Generalized to arbitrary (dynamical - bigravity) reference metrics by Hassan, Rosen 2011

Second Class Constraints

But why does it work??????

Theory requires **two** second class constraints

The first was difficult to show - the second was extremely difficult!!!

Upgrading from Second to First Class

In many systems it is more natural to formulate a system with two second class constraints as a system with one first class constraint

$$2 \times 1 \text{ second class} = 1 \text{ local symmetry} \;=$$
$$1 \text{ first class constraint} + 1 \text{ gauge choice}$$

We are looking for an extra local/gauge symmetry!

Example: Extra Dimensions

Massless Graviton in 5D has 5 degrees of freedom
because of the existence of 5=4+1 gauge symmetries

15-5 (constraint)-5 (gauge choice) = 5

5d symmetric matrix

DGP model:

More irrelevant More relevant

$$S = \int d^4x \sqrt{-g_4} \frac{M_4^2}{2} R_4 + \int d^4x \sqrt{-g_4} \mathcal{L}_M + \int d^5x \sqrt{-g_5} \frac{M_5^3}{2} R_5$$

Dominates in UV Dominates in IR

Finding the Hidden Symmetry

GOAL: Can we reformulation the 4D ghost-free massive gravity theories as theories possessing the same number of gauge symmetries as 5D gravity

Advantage:

1. Local symmetries protect against quantum corrections
2. Starting point for a consistent quantization
3. Proper understanding of helicity zero mode in full nonlinear theory
4. Guarantees the correct number of degrees of freedom
5. Coupling to matter must respect symmetry
6. Useful implications of Ward identities etc.

Main point: How to introduce helicity zero mode

DGP

$$ds^2 = N^2 dy^2 + g_{\mu\nu}(dx^\mu + N^\mu dy)(dx^\nu + N^\nu dy)$$

4 Stueckelberg fields =
4 ADM Shifts

Massive gravity

$$g_{\mu\nu} \qquad\qquad f_{\mu\nu} = \partial_\mu \phi^A \partial_\nu \phi^B \eta_{AB}$$

No analogue of N
No analogue of y coordinate transformations

Main point: How to introduce helicity zero mode

Need to introduce a NEW Stueckelberg field for a broken U(1) symmetry

Previous attempts:

Arkani-Hamed, Georgi, Schwartz 2003
Creminelli, Nicolis, Papucci, Trincherini 2005

$$\phi^A = V^A + \partial^A \pi$$

Introduces a new symmetry

$$V^A \to V^A + \partial^A \chi$$
$$\pi \to \pi - \chi$$

Gets correct decoupling limit

de Rham, Gabadadze 2010

π is a Galileon field $\quad \pi \to \pi + v_\mu x^\mu$

Main point: How to introduce helicity zero mode

Arkani-Hamed, Georgi, Schwartz 2003
Creminelli, Nicolis, Papucci, Trincherini 2005

Previous attempts:

$$\phi^A = V^A + \partial^A \pi$$

But it fails nonlinearly !!!!!!!!

Alberte, Chamseddine, Mukhanov 2010

Superficially a problem but not really .....

de Rham, Gabadadze, Tolley 2011
Hassan, Schmidt-May, von Strauss 2012

It only indicates we have not **correctly** introduced the helicity zero mode

Massive Gravity in the Vierbein Formalism

Nibblelink Groot, Peloso, Sexton 2006
Hinterbichler and Rosen 2012

Write metric in terms of vierbeins (vielbeins)

Minkowski metric

$$\eta = - + + +$$

$$g_{\mu\nu} = e^a_\mu e^b_\nu \eta_{ab}$$

Lorentz Transformation

Introduce Local Lorentz symmetry

$$e^a_\mu \to e^b_\mu \Lambda^a_b \qquad \Lambda^a_c \Lambda^b_d \eta_{ab} = \eta_{cd} \qquad g_{\mu\nu} \to g_{\mu\nu}$$

Massive Gravity in the Vierbein Formalism

Nibblelink Groot, Peloso, Sexton 2006
Hinterbichler and Rosen 2012

Write reference (Minkowski) metric in terms of vierbeins

$$f_{\mu\nu} = b^a_\mu b^b_\nu \eta_{ab}$$

Most general reference vierbein is ...

$$b^a_\mu = \partial_\mu \phi^b \lambda^a_b \qquad \lambda^a_c \lambda^b_d \eta_{ab} = \eta_{cd}$$

Massive Gravity in the Vierbein Formalism

Under a diff (coordinate) transformation

ϕ^a and λ^a_b transform as scalars

Under a local Lorentz transformation

$$b^a_\mu = \partial_\mu \phi^b \lambda^a_b \qquad b^a_\mu \to b^b_\mu \Lambda^a_b \qquad \lambda^a_c \to \lambda^b_c \Lambda^a_b$$

The λ^a_b are the Stueckelberg fields for the broken Lorentz invariance

Allowed dRGT mass terms

All the mass terms arise as characteristic polynomials in the expansion of a determinant

$$\mathcal{L}_{\text{mass}} = m^2 M_{\text{pl}}^2 \det\left(e_\mu^a + \mu\, \partial_\mu \phi^b \lambda_b^a\right)$$

Proof (Nibblelink et al.)

Varying w.r.t. λ_b^a gives constraint

$$e_a^\mu \partial_\mu \phi^c \lambda_{cb} = e_b^\mu \partial_\mu \phi^c \lambda_{ca}$$

Allowed dRGT mass terms

more abstractly this says

$$e^{-1}\partial\phi\lambda = \left(e^{-1}\partial\phi\lambda\right)^T = \lambda^T(\partial\phi)^T e^{T^{-1}}$$

but $\lambda\eta\lambda^T = \eta$ since the λ^a_b are Lorentz Stueckelbergs

and so ....

$$\left(e^{-1}(\partial\phi)\lambda\eta\right)^2 = e^{-1}(\partial\phi)\lambda\eta\lambda^T(\partial\phi)^T e^{T^{-1}}\eta$$
$$= e^{-1}(\partial\phi)\eta(\partial\phi)^T e^{T^{-1}}\eta$$

which implies ....

$$\left(e^{-1}(\partial\phi)\lambda\eta\right) = \sqrt{e^{-1}(\partial\phi)\eta(\partial\phi)^T e^{T^{-1}}\eta}$$

The Famous Square Root of the dRGT model

Introducing the Helicity Zero mode

Perturbatively spotted by Mirbabayi, 1112.1435
Nonperturbatively (AJT to appear)

$$\mathcal{L}_{\rm mass} = m^2 M_{\rm pl}^2 \det\left(e^a_\mu + \mu\, \partial_\mu \phi^b \lambda^a_b\right)$$

$$\phi^a \to \phi^a + e^b_\mu {\lambda^{-1}}^a_{\ b} \left(V^\mu + \nabla^\mu \pi\right)$$

Local Symmetries:

4 coordinate transformations

6 local Lorentz transformations

1 `previously hidden' U(1) symmetry

$$V_\mu \to V_\mu + \partial_\mu \chi \qquad \pi \to \pi - \chi$$

The answer

$$\mathcal{L}_{\text{mass}} = m^2 M_{\text{pl}}^2 \det\left(e^a_\mu + \mu\, \partial_\mu \phi^b \lambda^a_b\right)$$

$$\phi^a \to \phi^a + e^b_\mu {\lambda^{-1}}^a_b \left(V^\mu + \nabla^\mu \pi\right)$$

This only works if we can show that the equations of motion for the helicity zero mode π

After some hard work .... this can be proven

(AJT to appear)

π looks not dissimilar although not the same as a gauged covariant Galileon

Summary

Formulated dRGT massive gravity in D dimensions with the same number of first class constraints corresponding to the number of symmetries as in (D+1) dimensional General Relativity

In this form there are no second class constraints

The extra U(1) symmetry guarantees the correct number of degrees of freedom and absence of BD ghost

It provided a suitable starting point for a consistent quantization and coupling to matter

It allows us to define what we mean by the helicity zero mode about in an arbitrarily curved geometry