



# Studying the Effects of Tidal Corrections on Parameter Estimation

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# Outline

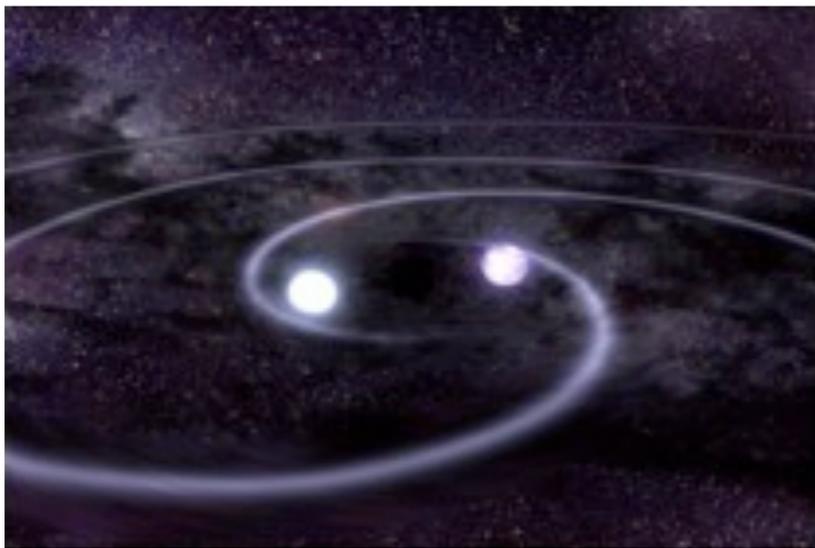
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- Background: *Physical description of tidal forces*
- Motivation: *Constraining NS equation of state*
- Gravitational Waveform: *Mathematical description of tidal corrections*
- Previous Work: *Focus on Fisher matrix*
- Proposed Contribution: *Full Markov Chain Monte Carlo runs and understand systematic biases*
- Summary

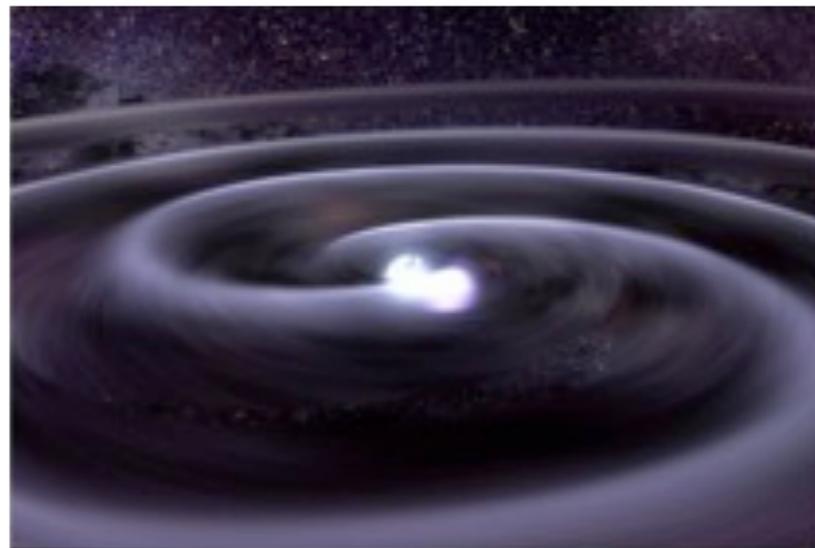
# Compact Binary Coalescence (CBC)

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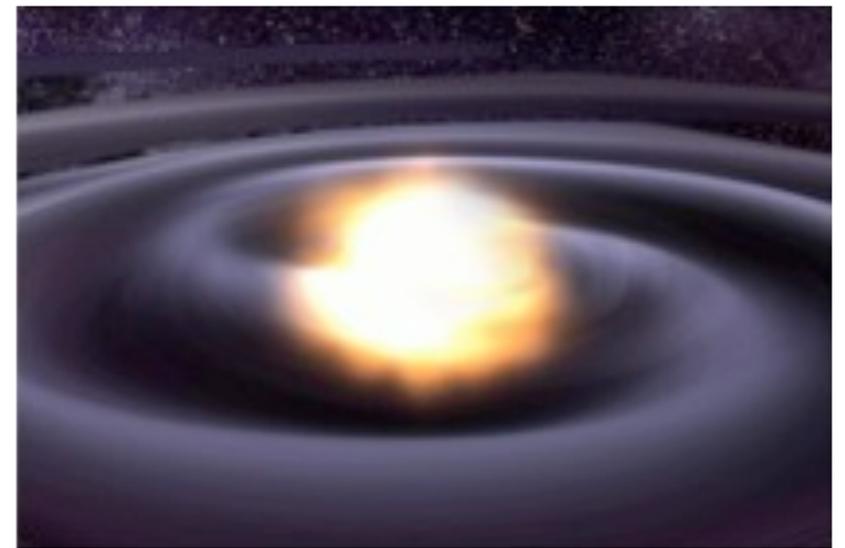
- When two massive bodies are in orbit, they will lose energy to gravitational-wave emission
- They will *inspiral* until *merger*, which is followed by a *ringdown*
- The gravitational waveform of the CBC inspiral can be approximated using post-Newtonian theory



Inspiral



Merger

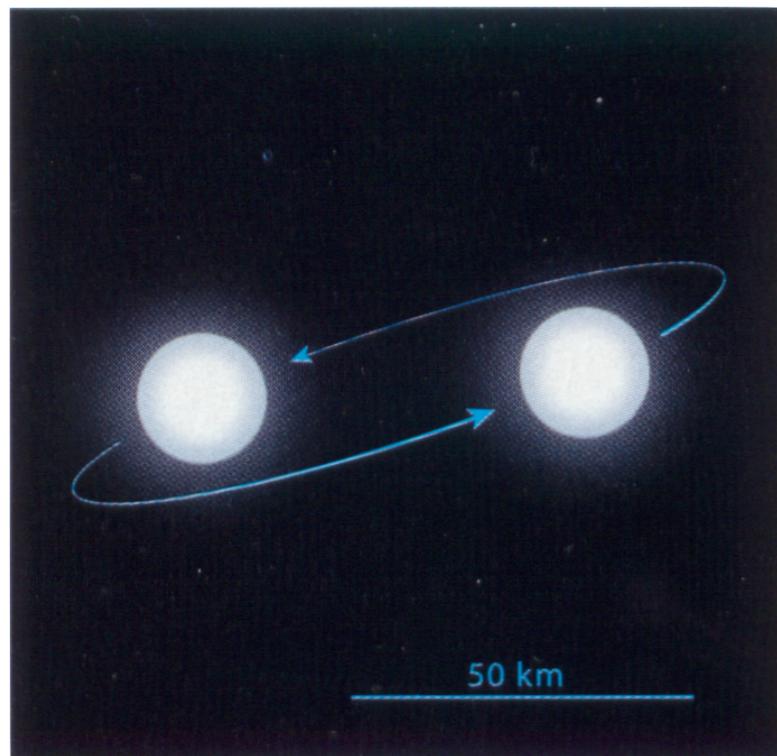


Ringdown

# NS-NS Binaries

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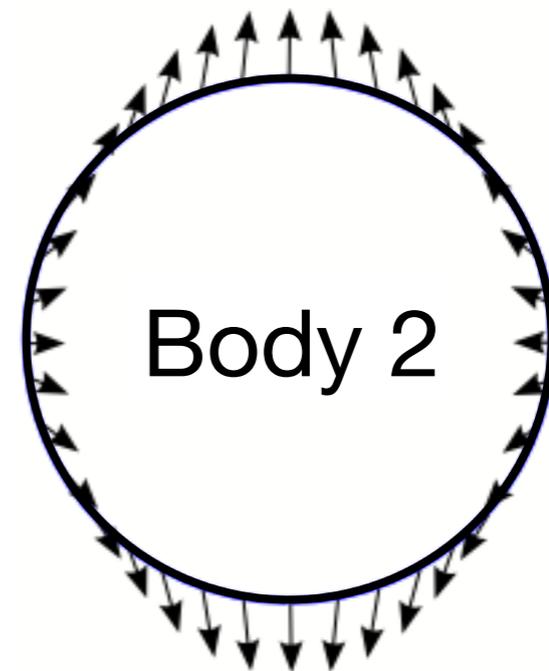
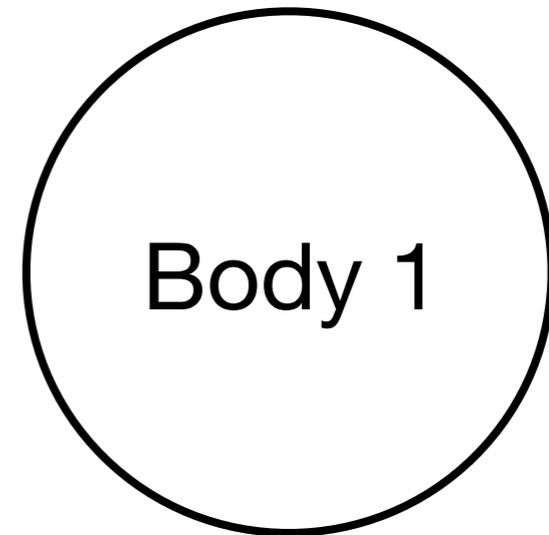
- To find the CBC waveform, each body is approximated as a point particle
- NS-NS binaries will depart from the point particle approximation during the late inspiral phase
- **Tidal forces** will deform NSs depending on their internal structure



# Physical Description of Tidal Forces

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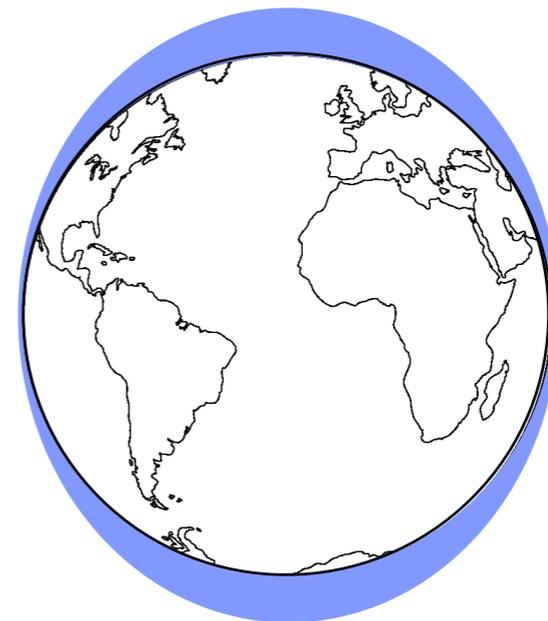
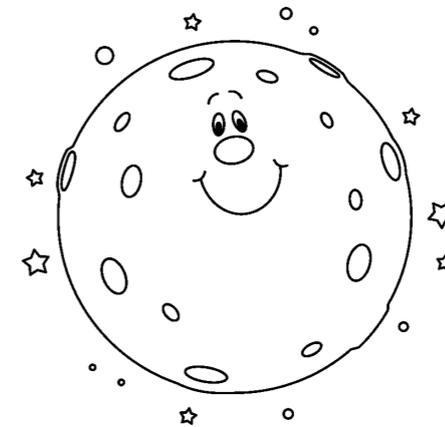
- The gravitational field of Body 1 falls off as  $1/r^2$
- Therefore, the gravitational field across Body 2 is not constant
- The ***tidal force*** is a result of the relative difference in strength of the gravitational field across Body 2
- The tidal ***love number***  $\lambda$  parameterizes how much a body will deform due to tidal forces



# Physical Description of Tidal Forces

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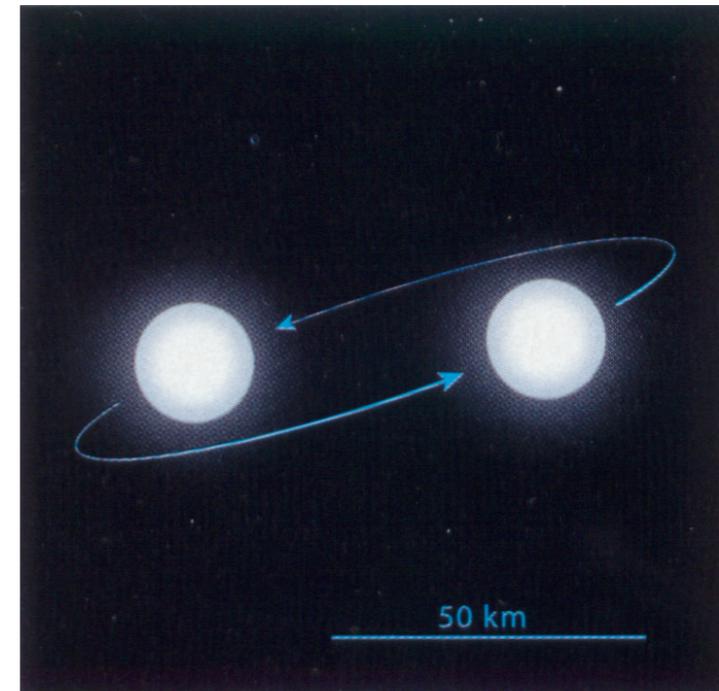
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# Motivation: Constraining NS Equation of State

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- Tidal corrections depend only on the love number
- Accurately determining the love number can constrain the NS equation of state (EOS)
- The NS EOS is useful for ***fundamental physics*** and ***astrophysics***
  - Fundamental physics: Better understand NS matter, which will expand our understanding of phase diagrams at high energies
  - Astrophysics: Gravitational detection can independently provide a distance-redshift relationship

# NS-NS Binary Waveform

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- The point particle approximation to the NS-NS binary waveform:

$$\tilde{h}(f) = \mathcal{A} f^{-7/6} \exp [i\psi(f)]$$

where the phase is

$$\psi(f) = \psi_{3.5} + \mathcal{O}(v^8)$$

where  $v = (\pi M f)^{1/3}$

# NS-NS Binary Waveform + Tidal Corrections

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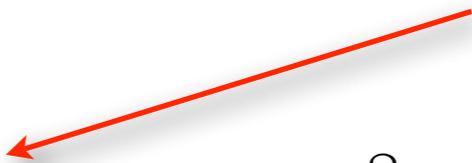
- The point particle approximation to the NS-NS binary waveform:

$$\tilde{h}(f) = \mathcal{A} f^{-7/6} \exp [i\psi(f)]$$

where the phase is

$$\psi(f) = \psi_{3.5} + \delta\psi_{\text{tidal}} + \mathcal{O}(v^8)$$

of order  $v^{10}$  beyond  
leading order



where  $v = (\pi M f)^{1/3}$  and

$$\delta\psi_{\text{tidal}} = -\frac{117}{8} \frac{\tilde{\lambda} v^5}{\eta M^5}$$

where  $\tilde{\lambda}$  is a linear combination of  $\lambda_1$  and  $\lambda_2$

# Notable Work with Tides

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- ***Early inspiral*** (low frequencies) with analytic waveforms
  - Flanagan and Hinderer (2008): Find that aLIGO will be able to constrain  $\lambda$  even though tidal influences are minimal at low frequencies.
  - Hinderer *et al.* (2010): Find that the measurement error in  $\lambda$  steeply increases with the total mass of the binary. They conclude that aLIGO will only be able to probe stiff EOSs while the proposed Einstein Telescope would likely see a clean tidal signature.
- ***Late inspiral*** (high frequencies) with numerically simulated waveforms
  - Read *et al.* (2009): Find that greater measurement accuracy can be achieved at high frequencies and produce quantitative estimates of NS EOS measurability with aLIGO.



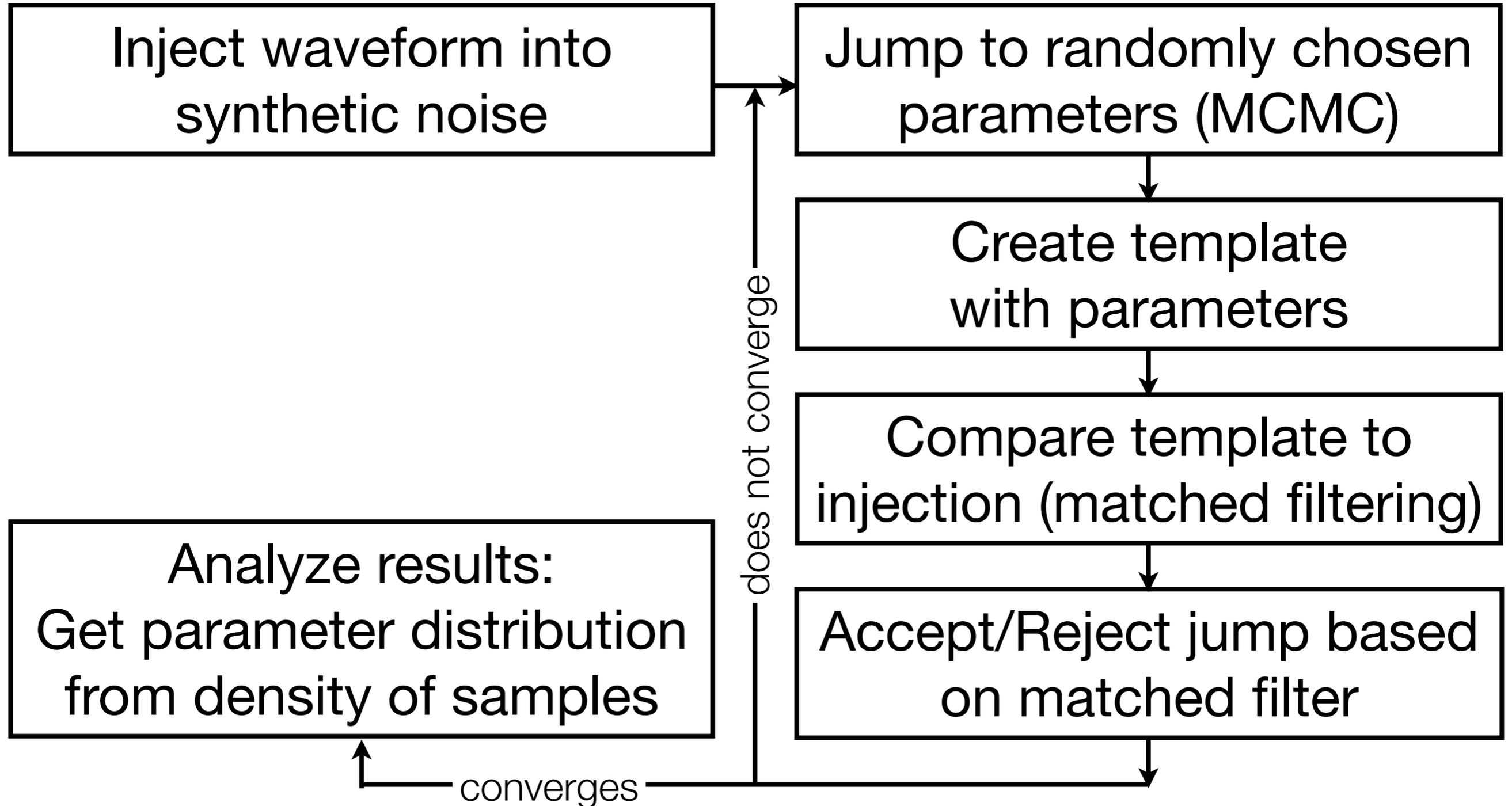
# Proposed Contribution

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- Full **Markov Chain Monte Carlo** (MCMC) runs instead of just Fisher matrix
  - Explore all signal parameters
  - Avoid Fisher matrix assumptions (particularly high signal-to-noise-ratio)
  - Work toward actual parameter estimation method for aLIGO
- Add tidal corrections to several more LAL waveform families
  - Use multiple waveform families to study **systematic biases** in our parameter estimation method

# Parameter Estimation

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# Summary

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- Gravitational wave detectors such as aLIGO may be able to significantly constrain the NS EOS through the addition of tidal corrections to our current waveforms
- Past efforts to determine the measurement error of the tidal love number with gravitational wave detectors have used the Fisher matrix
- We will run full MCMC runs which will produce much more realistic results
- We will use different post-Newtonian waveforms with tidal corrections to better understand systematic biases