Probing the Non-Linearity in Galaxy Clusters Through the Analysis of the Fractal Dimension

Loay Khalifa, Jesus Pando

DePaul University, Chicago, IL

DECAM-NFC, June 2018, Chicago, IL

What is covered?

1. Goals and motivations for the project
2. Previous work.
3. The Data: Baryonic Oscillations Spectroscopic Survey (BOSS) Galaxies and Mock Galaxy Catalogs
4. Our work (Algorithm and Results)
5. Conclusion and future research

Goals and motivations

- Testing whether the galaxy clusters distributions can be described as fractal system or not, i.e. self affine processes, by calculating the fractal dimension.
- The fractal dimension of a system describes how “irregular” or “homogenous/inhomogenous” is the distribution.
- In the context of galaxy distribution the fractal dimension indicates the galaxy clustering density and the dominance of voids.

Previous work

- Different authors reported different values of D according to the methods and assumptions.
- Davies and Pebbles reported D=1.2 using the correlation function at 5 Mpc.
- Irribarem et.al reported D=0.5 and D=1.4 using the galaxy volume number densities.
- Our method: Estimate the angular fractal dimension using the wavelet methods.

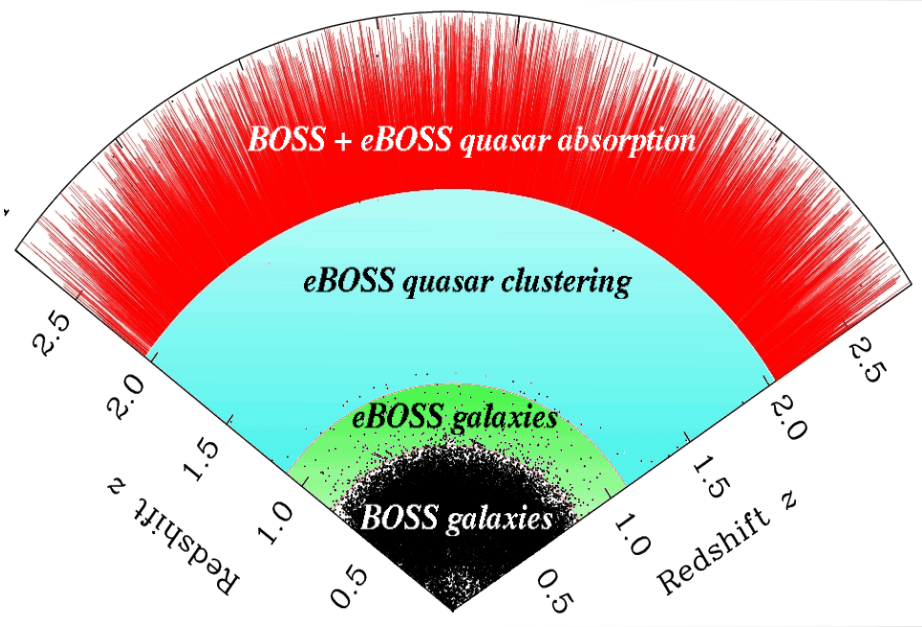
Data

- SDSS DR 13 BOSS: spatial distribution of galaxies.
- Mock Galaxy Catalogs were created using Particle Mesh Method with

$$\Omega_m = 0.274$$

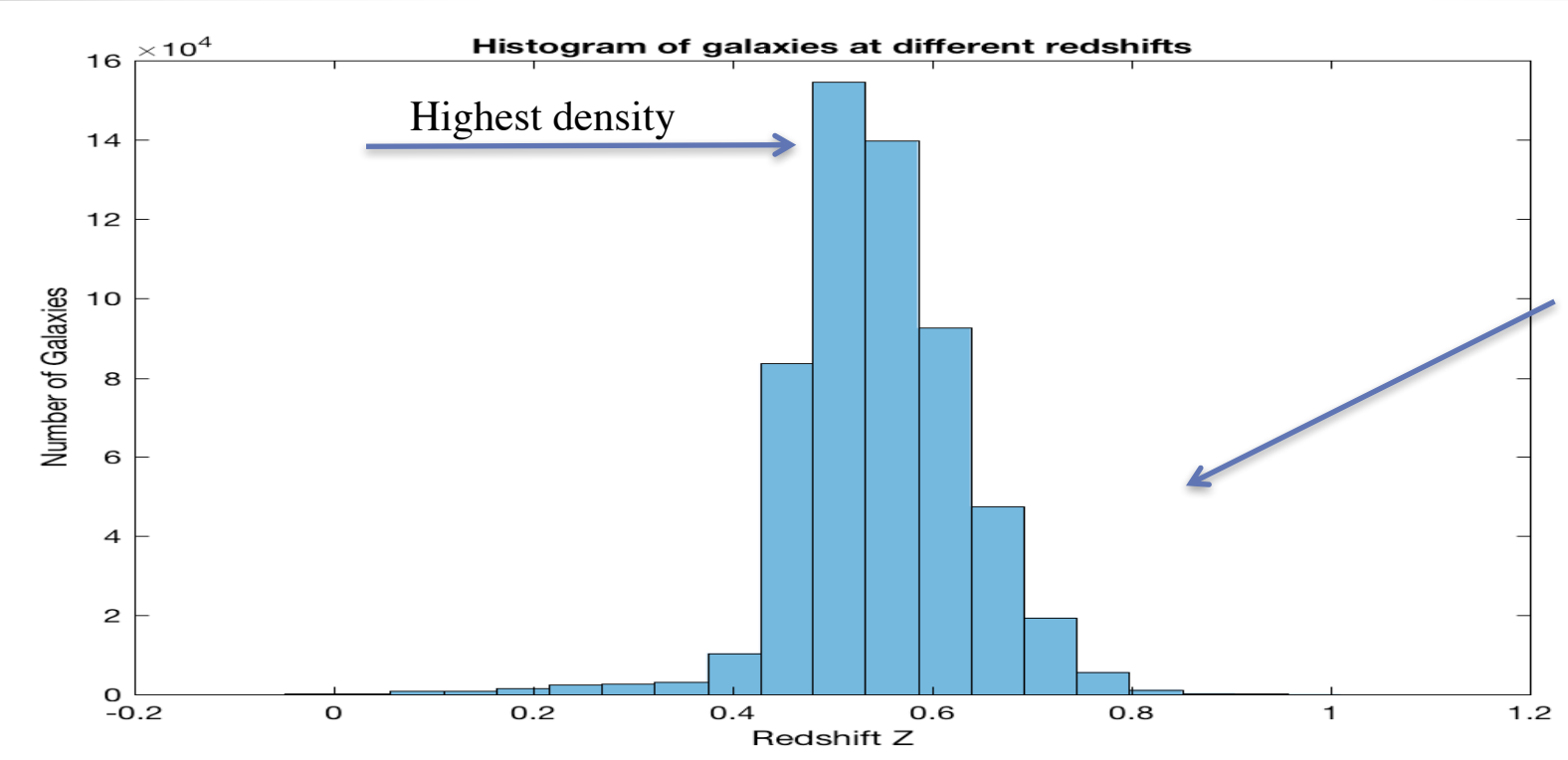
$$\Omega_\Lambda = 0.726$$

$$\Omega_b = 0.046$$



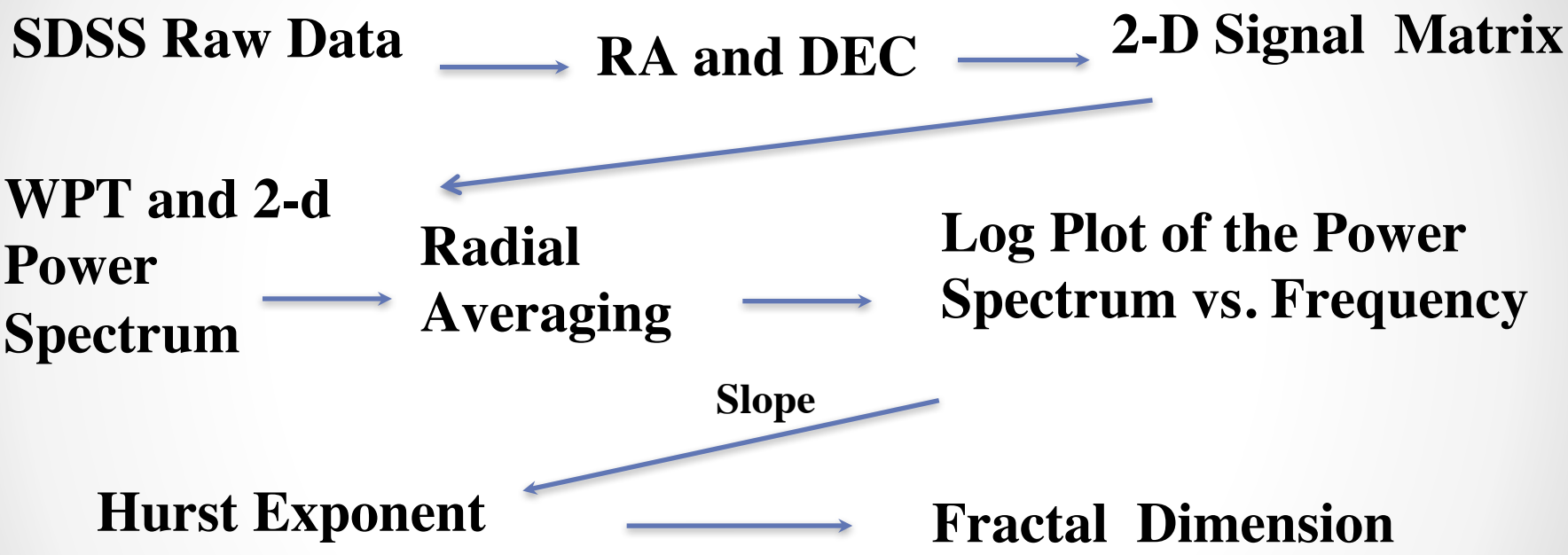
Source: SDSS

Galaxy clusters distribution



The number of galaxies decreased as Z increased

Work flow


SDSS Raw Data
RA and DEC
2-D Signal Matrix
WPT and 2-d Power Spectrum
Radial Averaging
Log Plot of the Power Spectrum vs. Frequency
Slope
Hurst Exponent
Fractal Dimension

Fractal Dimension estimation

- Find the power spectrum

$$var(d_j, k) = \frac{1}{2^j} \sum_{k=0}^{2^j-1} |d_j, k|^2 \qquad (1)$$

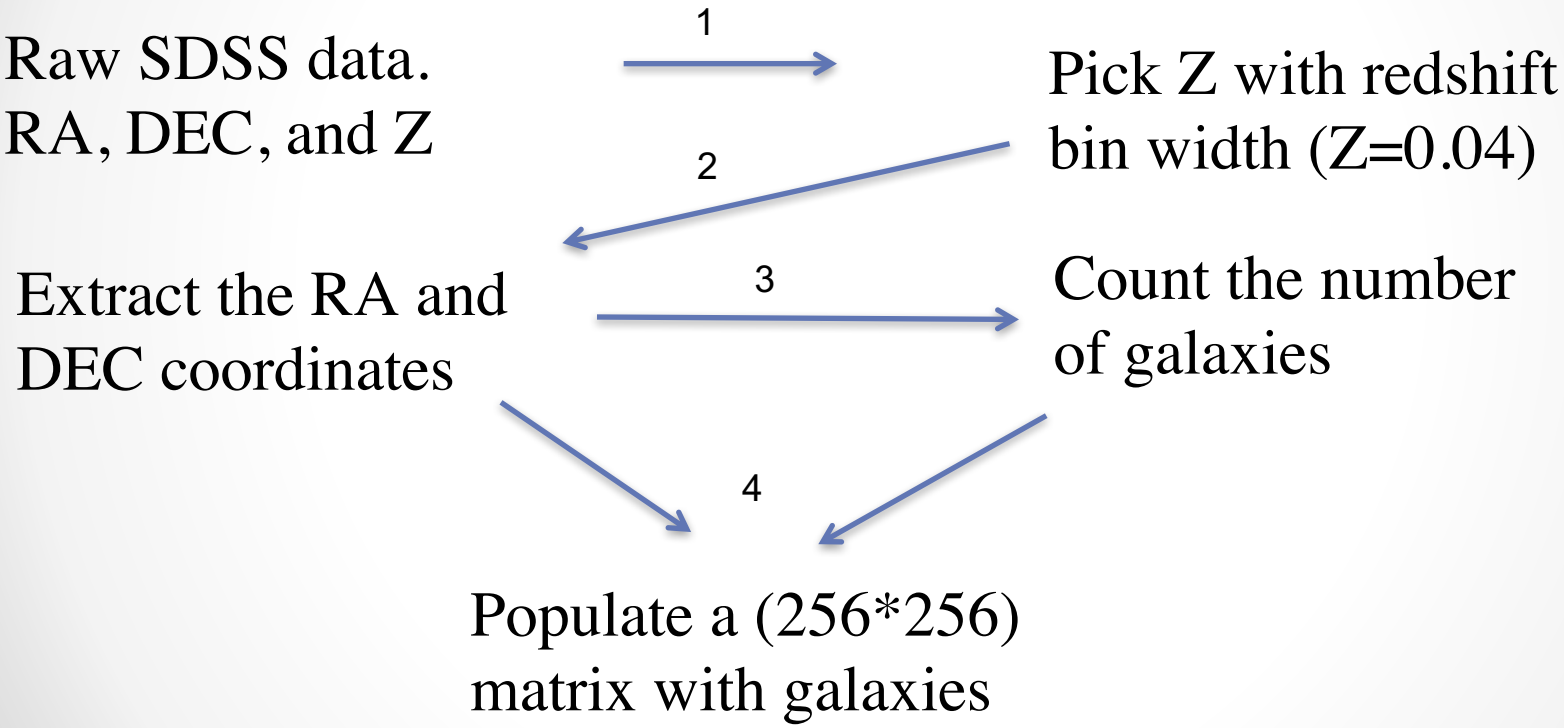
- The Hurst Exponent is estimated using the relation

$$H = \frac{\alpha + 3}{2} \qquad (2)$$

- The fractal dimension is calculated using the relation

$$D = d + 1 - H \qquad (3)$$

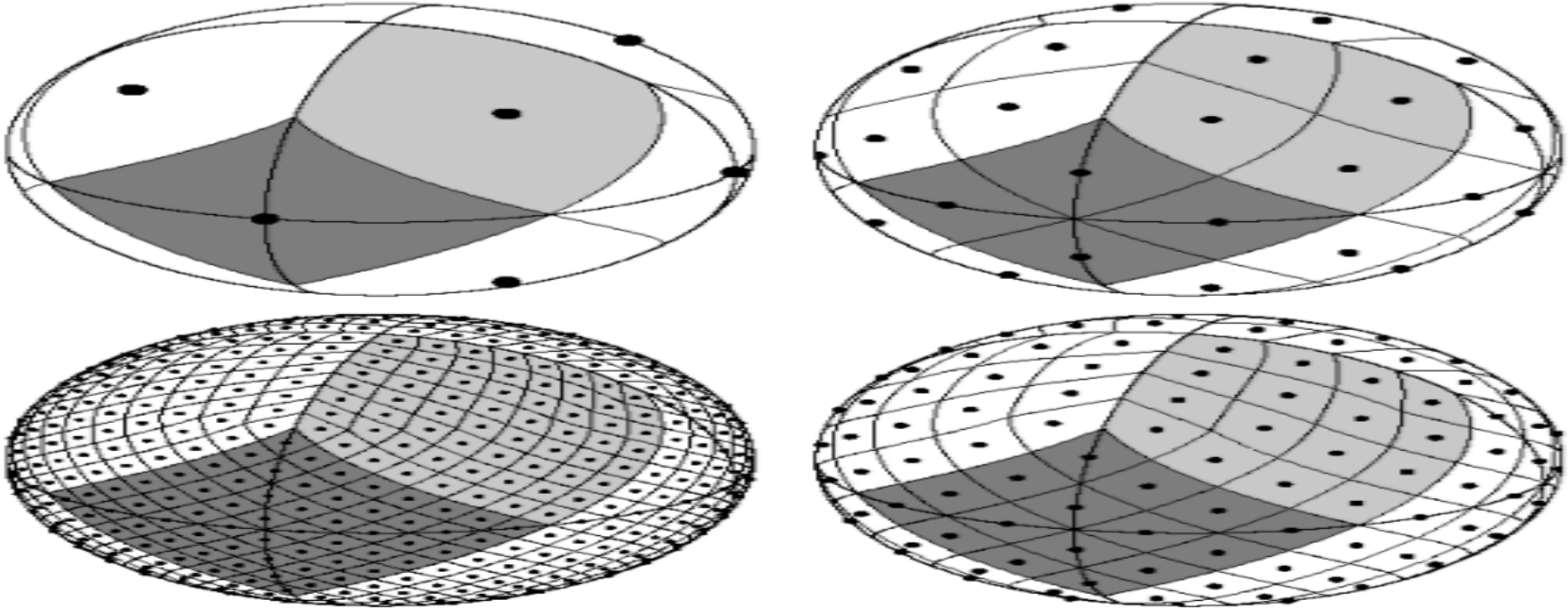
Creating the 2-d signal matrix



HealPix and SDSSPix

- Divide the sky into pixels of equal areas and equal sizes.
- Depending on the resolution, the higher the resolution, the larger the pixels.
- Using indexing schemes, each pixel is assigned 2 indices i and j.
- The two indices i and j are converted into row and column indices, then populated into the two dimensional matrix.
- Populate the two dimensional matrix with galaxies at different redshifts

HealPix and SDSSPix



Wavelets

- Wavelets are a tool which is used in signal analysis.
- Localized in the time as well as the frequency domain.
- Operates by partitioning the signal into different sub-signals with frequency components mapped to coefficients having different energies.
- The transform operates like a microscope for detail examination.

Wavelets

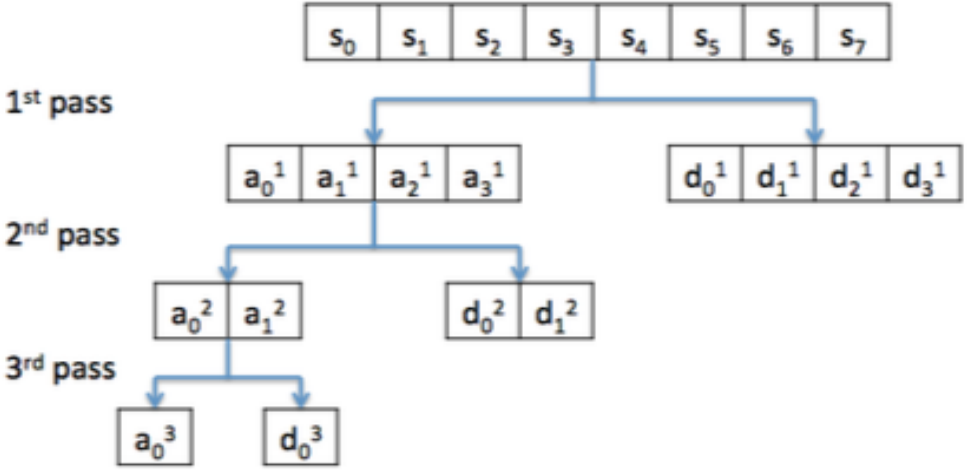
Wavelet Transform:

$$\sum_{j=1}^{2^N} s_j \phi(x) = \sum_{j=1}^{N} a_j \phi'(x) + \sum_{j=1}^{N} d_j \psi'(x)$$

Types:

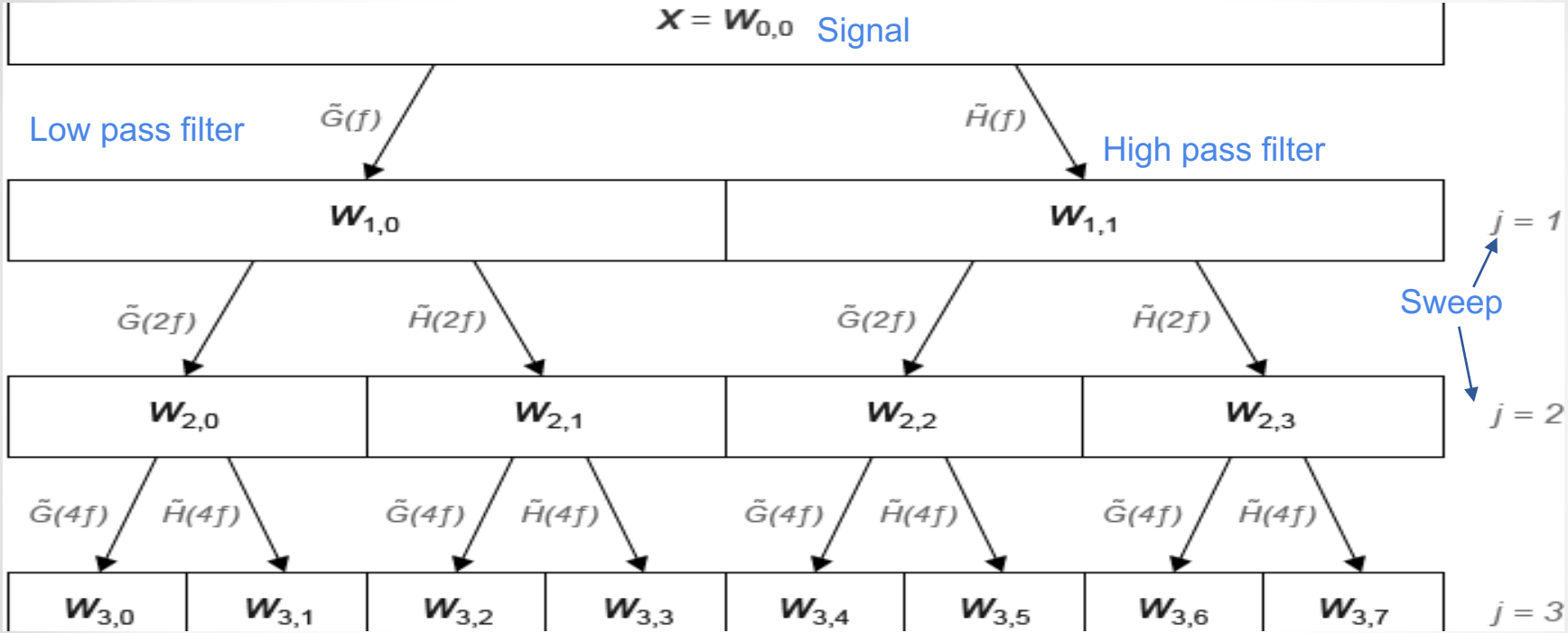
1- Discrete Wavelet Transform

2- Wavelet Packet Transform



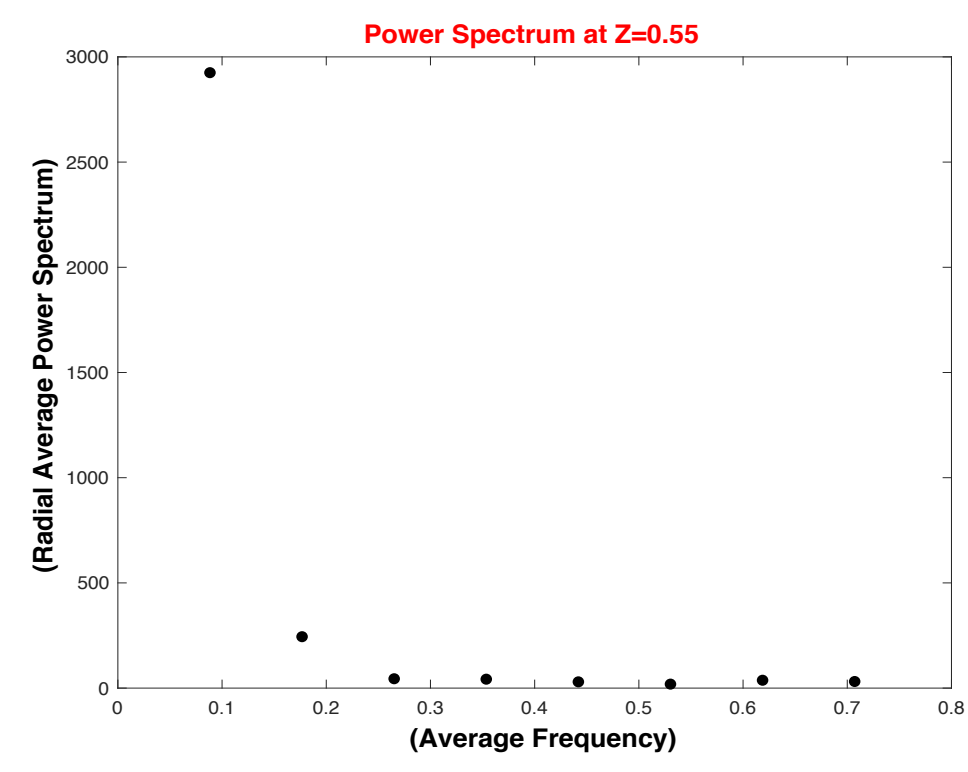
Discrete Wavelet Transform

Wavelet Packet Transform

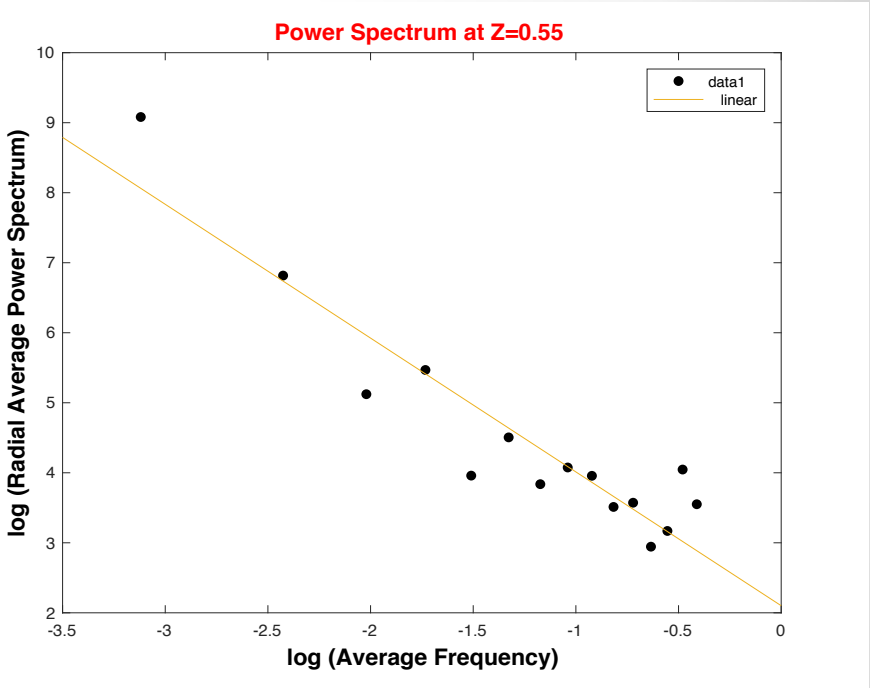


Source: Mathworks

Spectrum estimation



Power law distribution

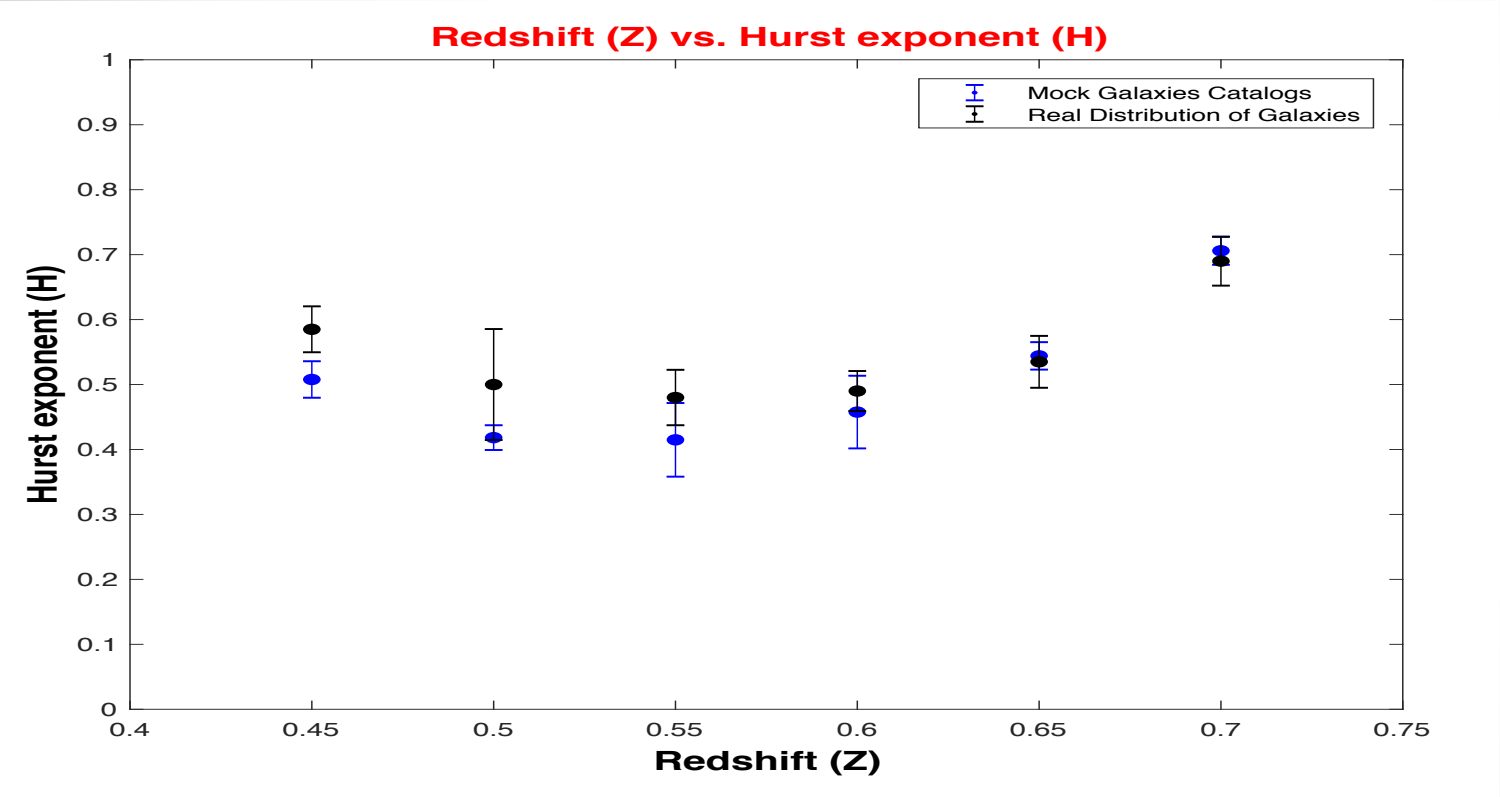


Fitting a straight line to the log-log plot

Hurst Exponent H

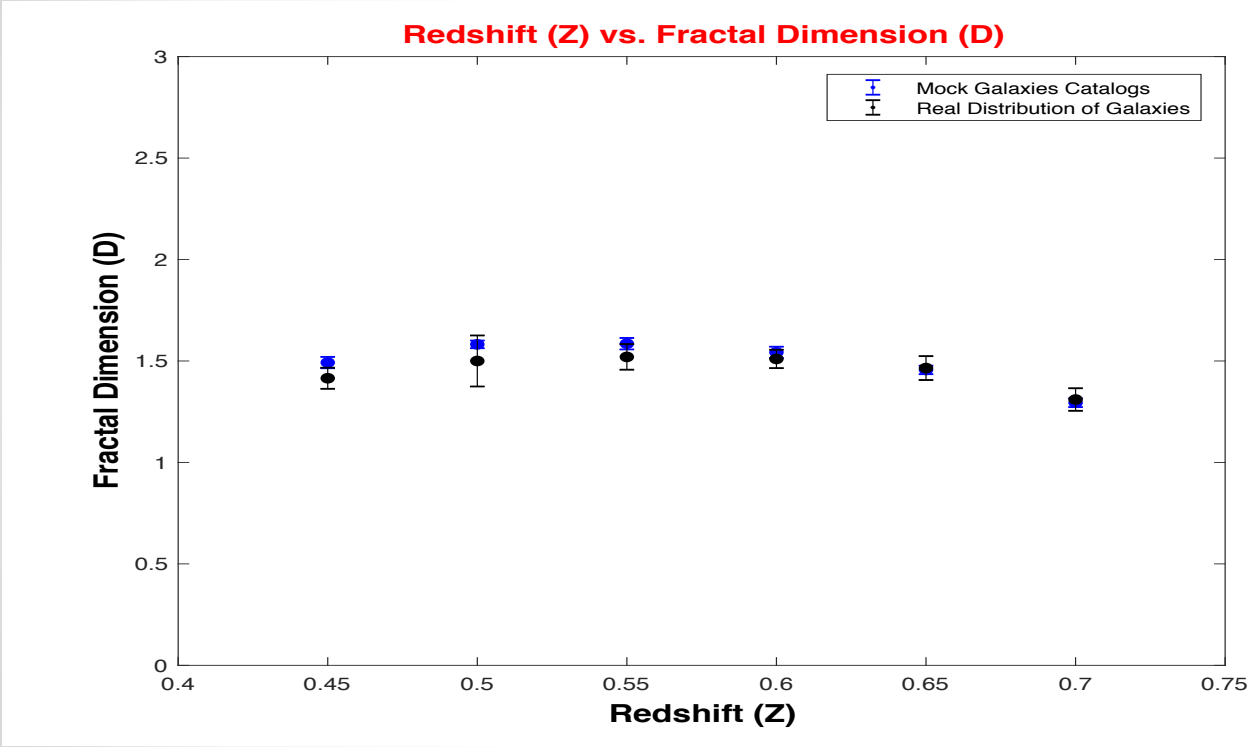
- Values between (0-1).
- (H= 0) implies surfaces of extreme irregularity.
- (H= 0.5) implies random process, i.e., Brownian motion.
- (H> 0.5 to H< 1) surfaces getting smoother.
- (H=1) smooth surface, i.e. homogenous or regular distribution.

Results



As H decrease, the surface is more irregular.

Results



D is constant around 1.5, single fractal system.

Inhomogenous distribution, D < 3.

Conclusion

- We developed a wavelet based algorithm to test whether the large scale structure of the universe can be modeled as fractal systems or not, by calculating the angular fractal dimension.
- Galaxy clusters behave as a power law against cosmological distances.
- The fractal dimension is constant, and the galaxy clusters distribution is inhomogenous.
- The large scale structure is dominated by voids.

Thank You!

- Q&A

References

1. Y. Nievergelt. Wavelets Made Easy. Boster: Birkh auser, 1999.
2. Steven Bradley Lowen, Malvin Carl Teich. Fractal Based Point Processes. John Wiley & Sons, Inc, 2005
3. G. Conde-Saavedraa, A. Iribarrema, Marcelo B. Ribeiro: Fractal analysis of the galaxy distribution in the redshift range $0.45 \leq z \leq 5.0$. Physica A 417: 332-344, 2015. doi:10.1016/j.physa.2014.09.044

References

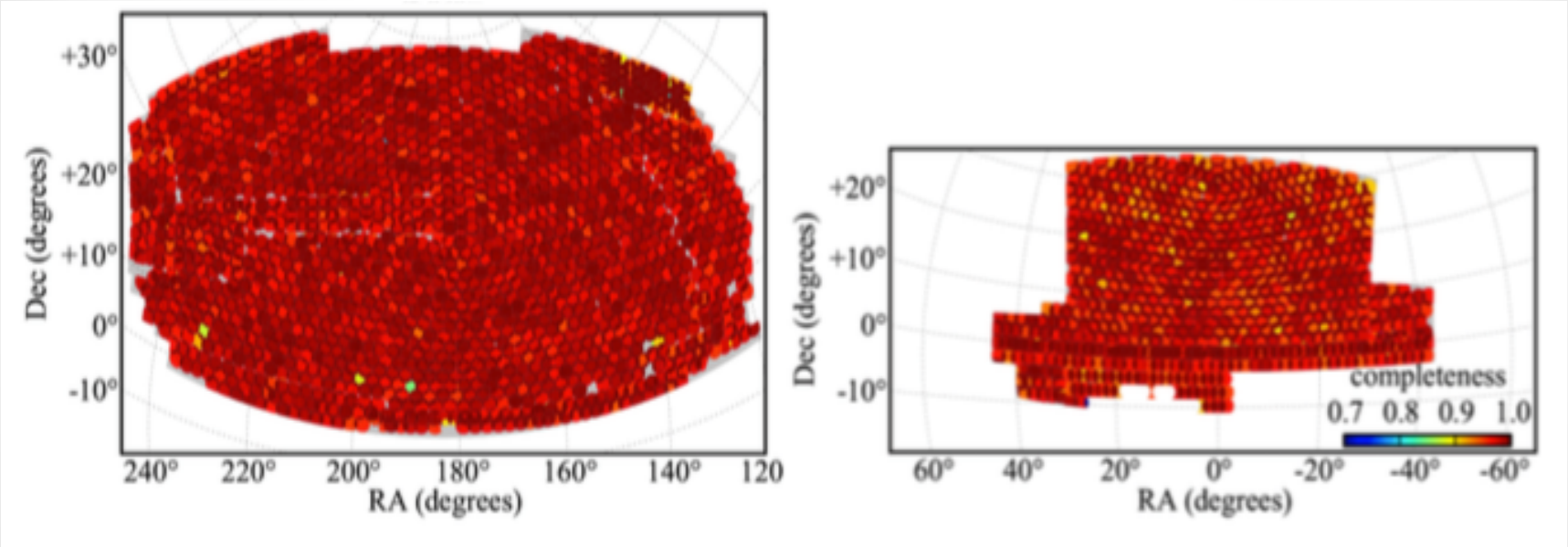
4. L.J. Rangel Lemos and M.B. Riberio: Spatial and observational homogeneities of the galaxy distribution in standard cosmologies. Astronomy and Astrophysics, arxiv:0805.3336v3

5. Marie-Noelle Celerier and Reuben Thieberger: Fractal dimensions of the galaxy distribution varying by steps? arxiv:astro-ph/0504442v1

Additional slides

Data: SDSS

Using SDSS IV DR 13



Source: SDSS

Northern Galactic Cap

Southern Galactic Cap

SDSS Instrumentation

- SDSS makes use of ‘plug plates.’ These are aluminum plates, 32 inches in diameter, in which more than 1000 holes are strategically drilled.
- Each hole corresponds to a target object, and when the plate is assembled to the SDSS 2.5 meter telescope, 62″ fiber-optic cables are plugged into each hole in the plate.
- During telescope operation, these fiber-optic cables then collect light from the target source and carry it to a spectroscopic analyzer.

SDSS Instrumentation

Plates