Probing the Non-Linearity in Galaxy Clusters Through the Analysis of the Fractal Dimension

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What is covered?

1. Goals and motivations for the project
2. Previous work.
3. The Data: Baryonic Oscillations Spectroscopic Survey (BOSS) Galaxies and Mock Galaxy Catalogs
4. Our work (Algorithm and Results)
5. Conclusion and future research
Goals and motivations

• Testing whether the galaxy clusters distributions can be described as fractal system or not, i.e. self affine processes, by calculating the fractal dimension.
• The fractal dimension of a system describes how “irregular” or “homogenenous/inhomogenous” is the distribution.
• In the context of galaxy distribution the fractal dimension indicates the galaxy clustering density and the dominance of voids.
Previous work

• Different authors reported different values of $D$ according to the methods and assumptions.
• Davies and Pebbles reported $D=1.2$ using the correlation function at 5 Mpc.
• Irribarem et.al reported $D=0.5$ and $D=1.4$ using the galaxy volume number densities.
• Our method: Estimate the angular fractal dimension using the wavelet methods.
Data

- SDSS DR 13 BOSS: spatial distribution of galaxies.
- Mock Galaxy Catalogs were created using Particle Mesh Method with
  \[ \Omega_m = 0.274 \]
  \[ \Omega_\Lambda = 0.726 \]
  \[ \Omega_b = 0.046 \]

Source: SDSS
Galaxy clusters distribution

The number of galaxies decreased as Z increased
Work flow

SDSS Raw Data → RA and DEC → 2-D Signal Matrix

WPT and 2-d Power Spectrum → Radial Averaging

Log Plot of the Power Spectrum vs. Frequency

Hurst Exponent → Slope → Fractal Dimension
Fractal Dimension estimation

- Find the power spectrum

\[
var(d_j, k) = \frac{1}{2^j} \sum_{k=0}^{2^j-1} |d_{j, k}|^2
\]  

(1)

- The Hurst Exponent is estimated using the relation

\[
H = \frac{\alpha + 3}{2}
\]  

(2)

- The fractal dimension is calculated using the relation

\[
D = d + 1 - H
\]  

(3)
Creating the 2-d signal matrix

1. Pick Z with redshift bin width (Z=0.04)
2. Extract the RA and DEC coordinates
3. Count the number of galaxies
4. Populate a (256*256) matrix with galaxies

Raw SDSS data. RA, DEC, and Z
HealPix and SDSSPix

• Divide the sky into pixels of equal areas and equal sizes.
• Depending on the resolution, the higher the resolution, the larger the pixels.
• Using indexing schemes, each pixel is assigned 2 indices $i$ and $j$.
• The two indices $i$ and $j$ are converted into row and column indices, then populated into the two dimensional matrix.
• Populate the two dimensional matrix with galaxies at different redshifts.
HealPix and SDSSSPix
Wavelets

• Wavelets are a tool which is used in signal analysis.
• Localized in the time as well as the frequency domain.
• Operates by partitioning the signal into different sub-signals with frequency components mapped to coefficients having different energies.
• The transform operates like a microscope for detail examination.
Wavelets

Wavelet Transform:

\[ \sum_{j=1}^{2^N} s_j \phi(x) = \sum_{j=1}^{N} a_j \phi'(x) + \sum_{j=1}^{N} d_j \psi'(x) \]

Types:
1- Discrete Wavelet Transform
2- Wavelet Packet Transform

Discrete Wavelet Transform
Wavelet Packet Transform

\[ x = W_{0,0} \quad \text{Signal} \]

Low pass filter \[ \tilde{G}(f) \] \[ W_{1,0} \]

High pass filter \[ \tilde{H}(f) \] \[ W_{1,1} \]

\[ \tilde{G}(2f) \] \[ \tilde{H}(2f) \] \[ W_{2,0} \] \[ W_{2,1} \] \[ W_{2,2} \] \[ W_{2,3} \]

\[ \tilde{G}(4f) \] \[ \tilde{H}(4f) \] \[ W_{3,0} \] \[ W_{3,1} \] \[ W_{3,2} \] \[ W_{3,3} \] \[ W_{3,4} \] \[ W_{3,5} \] \[ W_{3,6} \] \[ W_{3,7} \]

Sweep

Source: Mathworks
Fitting a straight line to the log-log plot

Power law distribution
Hurst Exponent H

- Values between (0-1).
- (H= 0) implies surfaces of extreme irregularity.
- (H= 0.5) implies random process, i.e., Brownian motion.
- (H> 0.5 to H< 1) surfaces getting smoother.
- (H=1) smooth surface, i.e. homogenous or regular distribution.
Results

As $H$ decrease, the surface is more irregular.
Results

D is constant around 1.5, single fractal system.

Inhomogenous distribution, D < 3.
Conclusion

• We developed a wavelet based algorithm to test whether the large scale structure of the universe can be modeled as fractal systems or not, by calculating the angular fractal dimension.
• Galaxy clusters behave as a power law against cosmological distances.
• The fractal dimension is constant, and the galaxy clusters distribution is inhomogenous.
• The large scale structure is dominated by voids.
Thank You!

• Q&A


References


Additional slides
Data: SDSS

Using SDSS IV DR 13

Source: SDSS

Northern Galactic Cap

Southern Galactic Cap
SDSS Instrumentation

- SDSS makes use of ‘plug plates.’ These are aluminum plates, 32 inches in diameter, in which more than 1000 holes are strategically drilled.

- Each hole corresponds to a target object, and when the plate is assembled to the SDSS 2.5 meter telescope, 62” fiber-optic cables are plugged into each hole in the plate.

- During telescope operation, these fiber-optic cables then collect light from the target source and carry it to a spectroscopic analyzer.
SDSS Instrumentation

Plates