Generalized Slow Roll
and
CMB Constraints on the Inflaton Potential

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References:
Ordinary slow roll (brief background)

Technique for computing the initial curvature power spectrum for inflationary models where the scalar field potential is sufficiently flat and slowly varying.

\[
\epsilon_H \equiv \frac{1}{2} \left( \frac{\dot{\phi}}{H} \right)^2
\]

\[
\eta_H \equiv -\left( \frac{\dddot{\phi}}{H\dot{\phi}} \right)
\]

\[
\delta_2 \equiv \frac{\dddot{\phi}}{H^2 \dot{\phi}}
\]

Linked to the shape of the potential

Slow-roll parameters: \( \epsilon_H, \eta_H \ll 1 \) and slowly varying

Slow roll approximation: \( \Delta^2_R \approx \left( 1 - (2C + 1)\epsilon_H - C\eta_H \right) \frac{H^2}{2\pi|\dot{\phi}|} \approx 1 \)
Why go further?

- Glitches in the temperature power spectrum of the CMB have led to interest in exploring models with features in the inflaton potential.

- These models require order unity variations in the curvature power spectrum: slow-roll parameters are not necessarily small or slowly varying.
Inflationary features

A step in the inflationary potential generates an oscillatory initial curvature power spectrum.

The slow-roll parameters are *neither small nor slowly varying.*
INFLATIONARY POTENTIAL

POWER SPECTRUM

TEMPERATURE

OBSERVABLES

POLARIZATION
How does the Generalized Slow Roll approximation work?


- **Field equation:**
  \[
  \frac{d^2 y}{dx^2} + \left( 1 - \frac{2}{x^2} \right) y = \frac{g(\ln x)}{x^2} y
  \]
  \[(y = \sqrt{2k} u_k; x = k \eta)\]

- **Perfect slow roll:**
  \[
  \frac{d^2 y_0}{dx^2} + \left( 1 - \frac{2}{x^2} \right) y_0 = 0
  \]

- **GSR approximation:**
  \[
  \frac{d^2 y}{dx^2} + \left( 1 - \frac{2}{x^2} \right) y = \frac{g(\ln x)}{x^2} y_0
  \]

Source function (linear in slow-roll parameters)

Solution can be constructed with Green function technique.
How does the Generalized Slow Roll approximation work?

\[ \frac{d^2 y}{dx^2} + \left(1 - \frac{2}{x^2}\right) y = \frac{g(\ln x)}{x^2} y \]

\[ \frac{d^2 y_0}{dx^2} + \left(1 - \frac{2}{x^2}\right) y_0 = 0 \]

Source function (linear in slow-roll parameters)

Solution can be constructed with Green function technique.

**BUT...**

- **Nodes** in the power spectrum.
- Curvature is **not constant** for modes outside the horizon.
Generalized Slow Roll for Large Deviations

The curvature power spectrum only depends on a single source function:

\[
\ln \Delta^2_R(k) = G(\ln \eta_{\text{min}}) + \int_{\eta_{\text{min}}}^{\eta_{\text{max}}} \frac{d\eta}{\eta} W(k\eta)G'(\ln \eta)
\]

\[
+ \ln \left[ 1 + \frac{1}{2} \left( \int_{\eta_{\text{min}}}^{\eta_{\text{max}}} \frac{d\eta}{\eta} X(k\eta)G'(\ln \eta) \right)^2 \right]
\]

C. Dvorkin, W. Hu, PRD (2009)
Generalized Slow Roll for Large Deviations

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\]

C. Dvorkin, W. Hu, PRD (2009)

- Constant curvature for modes outside the horizon.
- Well controlled at large values of the source: percent level errors. (time integrals are small even if the source is not small everywhere).

- Simple to relate to the Inflaton Potential:

\[ G' \approx 3 \left( \frac{V_{,\phi}}{V} \right)^2 - 2 \left( \frac{V_{,\phi\phi}}{V} \right) \]
Second order Generalized Slow Roll: Well controlled

M. Mortonson, C. Dvorkin, H. V. Peiris, W. Hu, PRD (2009)

Polarization can test the feature hypothesis at 2.5 sigma with Planck, 5-6 sigma with CMBpol.
Second order Generalized Slow Roll: Well controlled

C. Dvorkin, W. Hu, PRD (2009)
We can use features in the power spectrum to directly constrain features in the potential.

- Power spectrum
- Source
- Potential
Model independent constraints (PC’s) (Work in progress)

*Principal components*: Basis for a complete representation of observable properties of the source function.

\[ G' = 1 - n_s + \sum_{a=1}^{N} m_a S_a(\ln \eta) \]

C. Dvorkin, W. Hu (2010, in preparation)

GSR approximation allows us to go *beyond specific inflationary models* and think of the data as directly constraining the source function.
WMAP7 constraints from MCMC’s (Work in progress)

As a by-product:
We parallelized the WMAP likelihood evaluation for multi-core systems and we improved its speed by \(~ 5 \times N_{core}\)

C. Dvorkin, W. Hu (2010, in preparation)
Summary

• The Generalized Slow Roll approximation is accurate at the percent level for order unity deviations in the CMB temperature and polarization power spectra.

• There is a single source function responsible for observed features and it is simply related to the local slope and curvature of the Inflaton Potential.

• Applications (work in progress):
  - Use this technique for model independent constraints on the Inflaton Potential: think of the data as directly constraining the source function.