# On The Contribution of Discrete Sources to the Background Radiation


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With

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# Outline

I. Estimating the Background Radiation

II. The Luminosity Function and its Evolution

III. Data

IV. Luminosity-Luminosity Correlations

V. Determining Multivariate Distributions

VI. Some Results

# *I. Two Methods*

# *From Source Counts*
# *or*
# *Luminosity Function*

# I. Estimating the Background Radiation

A. Use directly source counts (so-called Log *N*-Log *S*)

Given the differential or cumulative source counts

$$n(f)df \quad \text{and} \quad N(f) = \int_f^\infty n(f')df'$$

then the contribution of sources with fluxes >*f* is

$$I(>f) = \frac{1}{4\pi}\int_f^\infty f' n(f')df' = \frac{1}{4\pi}\left(\int_f^\infty N(f')df' + fN(f)\right)$$

The total counts requires extrapolation to lower fluxes

$$I_{\rm tot} = \lim_{f\to 0} I(>f) = \int_0^\infty f' n(f')df' = \int_0^\infty N(f')df'$$

But: *Extrapolation uncertain; no physics to guide*

Also: *Spectral variation introduces a new dimension*

*See Singal, Petrosian and Ajello, 2012*

# I. Estimating the Background Radiation

B. Use the luminosity function and its evolution $\Psi(L, z)$

$$I_{\rm tot} = \frac{1}{4\pi}\int_0^\infty \rho(z)\frac{dV(z)}{dz}\int_{L_{\rm min}}^\infty \frac{L}{4\pi d_L^2}\Psi(L,z)dL$$

Here $L_{\rm min}$ is the minimum luminosity often not known:
This also requires uncertain extrapolations to regions of phase space not fully covered by observations

But: *there may be some physics to guide*

Also: Clearly a multivariate problem; optical properties that are needed to get redshift come into play. So we deal with *at least a tri-variate distribution;*

*e.g. for radio background* $\Psi(L_{\rm opt}, L_{\rm rad}, z)$

# *II. Finding Distributions*

# *Importance of Correlations*

# What Do We Need To Do

1. Data Requirements

Need a Large Sample of Sources with known

a. Redshifts

b. Optical and Radio Fluxes: *spectral indicies*

c. A Cosmological Model: *to calculate luminosities*

$$L = 4\pi d_L^2(\Omega_i, z) f / K(z)$$

d. Well Defined Selection: *e.g. Robust Flux Limits*

# What Do We Need To Do

2. Sounds Methods to Determine The Tri-Variate Distribution $\Psi(L_{\rm opt}, L_{\rm rad}, z)$

This requires Determination of

a Correlation between the Luminosities

b. Radio and Optical Luminosity Evolutions

c. Co-moving Density Evolution

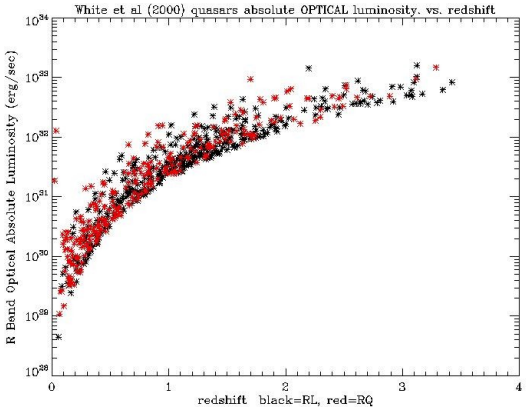
# *III. DATA*

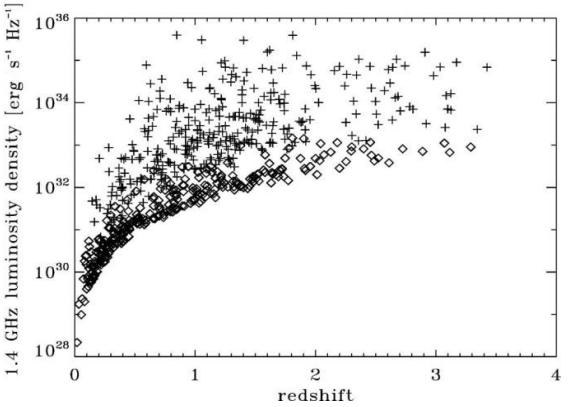
# *Radio and Optical Flux Limited Samples*

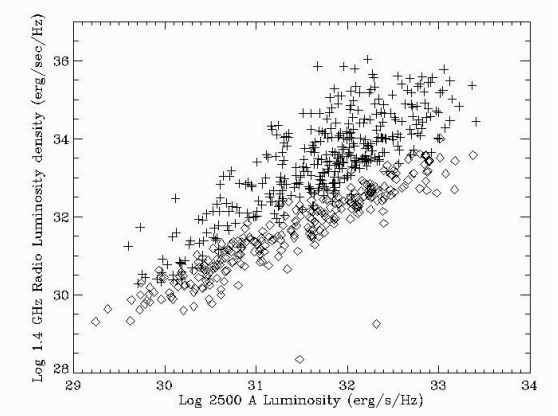
POSS-I, SDSS and FIRST

# II. Data: *Luminosities and Redshift*

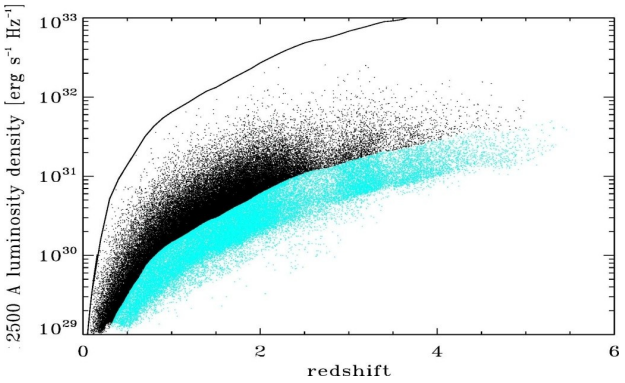
FIRST x POSS-I (636 sources)

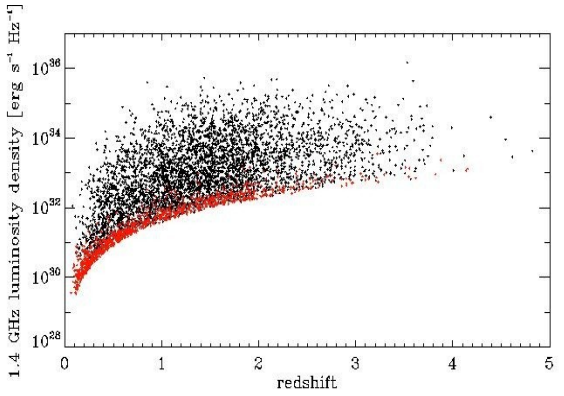


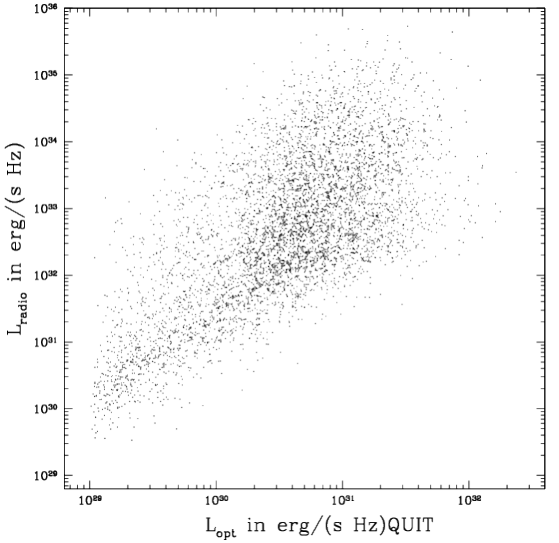




SDSSXFIRST (5445 sources)



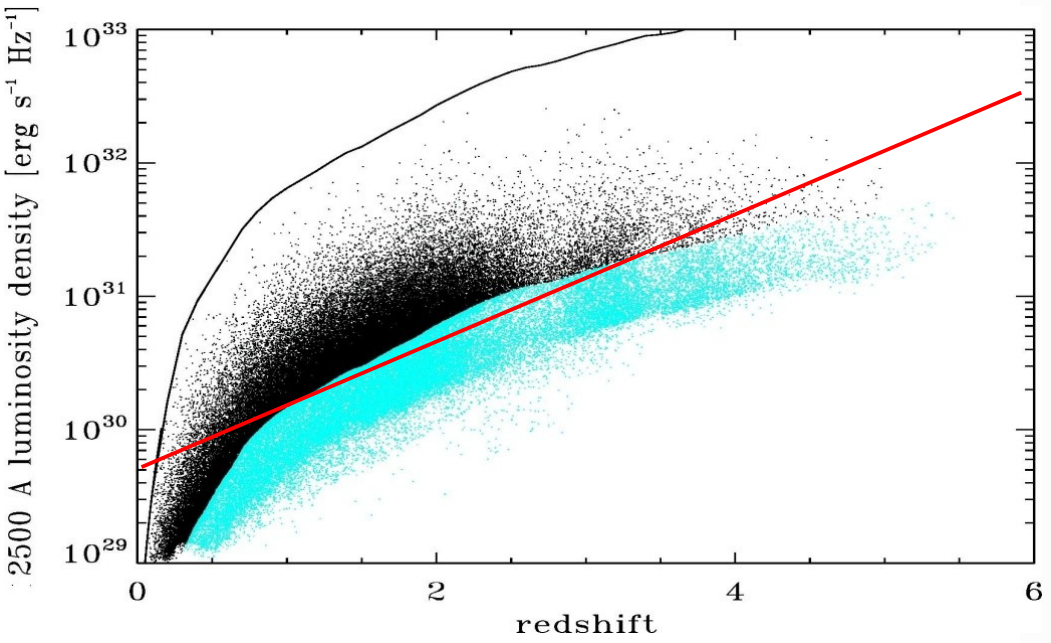



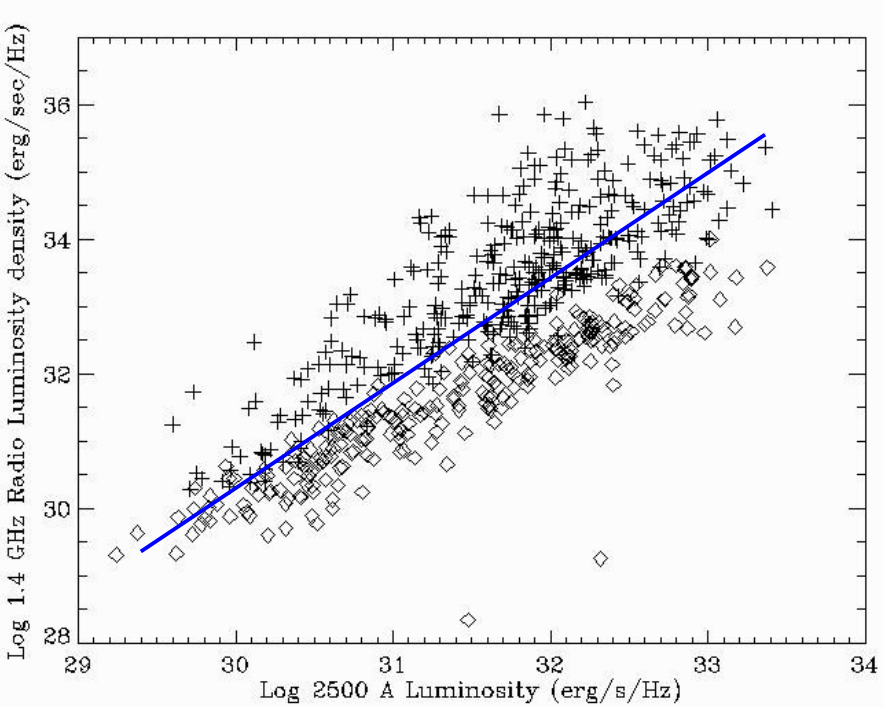
# Observed Correlations

There are clear and strong correlations

*Lo-z* correlation



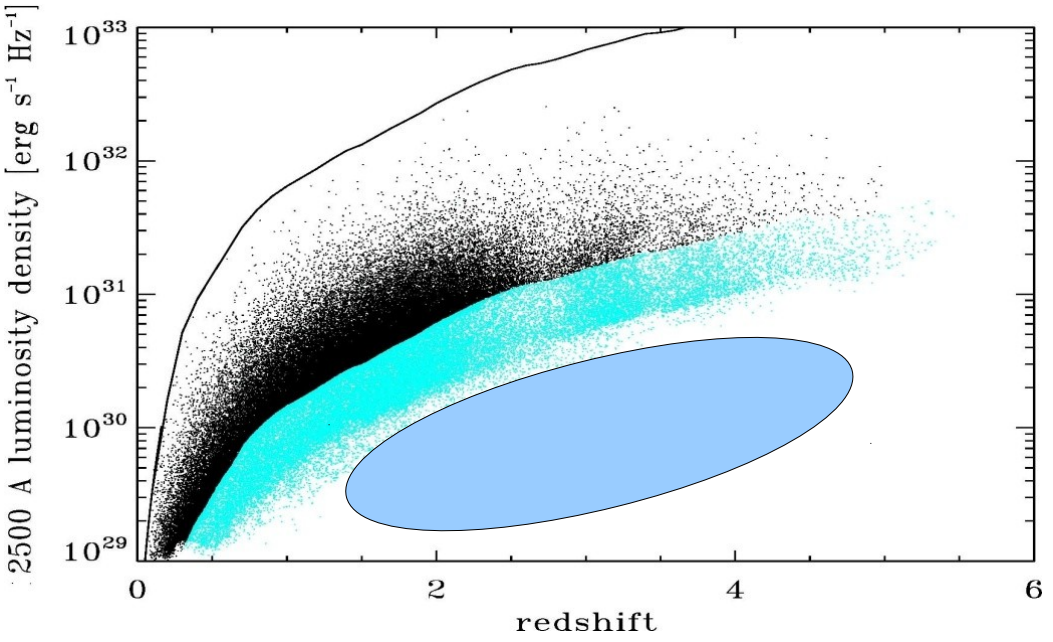

*Lo-Lr* correlation



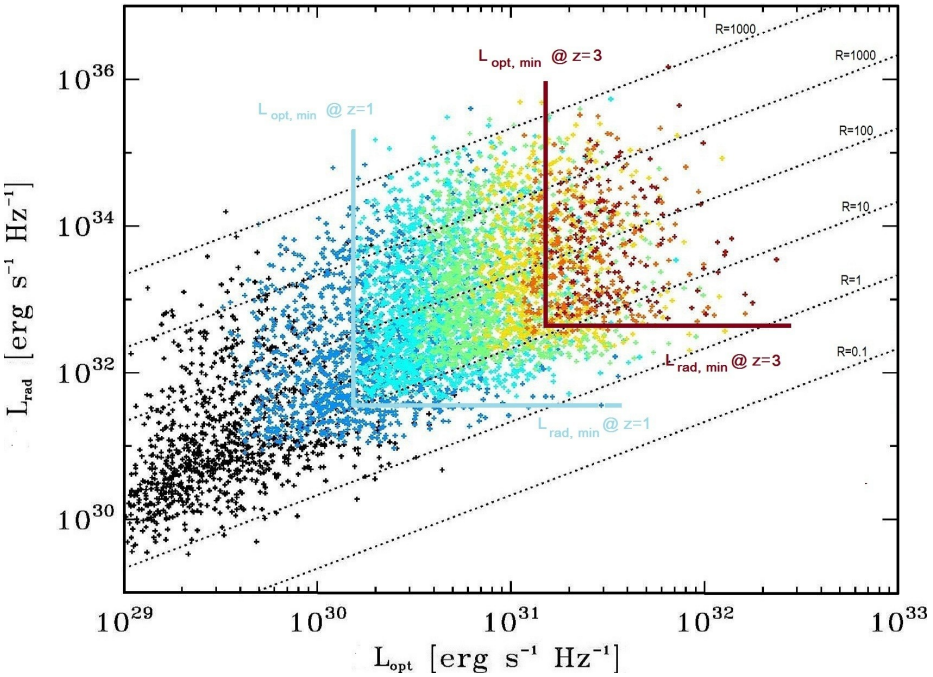
# Induced Correlations

## *Selection Effects and Redshift*

*Lo-z* correlation




*Lo-Lr* correlation



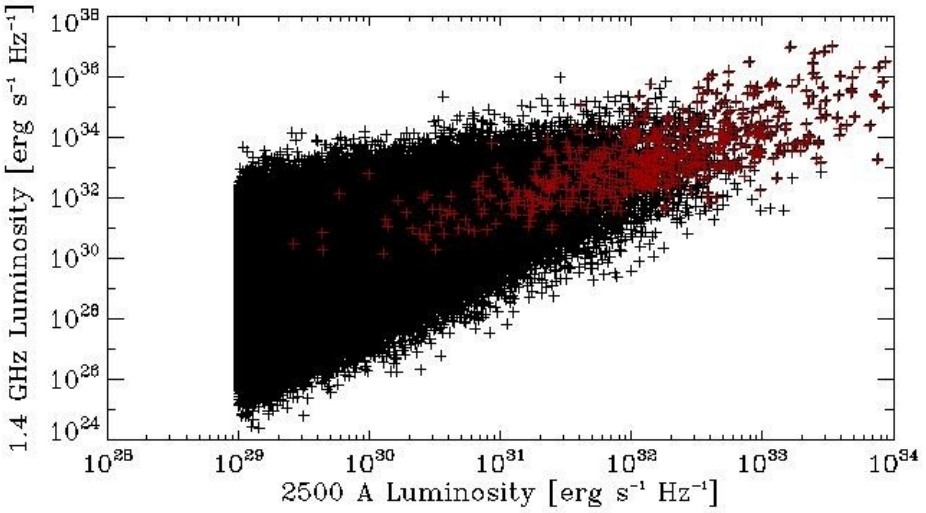
# *IV. Luminosity-Luminosity Correlation*

## *Simulations and Some Preliminary Result*

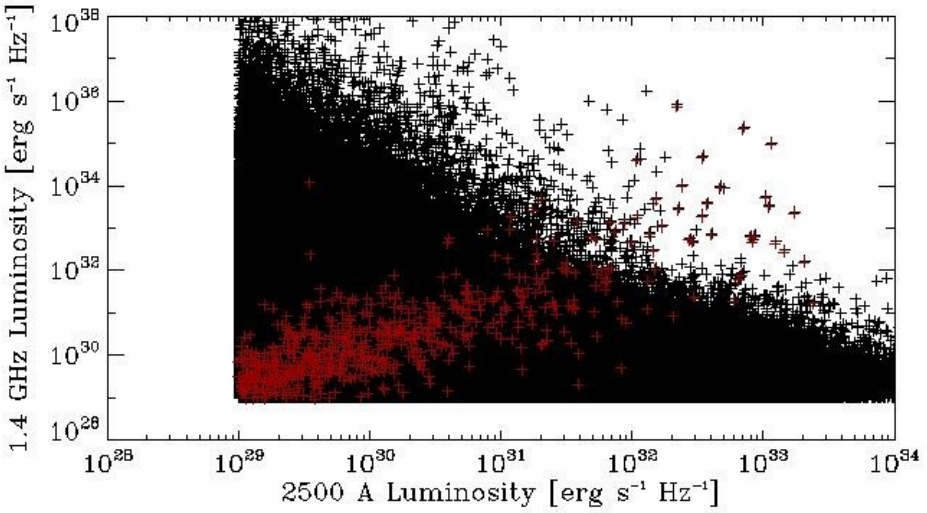
# 1. Luminosity-Luminosity Correlation

*Some preliminary results; simulations*

*correlated*




*uncorrelated*



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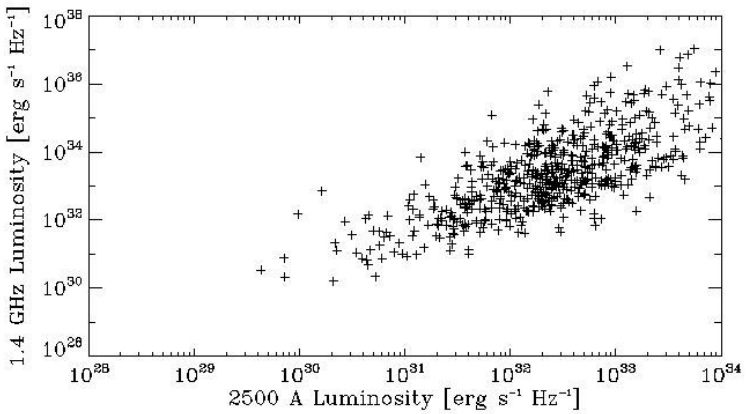
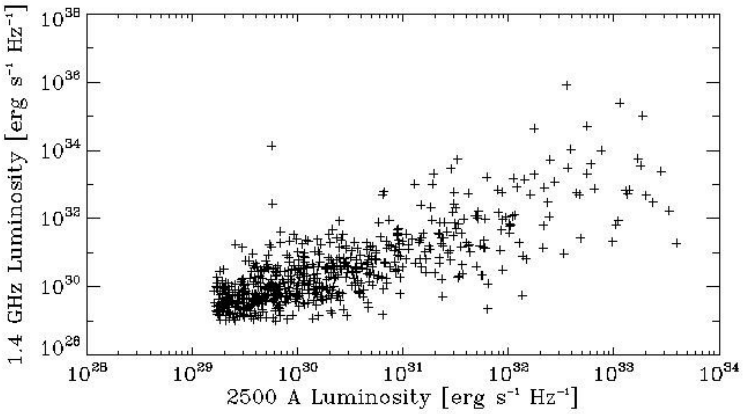



*uncorrelated*



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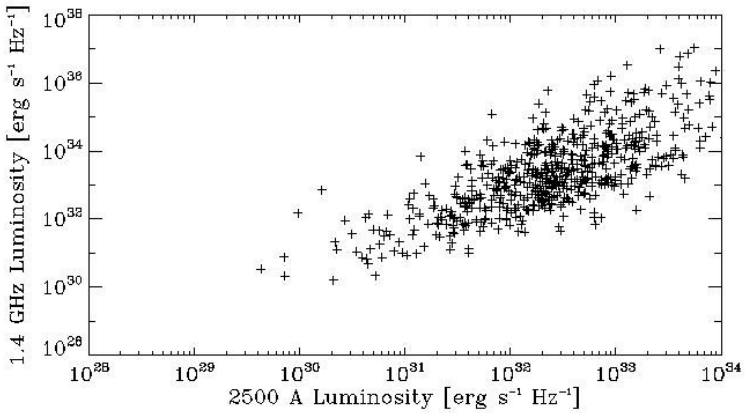


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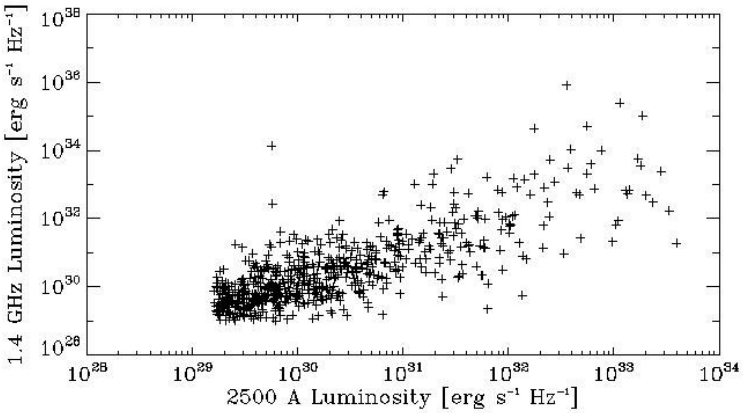


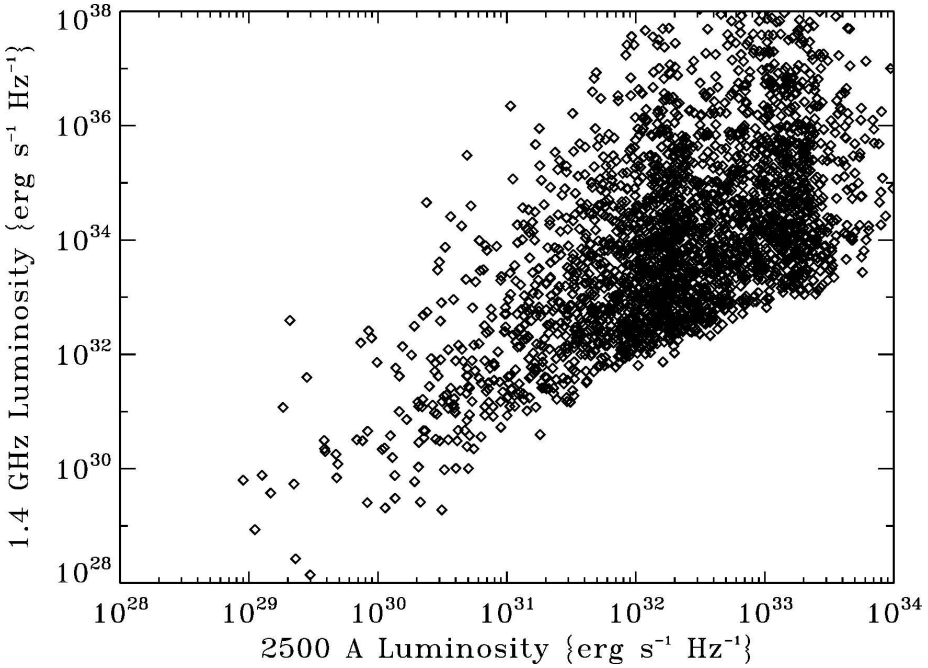

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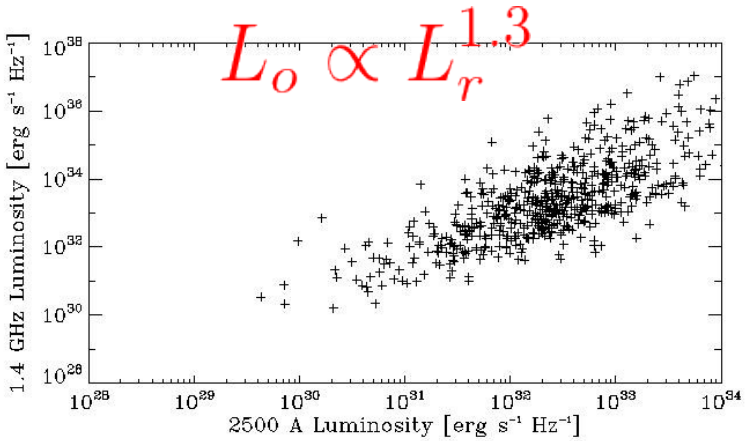


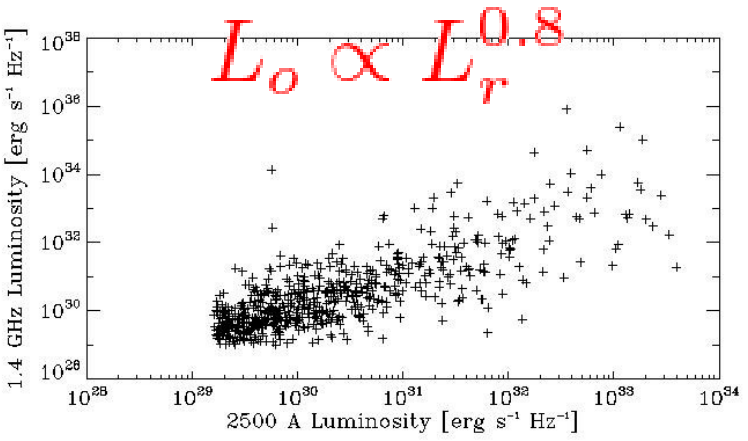


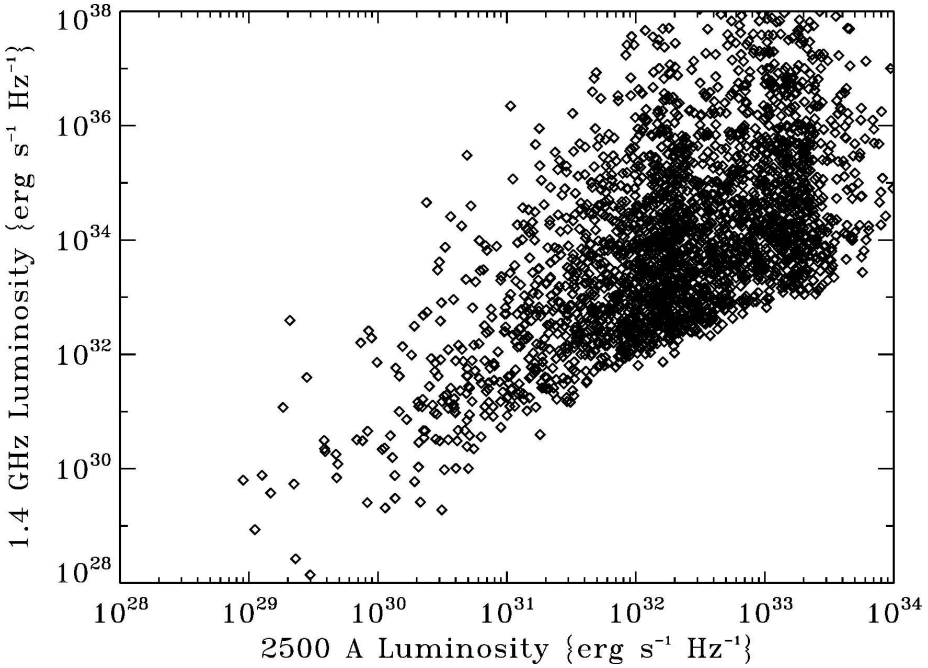

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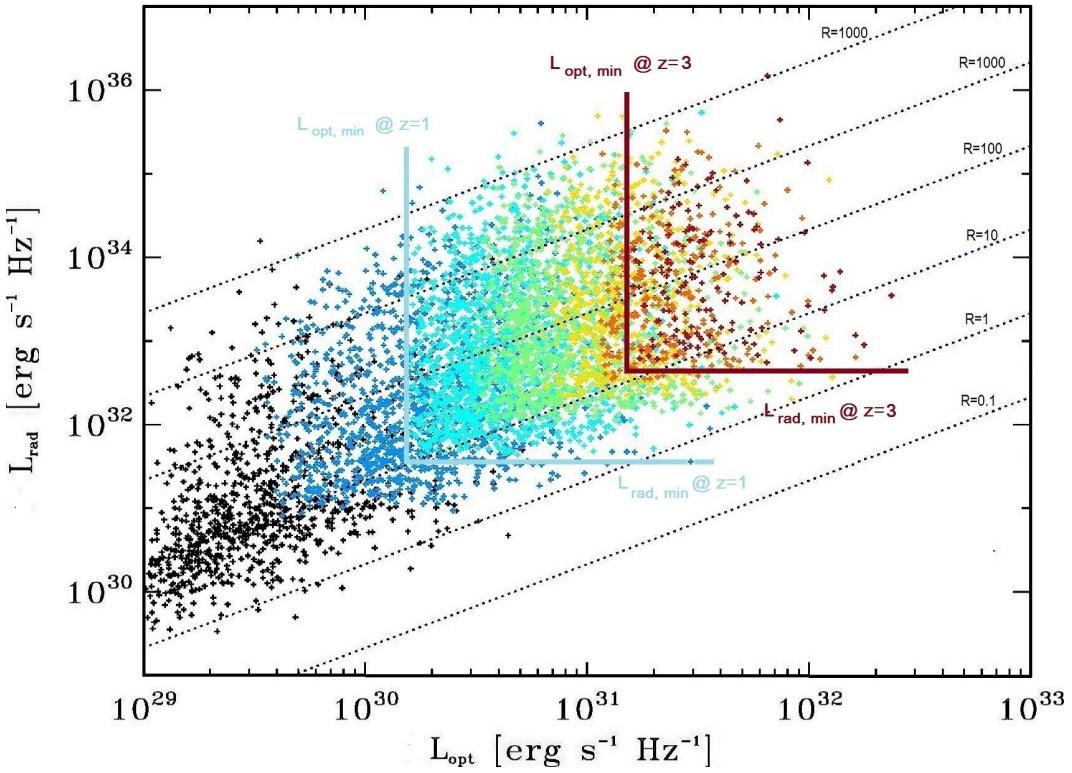


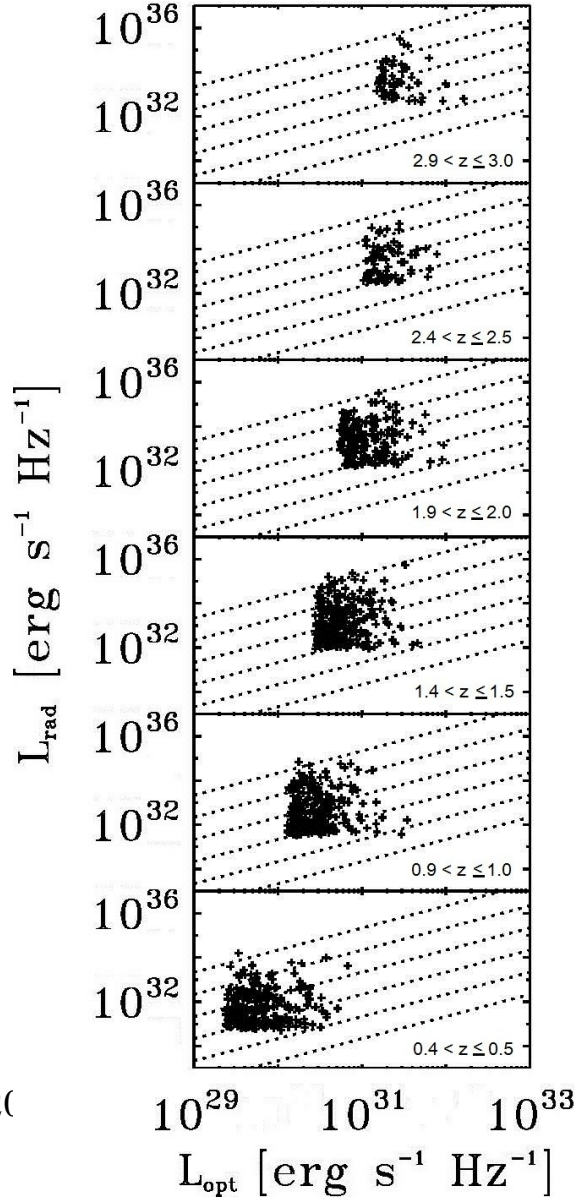

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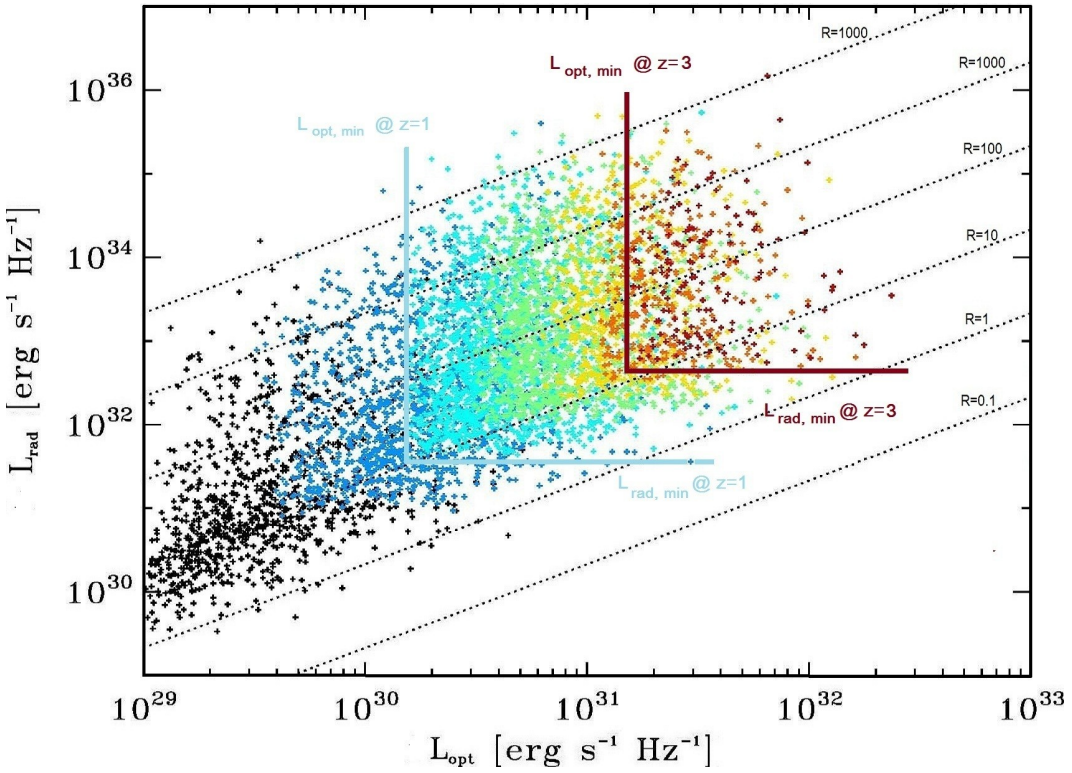


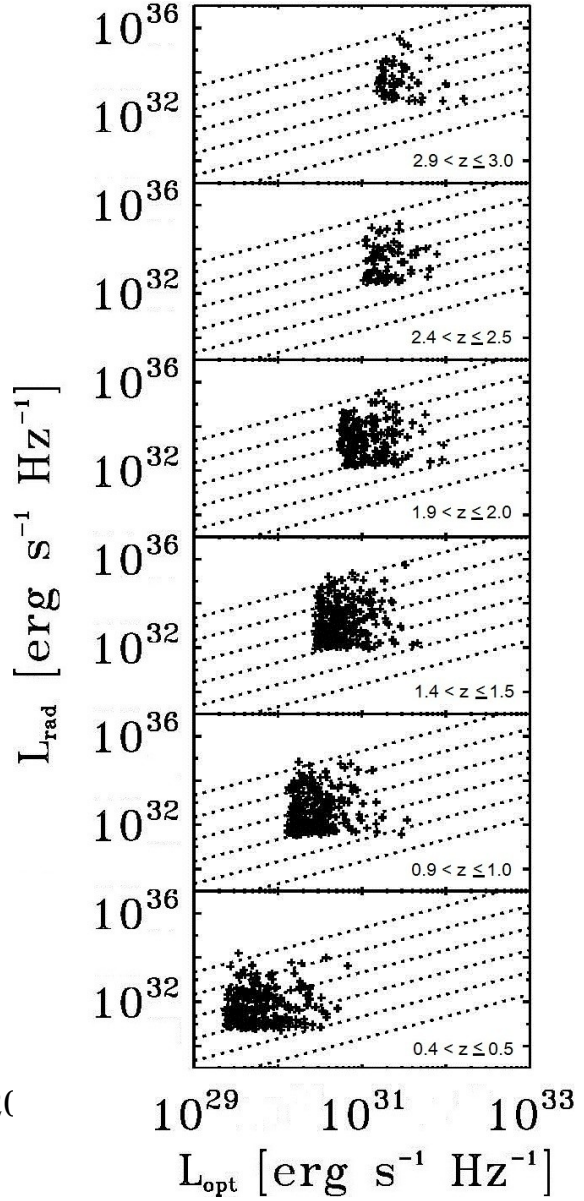

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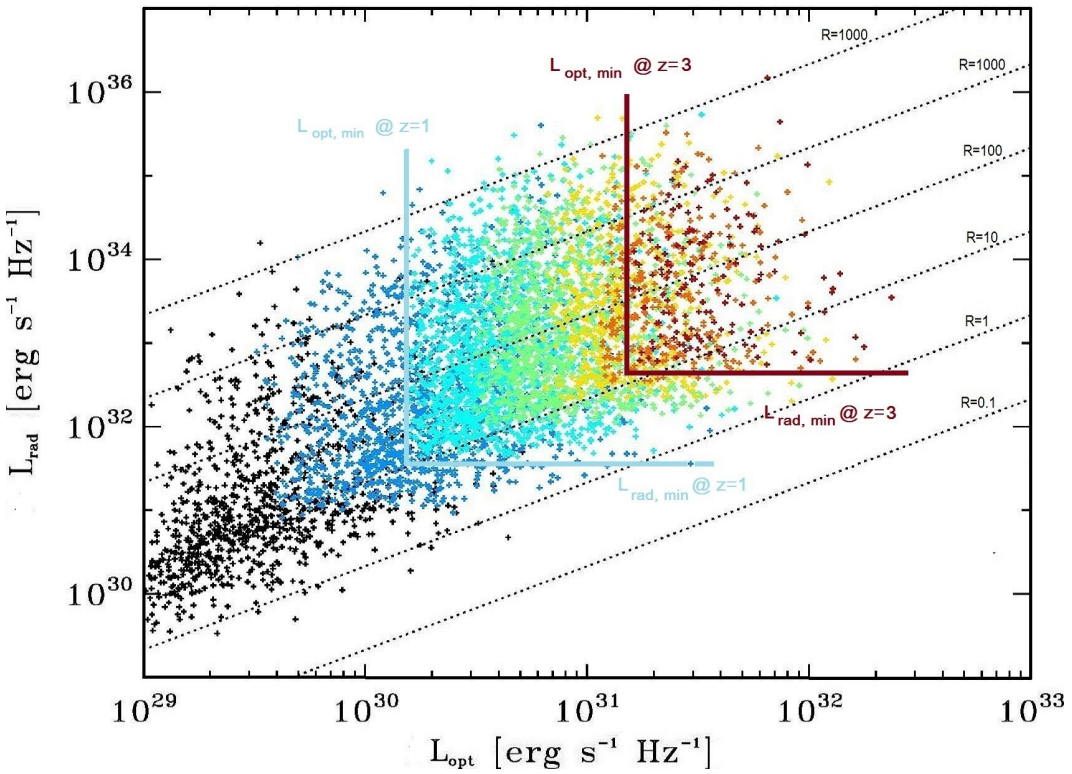
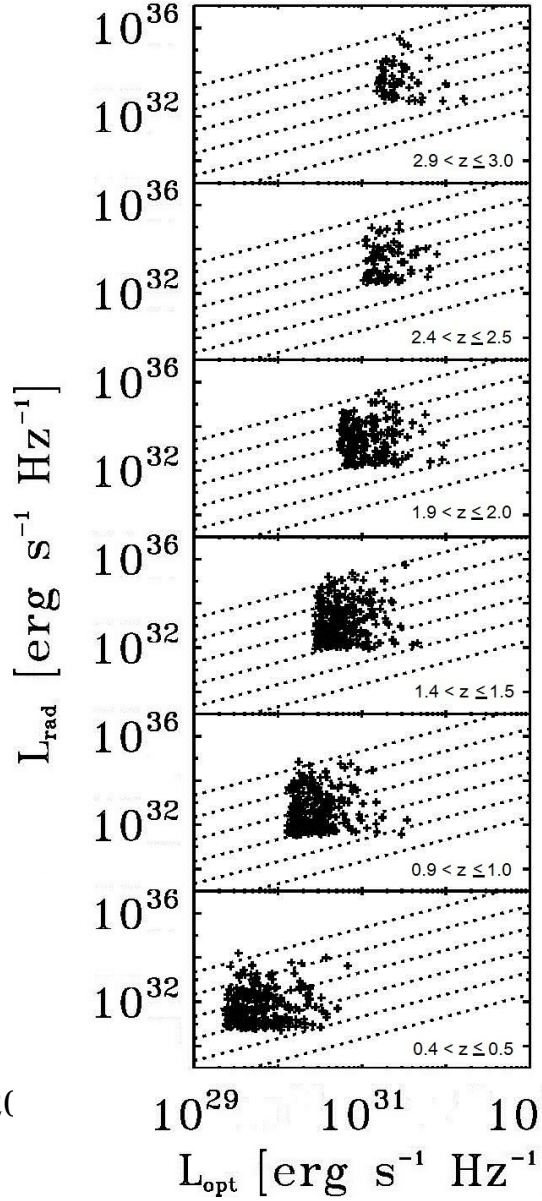



After correcting for redshift effect

*Correl. Coef. of whole sample = 0.11*




*Correl. Coef.*

-0.134

0.068

0.110

-0.003

0.270

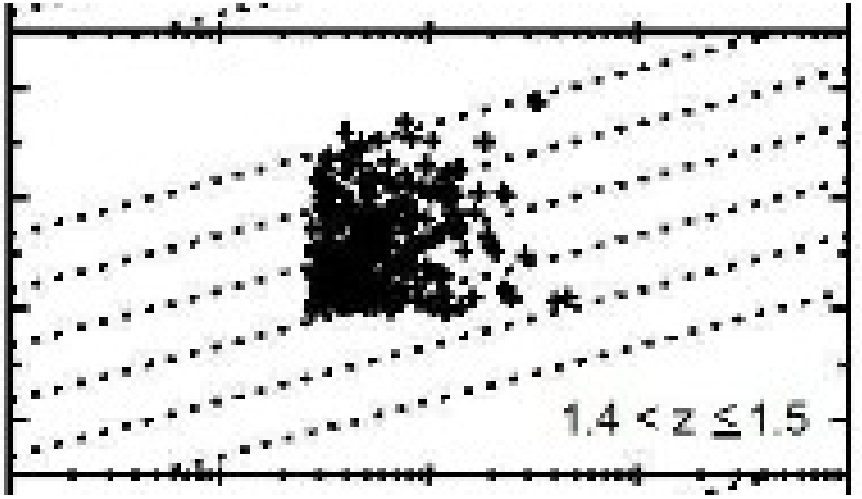
0.127

# 1. Luminosity-Luminosity Correlation Effects of truncation

Appears that we are seeing a corner of the distribution

Log-normal distribution with the center outside the observed range

Efron and Tibshirani 1996 Annals of Statistics



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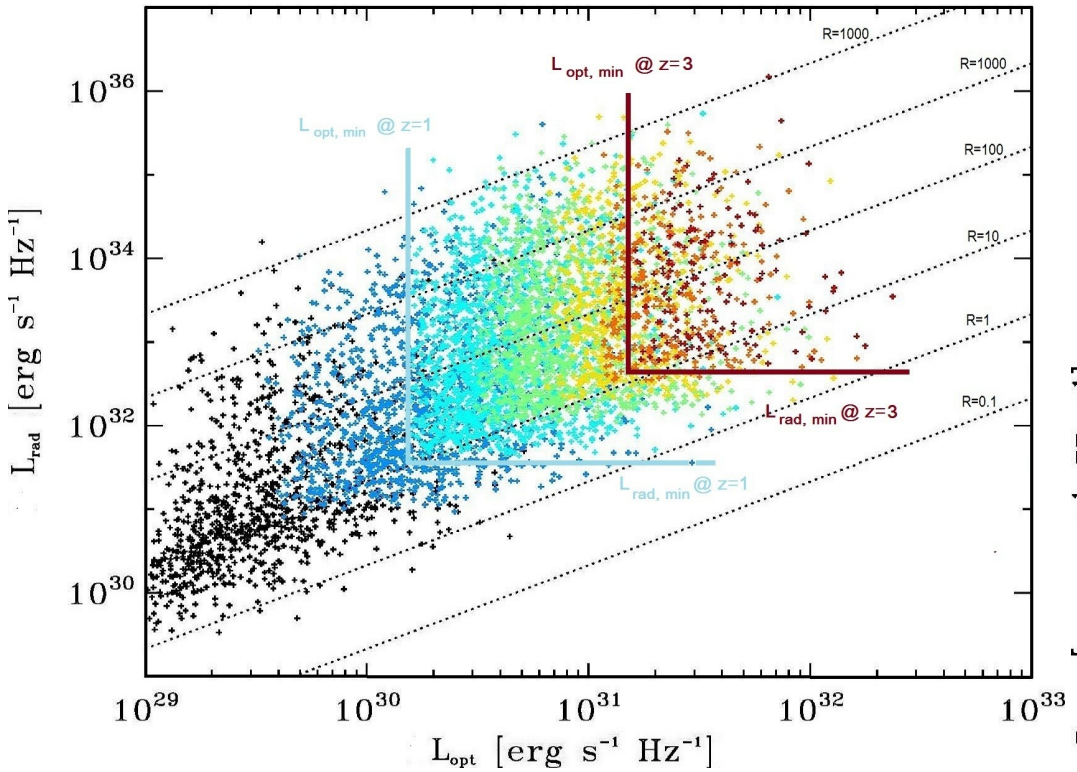


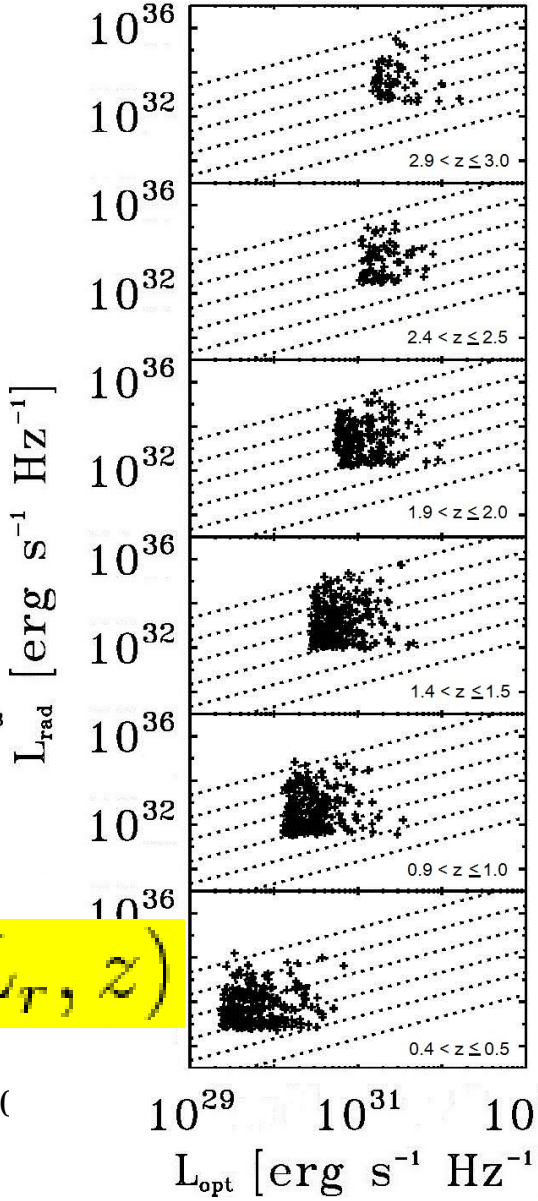


*Correl. Coef.*

-0.134

0.068

0.110

-0.003

0.270

0.127

After correcting for redshift effect

$$\Psi(L_o, L_r, z) = \psi(L_o, z)\psi(L_r, z)$$

# *V. Luminosity and Density Evolutions*

## *Published Results*

Singal, Petrosian et al. 2011,2012

## *based on*

## *Non-parametric Maximum Likelyhood Method*

Efron and Petrosian 1992,1994, 1999

# 1. Luminosity -Redshift Correlation OR *luminosity evolution*

This is usually done by forward Fitting:

a. Assume parametric form for Luminosity Function and Evolution

b. Fit to data to find best fit parameter values

*Not unique*

(especially when more than few parameters)

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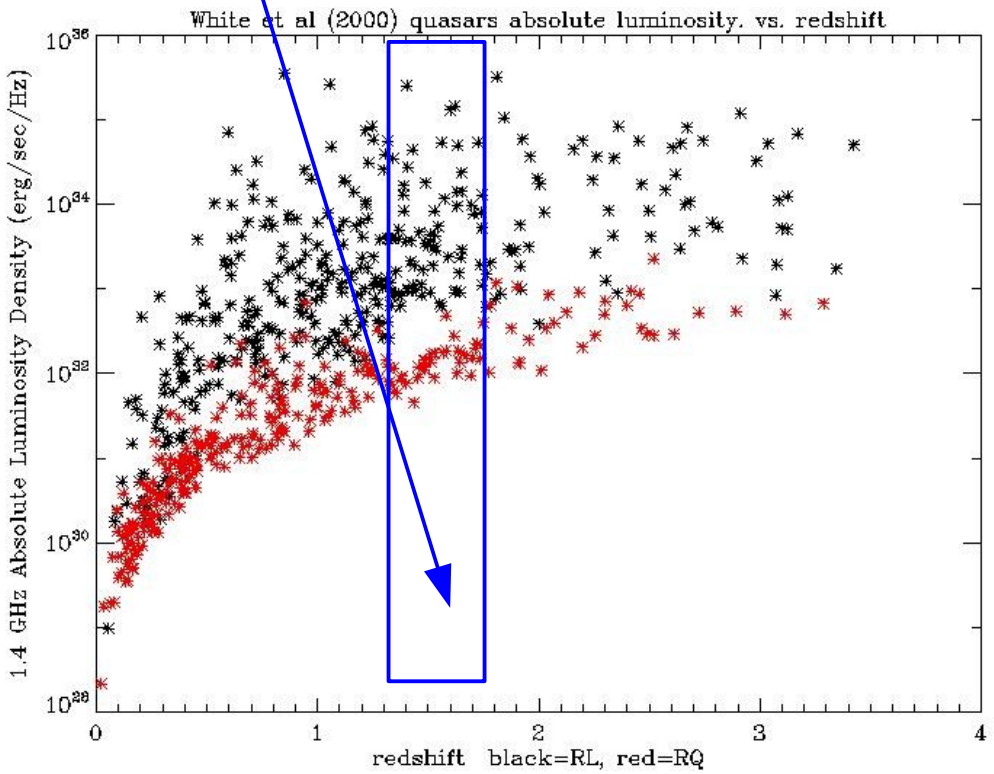
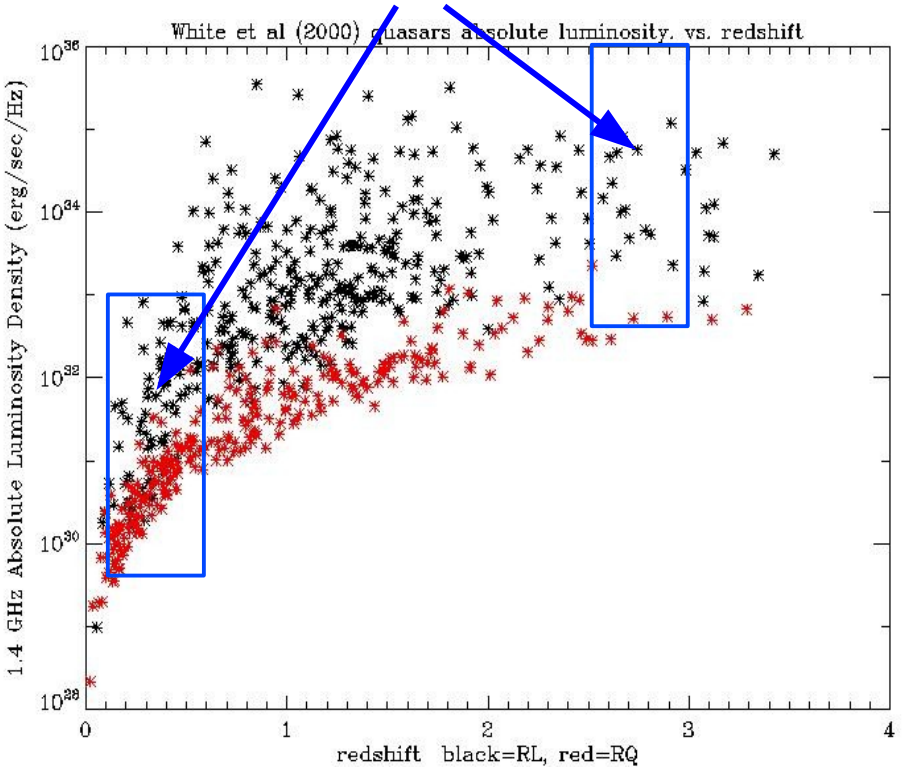
# 1. Luminosity -Redshift Correlation OR *luminosity evolution*

Binning:

*Empty Bins*




*with little overlap*



# 1. Luminosity -Redshift Correlation OR *luminosity evolution*

## Non-Parametric Methods:

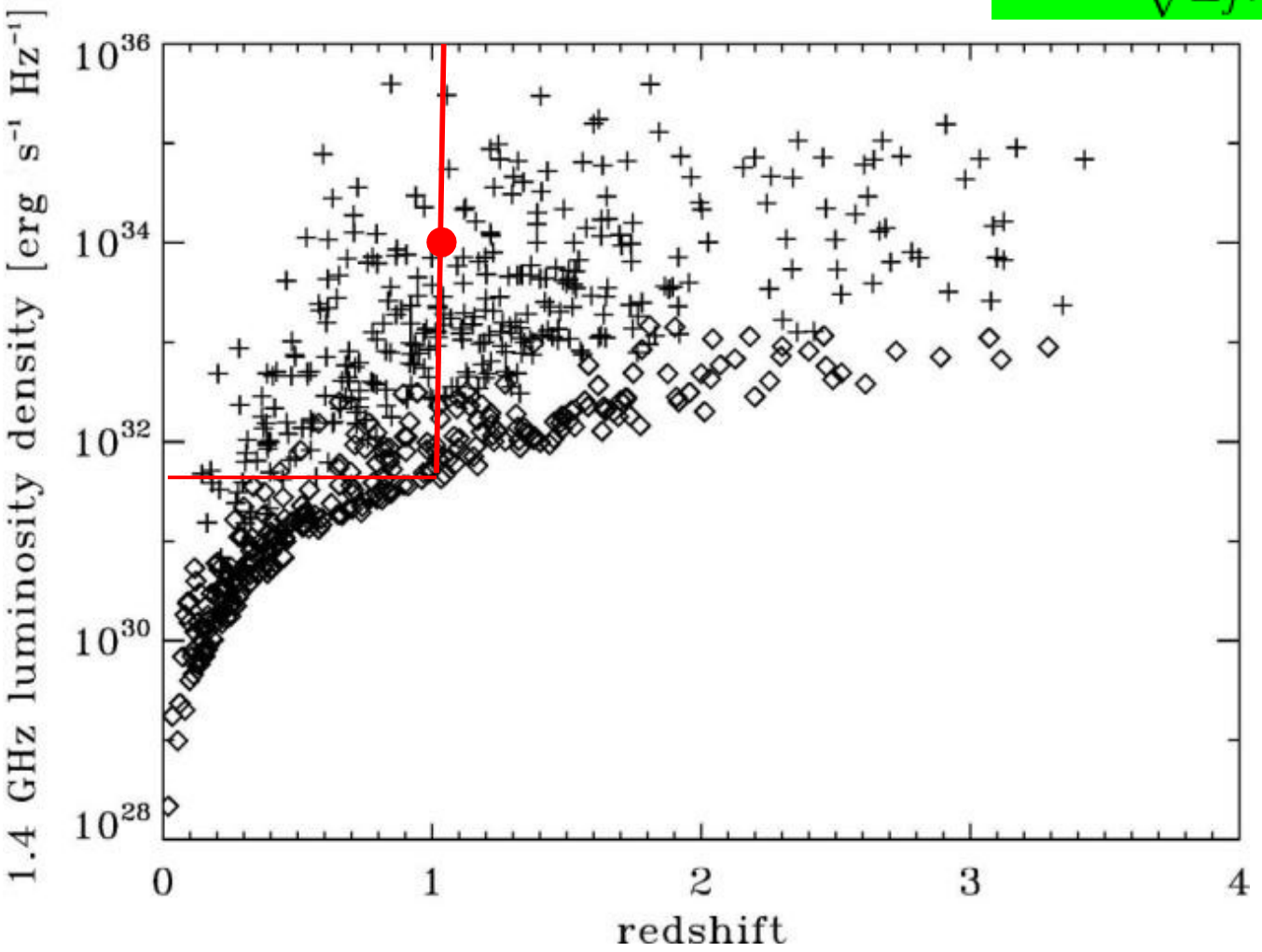
Efron & Petrosian 1992, 1994, 1999

Test for independence
Remove correlation if any

# Test of Independence

Spearman Rank Order Test: Distribution of Ranks $R_j$

Kendall's tau Statistic $\tau = \frac{\Sigma_j (R_j - E_j)}{\sqrt{\Sigma_j V_j}}$



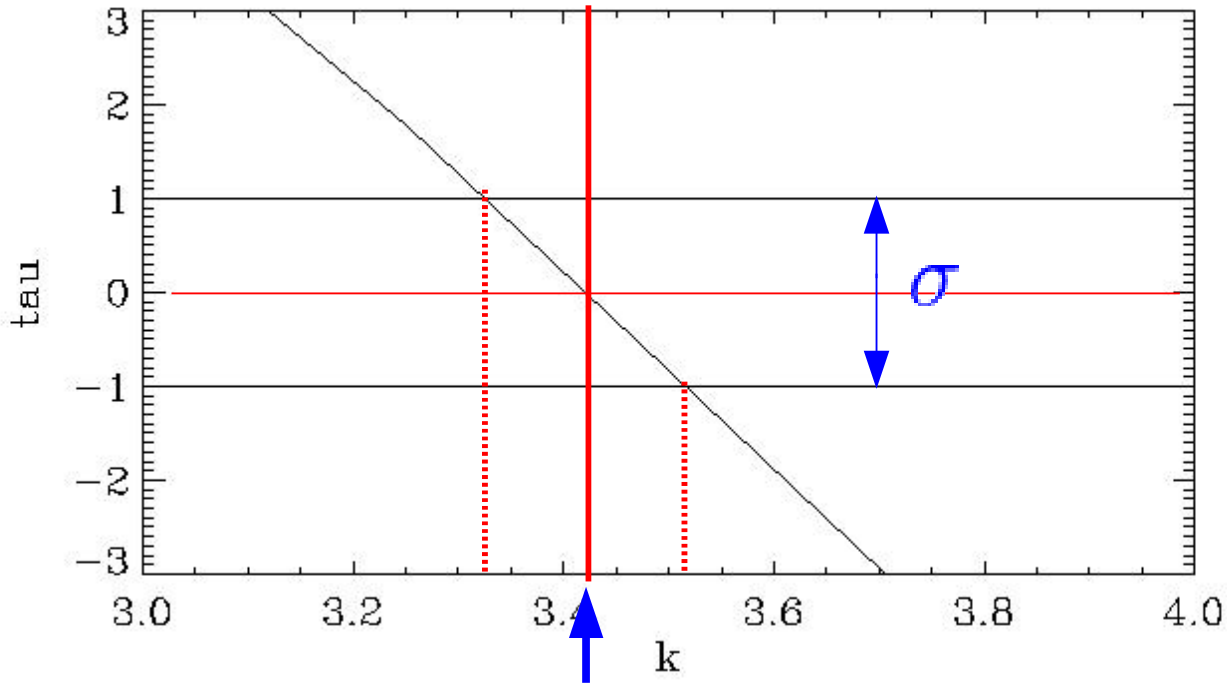

*associated set*

# Test of Independence

Remove the correlation by a variable transformation e.g.

$$L' = L \times g(z)$$

$$g(z) = (1+z)^k$$



# 3. Luminosity Function and Density Evolution

Determine mono-variate distributions of independent variables

*i.e. The Luminosity Function and Density Evolution*

$$\psi(L') \quad \text{and} \quad \rho(z)$$

# 2. Luminosity Function and Density Evolution

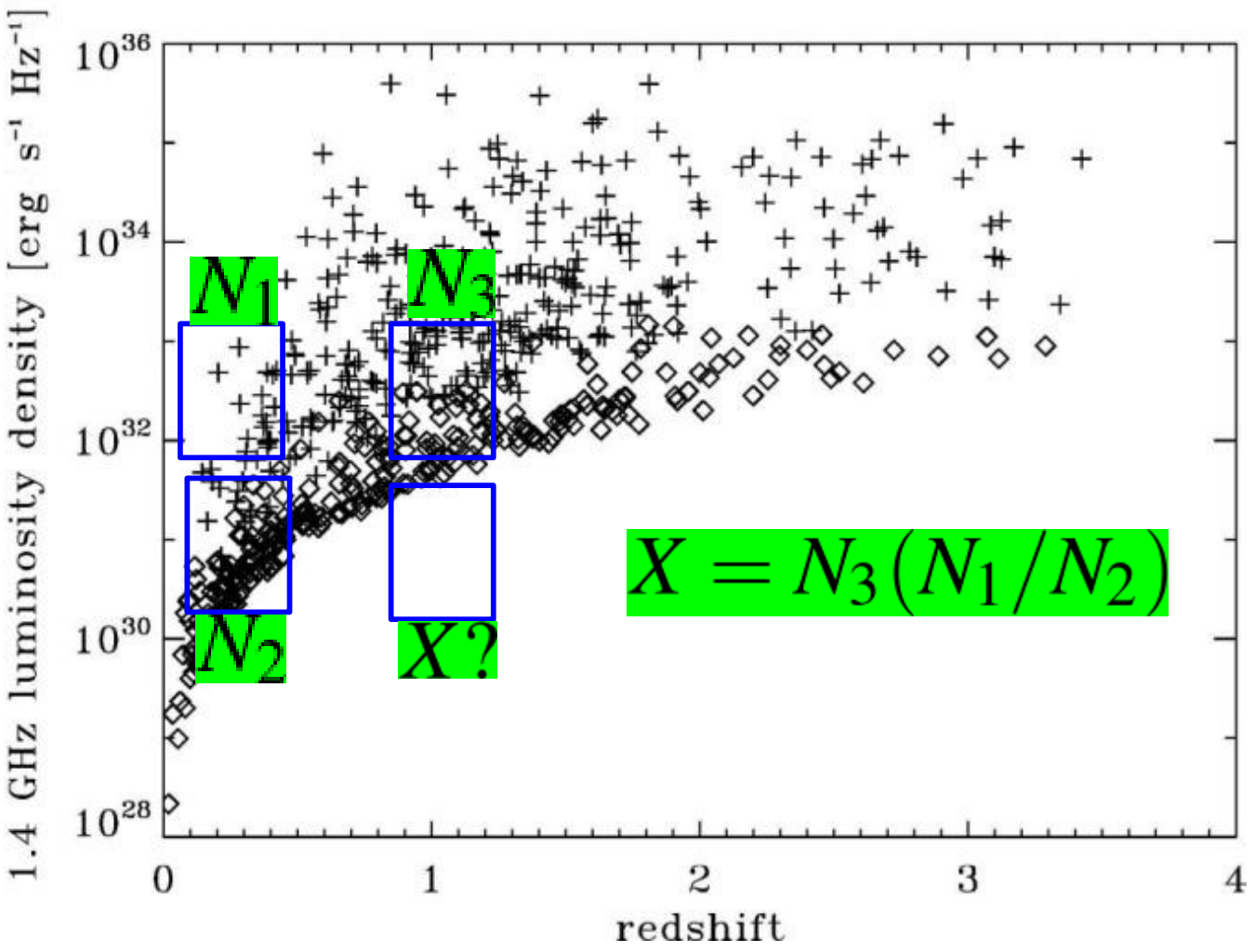
Determine mono-variate distributions of independent variables

*i.e. The Luminosity Function and Density Evolution*

$$\psi(L') \quad \text{and} \quad \rho(z)$$

# Must account for truncation

*given uncorrelated or independent variables*

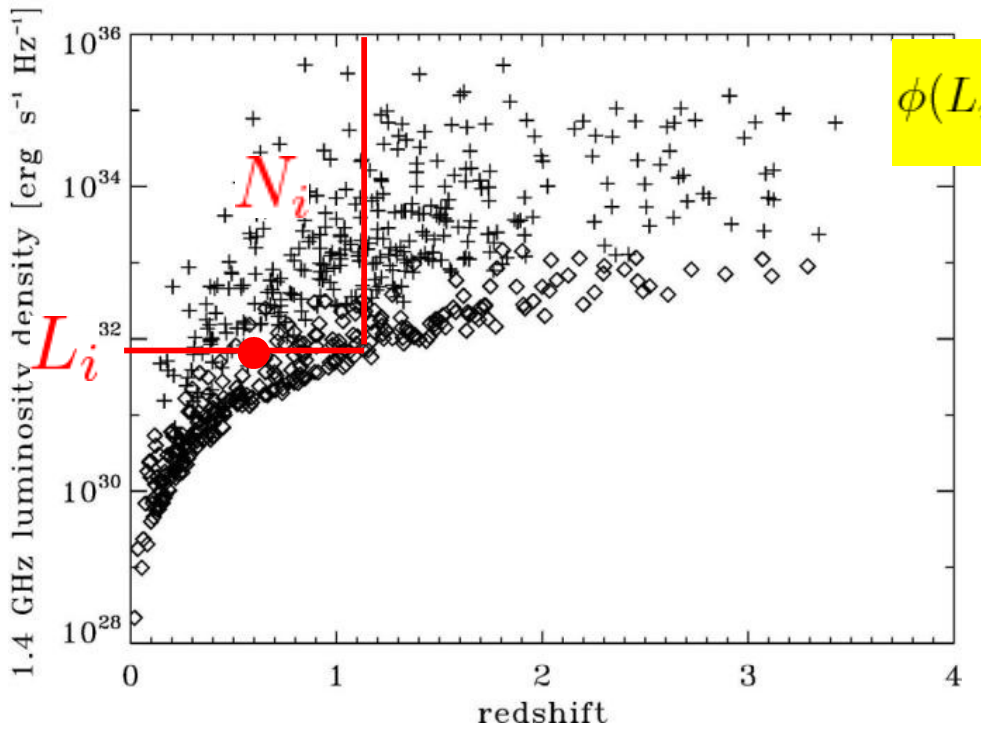



*Petrosian, 1993*

# Must account for truncation

## *If uncorrelated or independent*

$N_i$ *The associated set of* $L_i$




$$\phi(L_i) = \int_{L_i}^{\infty} \psi(L')dL' = \phi(L_1) \times \Pi_{j=1}^{i}(1 + 1/N_j)$$

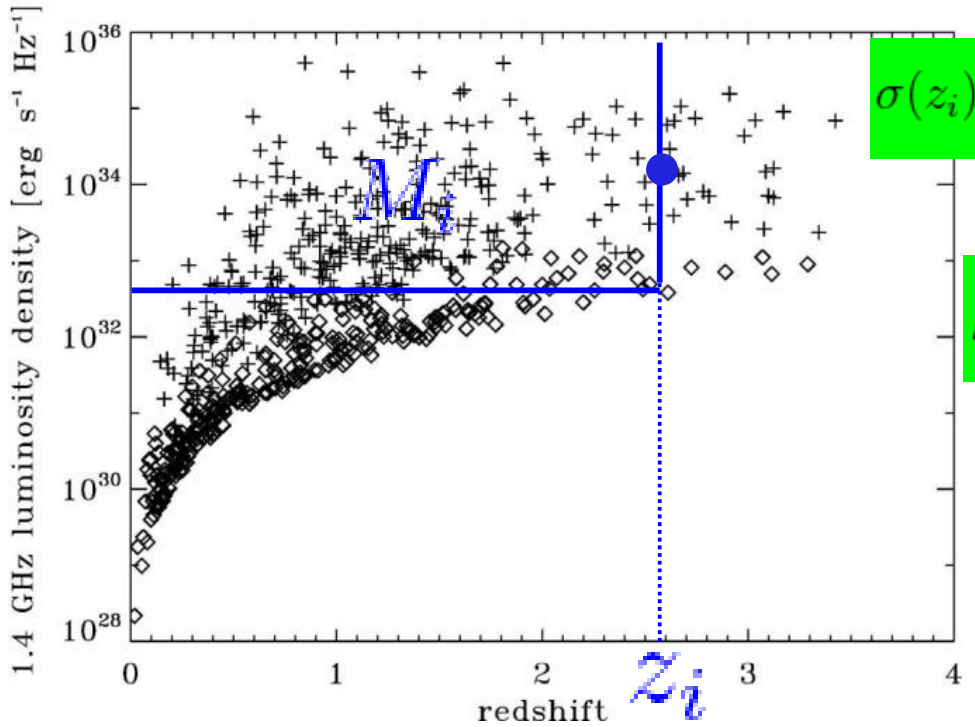
$$\psi(L_i) \sim \frac{\phi(L_i)}{L_{i-1} - L_i} \times \frac{1}{N_i} \quad \text{when} \quad N_i \gg 1$$

*Petrosian, 1993*

# Must account for truncation

*If uncorrelated or independent*

$M_i$ *The associated set of* $z_i$




$$\sigma(z_i) = \int_0^{z_i} \rho(z)(dV/dz)dz = \sigma_1 \Pi_{j=1}^{i}(1 + 1/M_j)$$

$$\rho(z_i) \sim \frac{dz}{dV}\frac{\sigma(z_i)}{z_i - z_{i-1}} \times \frac{1}{M_i} \quad \text{when} \quad M_i \gg 1$$

*Petrosian, 1993*

# *VI. Application to SDSS X FIRST Sources*

Schneider et al., 2010, *AJ*, 139, 2360 (i mag < 19.1; 65,000 quasars)
Becker et al. 1995, *ApJ*, 450, 559 (flux $_{1.4\ GHz}$>1 mJy; 300,000 objects)
Joint quasars 5,445

# RESULTS

Step 1. $L_{\rm rad} - L_{\rm opt}$ Correlation:

In our case, $L_{opt}$ and $L_{rad}$ are observed to be correlated.
Is this intrinsic or induced by redshift dependence and evolution?

a. Assume no intrinsic $L_{\rm rad} - L_{\rm opt}$ correlation

b. Assume the correlation induced by redshift

Define $L' = L_{\rm crr} = L_{\rm rad} \times (L_{\rm opt}/L_0)^{-\alpha}$

Find $\alpha = 1.2 \pm 0.1$

# RESULTS

Step 2. Luminosity Evolution

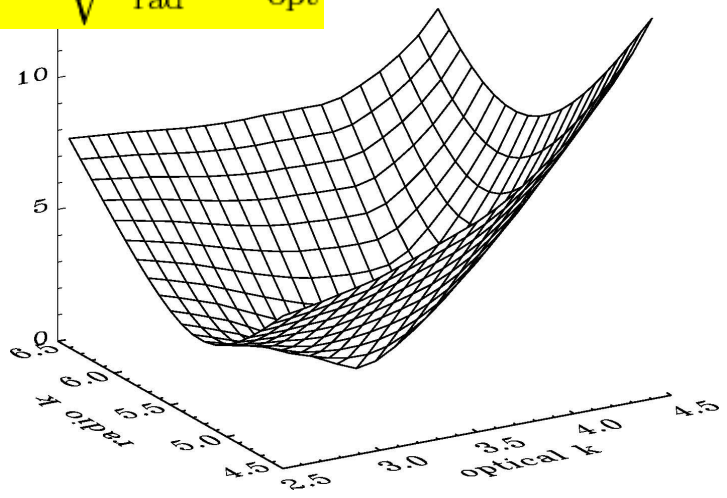
$$L' = L \times g(z)$$

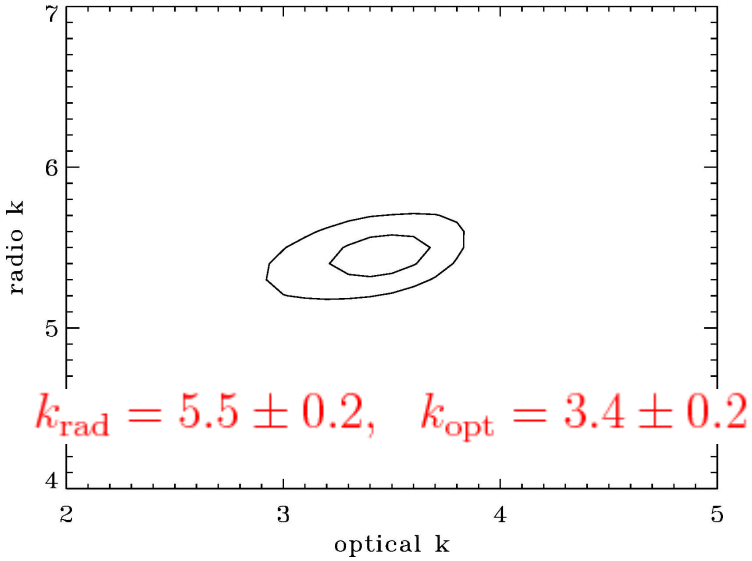
$$g(z) = (1+z)^k \text{ or } g(z) = \frac{(1+z)^k}{1+\left(\frac{1+z}{1+z_c}\right)^k}$$

But we have tri-variate function $\Psi(L_{\rm opt}, L_{\rm rad}, z)$

And two evolutions parameters $k_{\rm rad}$ and $k_{\rm opt}$

$$\tau = \sqrt{\tau_{\rm rad}^2 + \tau_{\rm opt}^2}$$

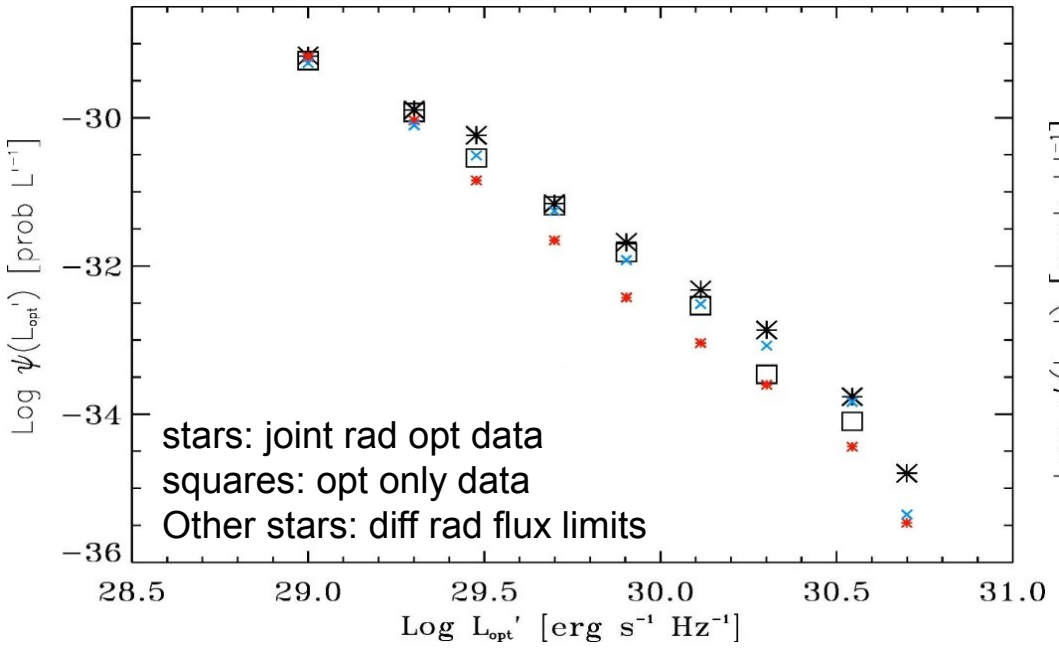


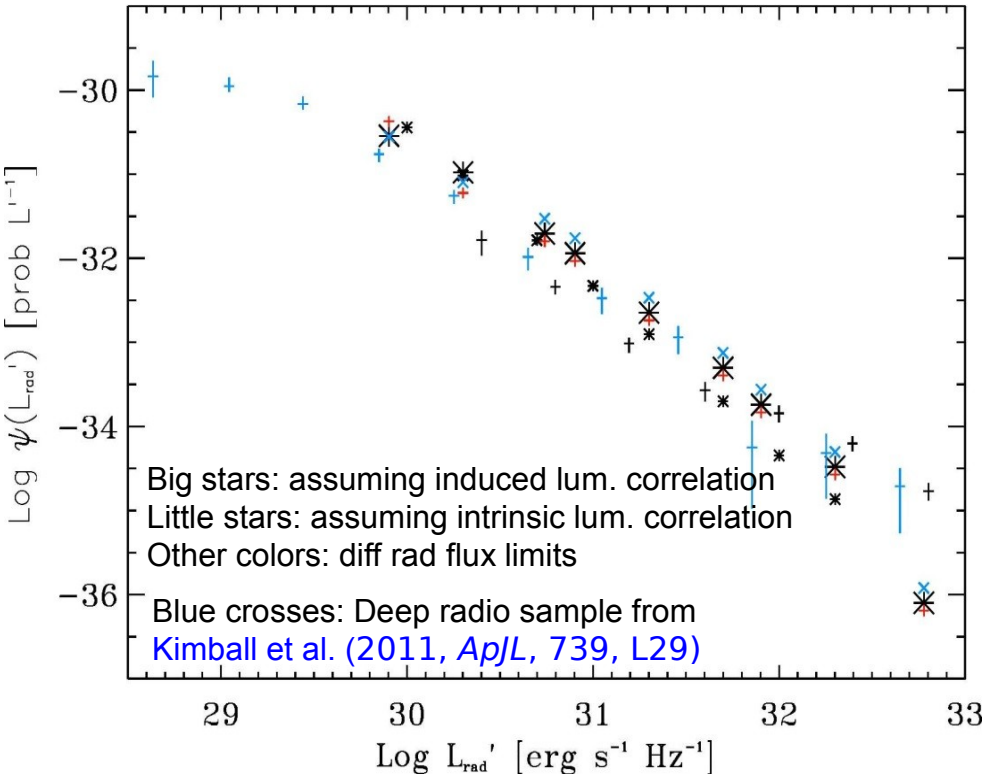




$k_{\rm rad} = 5.5 \pm 0.2, \quad k_{\rm opt} = 3.4 \pm 0.2$

# VI. RESULTS

## Step 3. Local Luminosity Functions



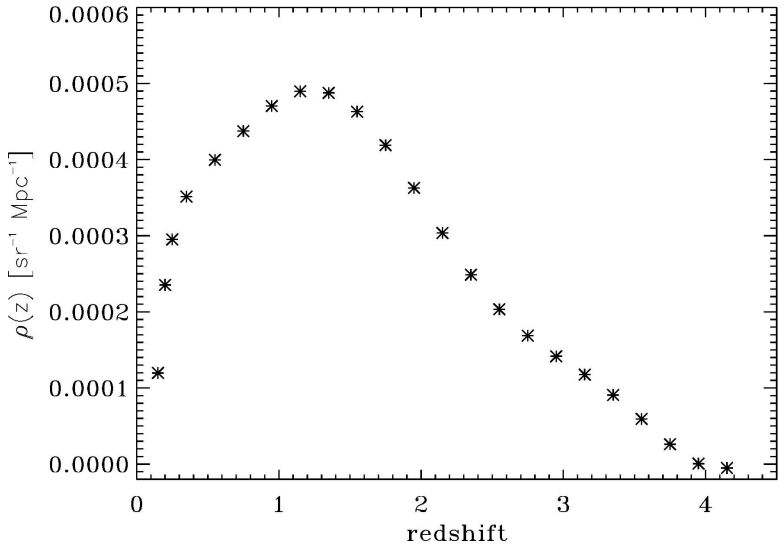




Reasonable agreement with
Boyle et al. 2000, *MNRAS*, 317, 1014

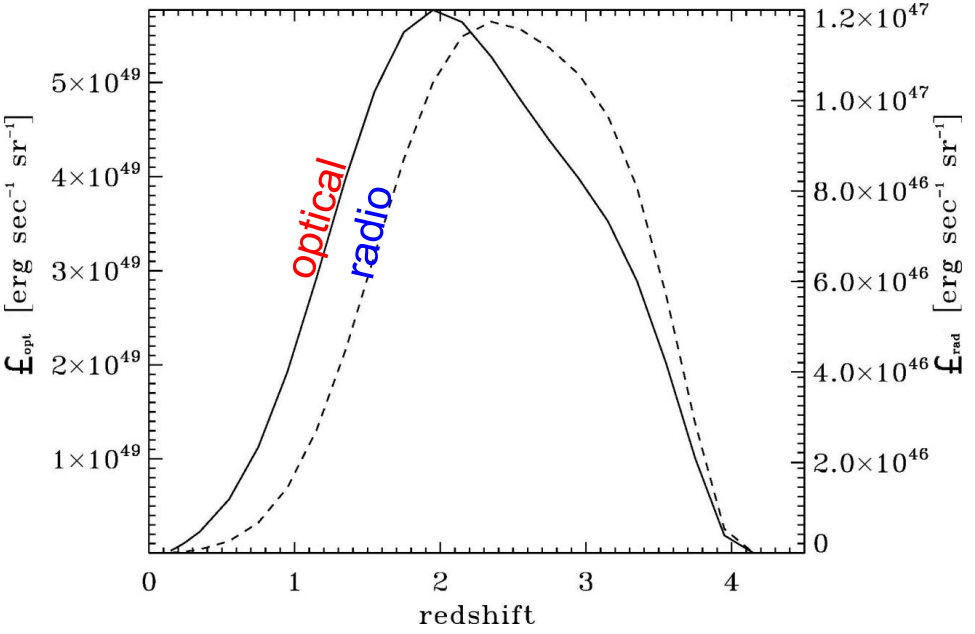
# VI. RESULTS

## Step 4. Co-moving Number Density and "Light" Evolution

$\rho(z)$




$\mathcal{L}(z) = \langle L \rangle g(z) \times \rho(z)(dV/dz)$



# SUMMARY and CONCLUSIONS

$$I_{\rm tot} = \frac{1}{4\pi}\int_0^\infty \rho(z)\frac{dV(z)}{dz}\int_{L_{\rm min}}^\infty \frac{L}{4\pi d_L^2}\Psi(L,z)dL$$

1.

# What Do We Need To Do

2. Use Sounds Methods to Determine The Tri-Variate Distribution $\Psi(L_{\rm opt}, L_{\rm rad}, z)$

Not difficult if variables are *uncorrelated or independent*

$$\Psi(L_o, L_r, z) = \psi_o(L_o)\psi_r(L_r)\rho(z)$$

Thus we need to

a. Determine the correlations; *Lo-Lr, Lo-z, Lr-z*

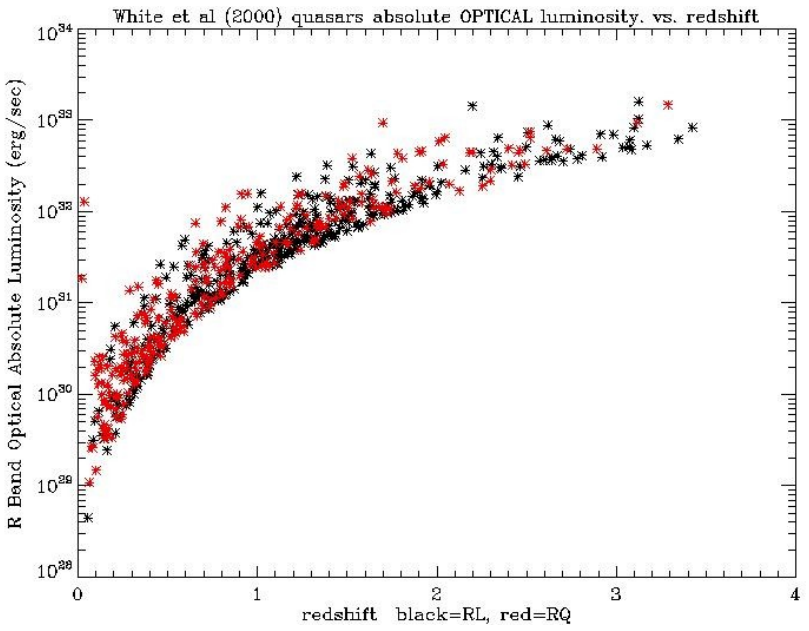
b. Remove these correlation: *via variable transformation*

c. Determine single luminosity functions and the co-moving density evolution

# II. Data: *Optical Luminsity and Redshift*

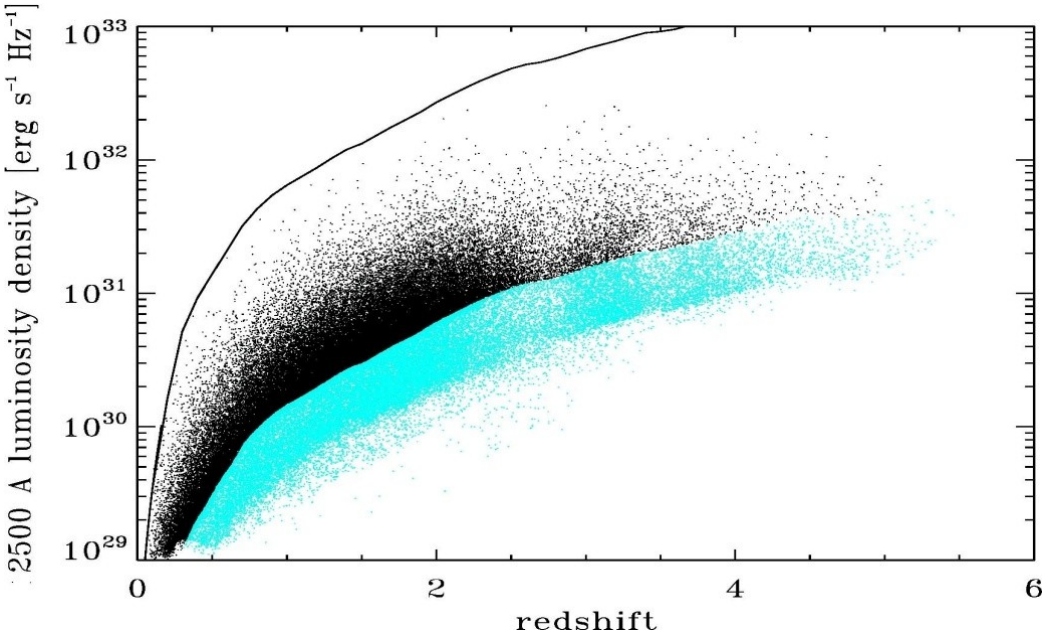
POSS-I (636 sources)

White et al. (2000)



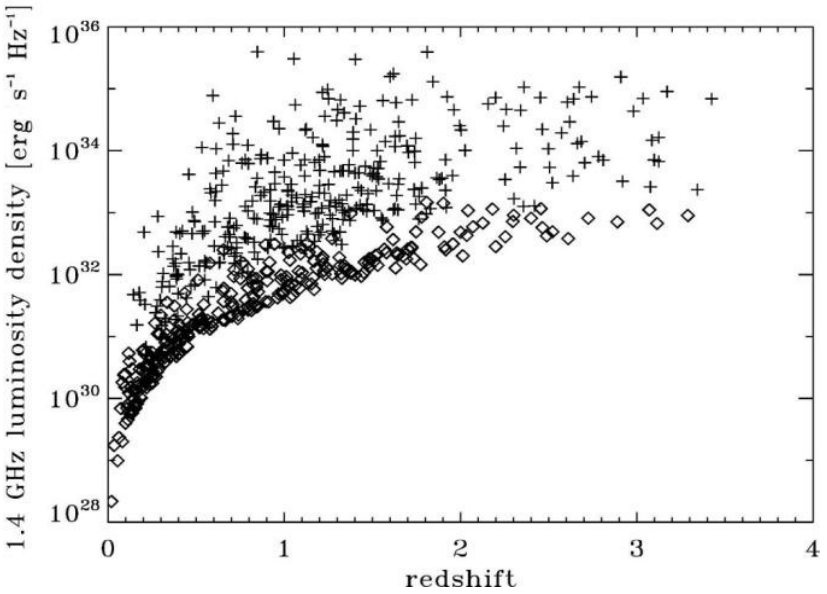

SDSS DR-7 (105 ksources)

Abazajian et al. (2009)

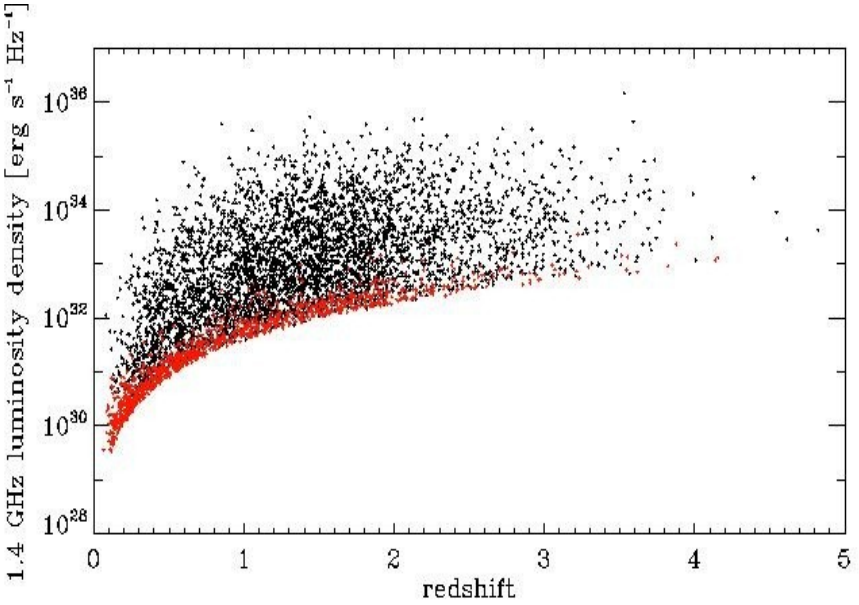


# II. Data: *Radio Luminsity and Redshift*

FIRST x POSS-I (636 sources)




SDSSXFIRST (5445 sources)

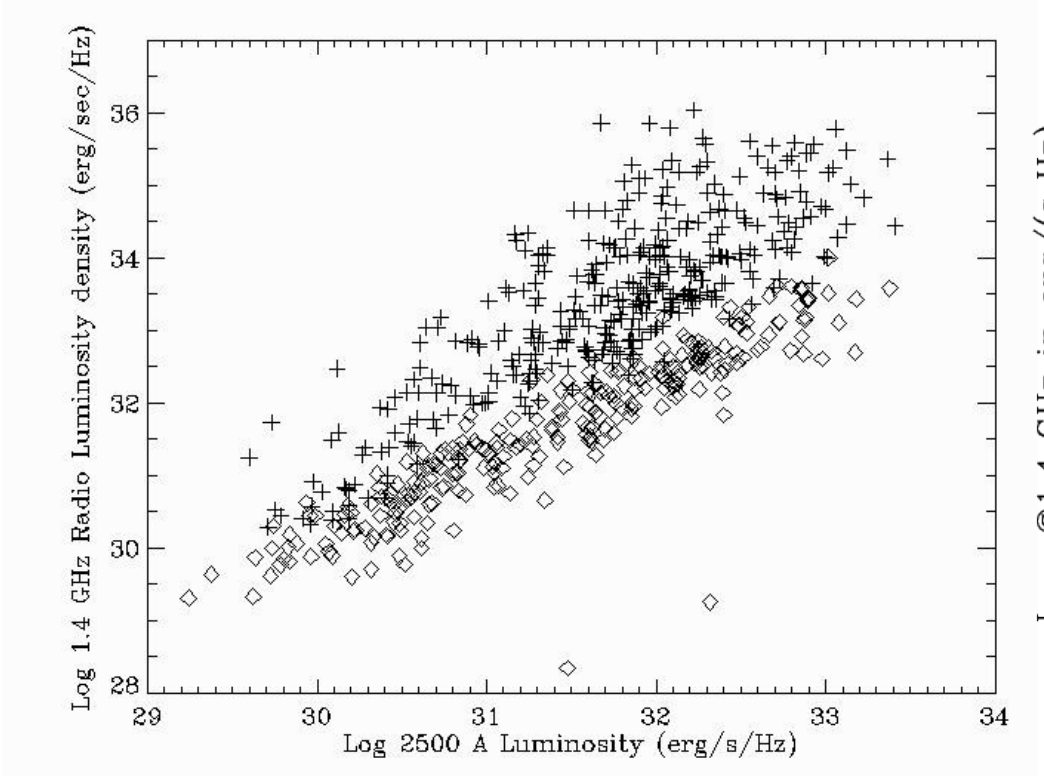



J. Singal, V. Petrosian, A. Lawrence, & L. Stawarz

2011, *ApJ*, 743, 104 2013, *ApJ*, 764, 43

# II. Data: *Optical and Radio Luminsities*

FIRST x POSS-I (636 sources)




SDSSXFIRST (5445 sources)