Challenges for Inflation

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Aug. 23, 2014

Outline

Plan

Current Paradigm for Early Universe Cosmology

The Inflationary Universe Scenario is the current paradigm of early universe cosmology.
Successes:

- Solves horizon problem
- Solves flatness problem
- Solves size/entropy problem
- Provides a causal mechanism of generating primordial cosmological perturbations (Chibisov & Mukhanov, 1981).

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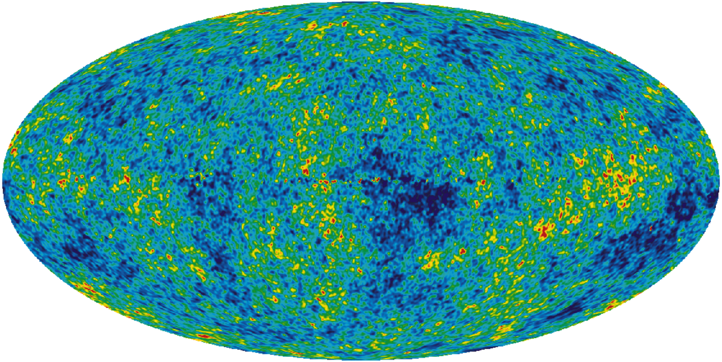
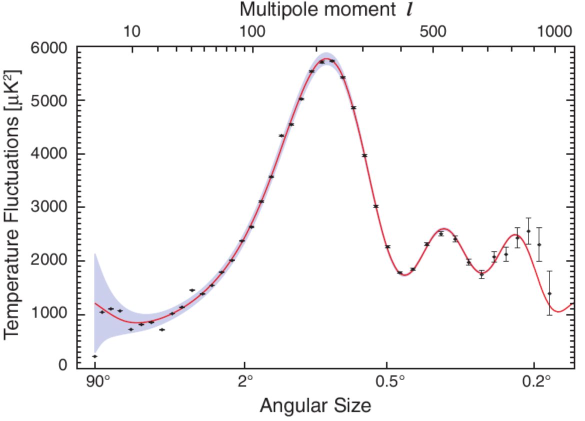


Credit: NASA/WMAP Science Team



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Historical Footnote

1970Ap&SS...7....3S

1970 paper

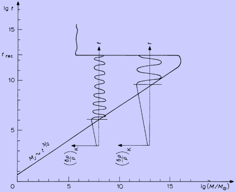

Fig. 1a. Diagram of gravitational instability in the 'big-bang' model. The region of instability is located to the right of the line $M_J(t)$; the region of stability to the left. The two additional lines of the graph demonstrate the temporal evolution of density perturbations of matter: growth until the moment when the considered mass is smaller than the Jeans mass and oscillations thereafter. It is apparent that at the moment of recombination perturbations corresponding to different masses correspond to different phases.

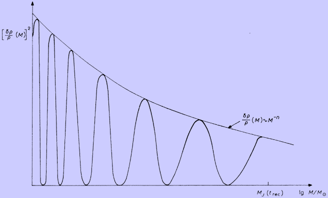

Fig. 1b. The dependence of the square of the amplitude of density perturbations of matter on scale. The fine line designates the usually assumed dependence $(\delta\varrho/\varrho)_M \sim M^{-n}$. It is apparent that fluctuations of relic radiation should depend on scale in a similar manner.

R. Sunyaev & Ya. Zeldovich, Astrophysics and Space Science 7 3-19 (1970)

Key Realization

R. Sunyaev and Y. Zel'dovich, Astrophys. and Space Science **7**, 3 (1970); P. Peebles and J. Yu, Ap. J. **162**, 815 (1970).

- Given a scale-invariant power spectrum of adiabatic fluctuations on "super-horizon" scales before t_{eq}, i.e. standing waves.
- → "correct" power spectrum of galaxies.
- → acoustic oscillations in CMB angular power spectrum.
- → baryon acoustic oscillations in matter power spectrum.
- Inflation is the first scenario based on causal physics which yields such a spectrum.
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Criteria for a Successful Early Universe Scenario

- Horizon $\gg$ Hubble radius in order for the scenario to solve the "horizon problem" of Standard Big Bang Cosmology.
- Scales of cosmological interest today originate inside the Hubble radius at early times in order for a causal generation mechanism of fluctuations to be possible.
- Squeezing of fluctuations on super-Hubble scales in order to obtain the acoustic oscillations in the CMB angular power spectrum.
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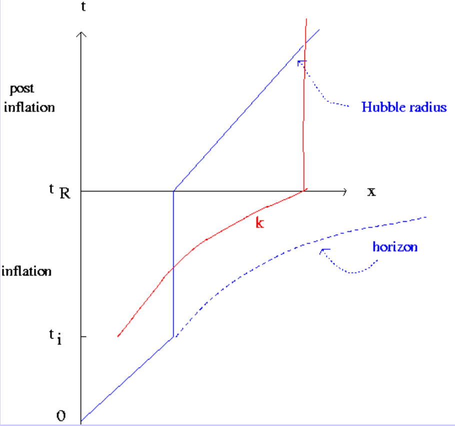
Inflation as a Solution



Matter Bounce as a Solution

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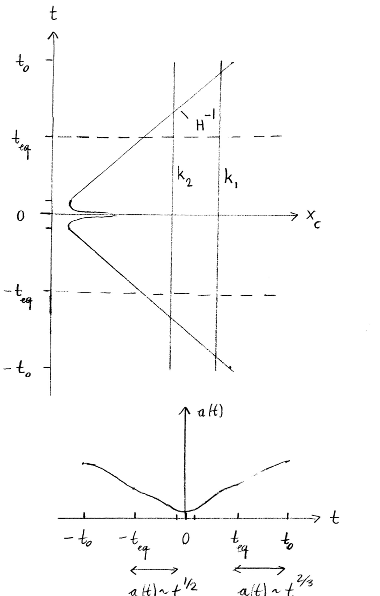
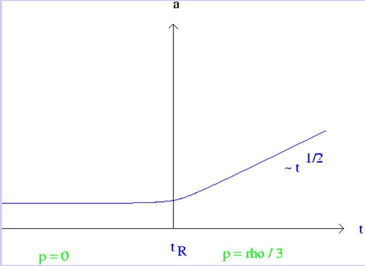


Emergent Universe

R.B. and C. Vafa, *Nucl. Phys. B316:391 (1989)*



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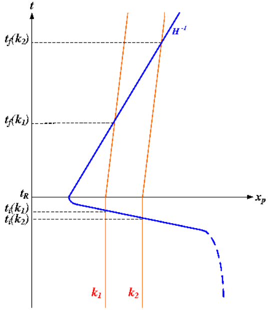
A. Nayeri, R.B. and C. Vafa, *Phys. Rev. Lett. 97:021302 (2006)*



Plan

Quantum Theory of Linearized Fluctuations

V. Mukhanov, H. Feldman and R.B., *Phys. Rep. 215:203 (1992)*

Step 1: Metric including fluctuations

$$\begin{aligned} ds^2 &= a(t)^2[(1+2\Phi(x,t))d\eta^2 - (1-2\Phi(x,t))d\mathbf{x}^2] \\ \varphi &= \varphi_0(t) + \delta\varphi(x,t) \end{aligned}$$

Note: Φ and $\delta\varphi$ related by Einstein constraint equations

Step 2: Expand the action for matter and gravity to second order about the cosmological background:

$$\begin{aligned} S^{(2)} &= \frac{1}{2}\int d^4x\left((v')^2 - v_{,i}v^{,i} + \frac{z''}{z}v^2\right) \\ v &= a\left(\delta\varphi + \frac{z}{a}\Phi\right) \\ z &= a\frac{\varphi_0'}{\mathcal{H}} \end{aligned}$$

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Step 3: Resulting equation of motion (Fourier space)

$$v_k'' + (k^2 - \frac{z''}{z})v_k = 0$$

Features:

- oscillations on sub-Hubble scales
- squeezing on super-Hubble scales $v_k \sim z$

Quantum vacuum initial conditions:

$$v_k(\eta_i) = (\sqrt{2k})^{-1}$$

Connection with curvature perturbation in comoving gauge:

$$\zeta = z^{-1} v \qquad (1)$$

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Gravitational Waves

$$ds^2 = a^2\left[(1+2\Phi)d\eta^2 - [(1-2\Phi)\delta_{ij} + h_{ij}]dx^i dx^j\right]$$

- $h_{ij}(\mathbf{x}, t)$ transverse and traceless
- Two polarization states

$$h_{ij}(\mathbf{x}, t) = \sum_{a=1}^{2} h_a(\mathbf{x}, t)\epsilon_{ij}^a$$

- At linear level each polarization mode evolves independently.

Gravitational Waves II

Canonical variable for gravitational waves:

$$u(\mathbf{x}, t) = a(t) h(\mathbf{x}, t)$$

Equation of motion for gravitational waves:

$$u_k^{''} + (k^2 - \frac{a^{''}}{a}) u_k = 0 \, .$$

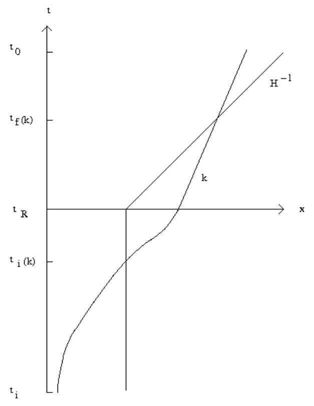
Squeezing on super-Hubble scales, oscillations on sub-Hubble scales.

Consequences for Tensor to Scalar Ratio r

Assuming adiabatic fluctuations:

- If EoS of matter is time independent, then $z \propto a$ and $u \propto v$.
- In this case $r \sim 1$.
- During a phase transition EoS changes and u evolves differently than v (z evolves differently than a).
- $\rightarrow$ Suppression of r.
- Example 1: Inflationary reheating transition.
- Example 2: nonsingular bounce phase in a bouncing cosmology.

Structure formation in inflationary cosmology



N.B. Perturbations originate as quantum vacuum fluctuations.

Origin of Scale-Invariance in Inflation

- Initial vacuum spectrum of ζ ($\zeta \sim v$): (Chibisov and Mukhanov, 1981).

$$P_{\zeta}(k) \equiv k^3 |\zeta(k)|^2 \sim k^2$$

- $v \sim z \sim a$ on super-Hubble scales
- At late times on super-Hubble scales

$$P_{\zeta}(k,t) \equiv P_{\zeta}(k,t_i(k)) \left(\frac{a(t)}{a(t_i(k))}\right)^2 \sim k^2 a(t_i(k))^{-2}$$

- Hubble radius crossing $t_H(k)$: $ak^{-1} = H^{-1}$
- $\rightarrow P_{\zeta}(k,t) \sim \left(\frac{H}{m_{pl}}\right)^2 \left(\frac{aH}{z}(t_H(k))\right)^2$

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Spectrum of Gravitational Waves

- Gravitational waves obey same EoM as cosmological perturbations.
- Scale-invariant spectrum of gravitational waves emerges.
- $P_h(k, t) \sim \left(\frac{H}{m_{pl}}\right)^2$
- H decreasing during inflation → red tilt of gravitational waves.
- Value of r set by the ratio of a and z during inflation.

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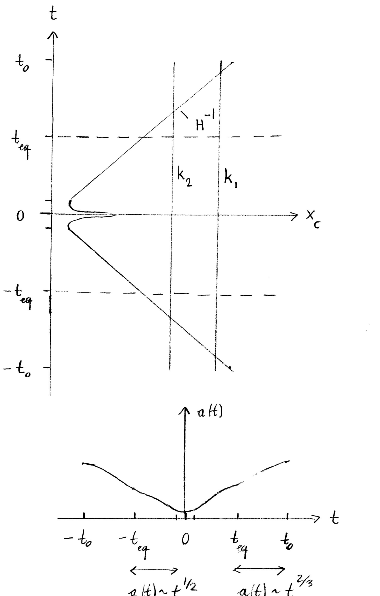

Origin of Scale-Invariant Spectrum

- The initial vacuum spectrum is blue:

$$P_\zeta(k) \,=\, k^3|\zeta(k)|^2 \,\sim\, k^2$$

- The curvature fluctuations grow on super-Hubble scales in the contracting phase:

$$v_k(\eta) \,=\, c_1\eta^2 + c_2\eta^{-1}\,,$$

- For modes which exit the Hubble radius in the matter phase the resulting spectrum is scale-invariant:

$$\begin{aligned} P_\zeta(k,\eta) &\sim k^3|v_k(\eta)|^2 a^{-2}(\eta) \\ &\sim k^3|v_k(\eta_H(k))|^2\Big(\frac{\eta_H(k)}{\eta}\Big)^2 \sim k^{3-1-2} \\ &\sim \text{const}\,, \end{aligned}$$

Spectrum of Gravitational Waves

- u satisfies the same equation of motion as v during the contracting phase.
- $z = a$ during the contracting phase.
- $\rightarrow r \sim 1$ at the end of the contracting phase.
- Scalar modes can be amplified during the non-singular transition phase (Y. Cai, E. McDonough, F. Duplessis and R.B. arXiv:1305.5259)
- $r < 1$ is possible.
- Note: Large r is not a special signature of inflation!

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Signature in the Bispectrum: formalism

$$< \zeta(t, \vec{k}_1)\zeta(t, \vec{k}_2)\zeta(t, \vec{k}_3) > \\ = i \int_{t_i}^{t} dt' < [\zeta(t, \vec{k}_1)\zeta(t, \vec{k}_2)\zeta(t, \vec{k}_3), L_{int}(t')] > ,$$

$$< \zeta(\vec{k}_1)\zeta(\vec{k}_2)\zeta(\vec{k}_3) > \quad = \quad (2\pi)^7 \delta(\sum \vec{k}_i) \frac{P_\zeta^2}{\prod k_i^3} \\ \times \mathcal{A}(\vec{k}_1, \vec{k}_2, \vec{k}_3) \, ,$$

$$|\mathcal{B}|_{NL}(\vec{k}_1, \vec{k}_2, \vec{k}_3) = \frac{10}{3} \frac{\mathcal{A}(\vec{k}_1, \vec{k}_2, \vec{k}_3)}{\sum_i k_i^3} \, .$$

Signature in the Bispectrum: Results

Y. Cai, W. Xue, R.B. and X. Zhang, *JCAP 0905:011 (2009)*

If we project the resulting shape function $\mathcal{A}$ onto some popular shape masks we get

$$|\mathcal{B}|_{NL}^{\mathrm{local}} = -\frac{35}{8} \, ,$$

for the local shape ($k_1 \ll k_2 = k_3$). This is negative and of order $O(1)$.

For the equilateral form ($k_1 = k_2 = k_3$) the result is

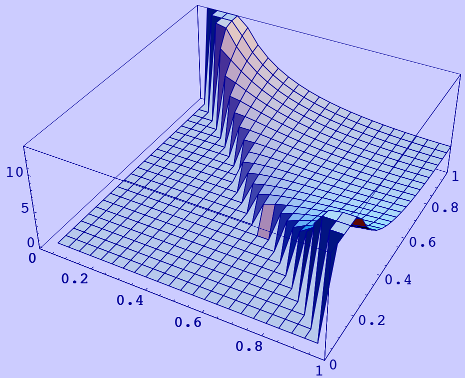
$$|\mathcal{B}|_{NL}^{\mathrm{equil}} = -\frac{255}{64} \, ,$$

For the folded form ($k_1 = 2k_2 = 2k_3$) one obtains the value

$$|\mathcal{B}|_{NL}^{\mathrm{folded}} = -\frac{9}{4} \, .$$

Bispectrum of the Matter Bounce Scenario

Y. Cai, W. Xue, R.B. and X. Zhang, *JCAP 0905:011 (2009)*



Comments on the Matter Bounce

Scenario

- Horizon problem: absent.
- Flatness problem: absent.
- Size and entropy problems: not present if we assume that the universe begins cold and large.
- Anisotropy problem (BKL instability): key challenge.

Note: Realization in String Theory: S-brane Bounce (R.B., C. Kounnas, H. Partouche, S. Patil and N. Toumbas, arXiv:1312.2524).

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String Gas cosmology as a Realization of an Emergent Universe

R.B. and C. Vafa, *Nucl. Phys. B316:391 (1989)*

Idea: make use of the new symmetries and new degrees of freedom which string theory provides to construct a new theory of the very early universe.
Assumption: Matter is a gas of fundamental strings
Assumption: Space is compact, e.g. a torus.
Key points:

- New degrees of freedom: string oscillatory modes
- Leads to a maximal temperature for a gas of strings, the Hagedorn temperature
- New degrees of freedom: string winding modes
- Leads to a new symmetry: physics at large R is equivalent to physics at small R

T-Duality

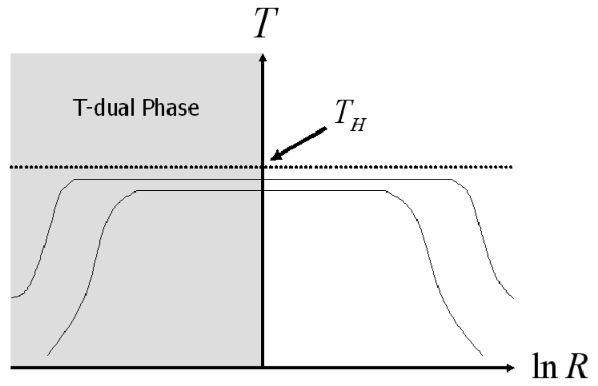
T-Duality

- Momentum modes: $E_n = n/R$
- Winding modes: $E_m = mR$
- Duality: $R \rightarrow 1/R \quad (n, m) \rightarrow (m, n)$
- Mass spectrum of string states unchanged
- Symmetry of vertex operators
- Symmetry at non-perturbative level → existence of D-branes

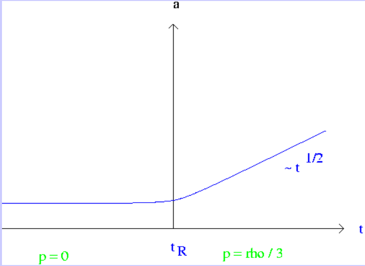
Adiabatic Considerations

R.B. and C. Vafa, *Nucl. Phys. B316:391 (1989)*

Temperature-size relation in string gas cosmology



Background for string gas cosmology



Structure formation in string gas cosmology

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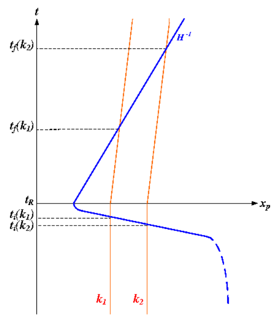

N.B. Perturbations originate as thermal string gas fluctuations.

Method

- Calculate matter correlation functions in the Hagedorn phase (neglecting the metric fluctuations)
- For fixed k, convert the matter fluctuations to metric fluctuations at Hubble radius crossing $t = t_i(k)$
- Evolve the metric fluctuations for $t > t_i(k)$ using the usual theory of cosmological perturbations

Extracting the Metric Fluctuations

Ansatz for the metric including cosmological perturbations and gravitational waves:

$$ds^2 = a^2(\eta)\big((1+2\Phi)d\eta^2 - [(1-2\Phi)\delta_{ij} + h_{ij}]dx^i dx^j\big)\,.$$

Inserting into the perturbed Einstein equations yields

$$\langle|\Phi(k)|^2\rangle = 16\pi^2 G^2 k^{-4} \langle \delta T^0{}_0(k) \delta T^0{}_0(k) \rangle\,,$$

$$\langle|h(k)|^2\rangle = 16\pi^2 G^2 k^{-4} \langle \delta T^i{}_j(k) \delta T^i{}_j(k) \rangle\,.$$

Power Spectrum of Cosmological Perturbations

Key ingredient: For thermal fluctuations:

$$\langle \delta\rho^2 \rangle = \frac{T^2}{R^6} C_V .$$

Key ingredient: For string thermodynamics in a compact space

$$C_V \approx 2 \frac{R^2/\ell_s^3}{T(1 - T/T_H)} .$$

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Power spectrum of cosmological fluctuations

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- Static Hagedorn phase (including static dilaton) → new physics required.
- $C_V(R) \sim R^2$ obtained from a thermal gas of strings provided there are winding modes which dominate.
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Comments on the Emergent Scenario

- Horizon problem: absent if the loitering phase lasts sufficiently long.
- Size and entropy problems: absent since there is no initial singularity.
- Flatness: not addressed.
- Key challenge: Dynamics of the emergent phase?

Plan

Tilt of the Tensor Mode Spectrum

- Inflationary cosmology (with matter obeying the energy conditions) yields a red tilt of the tensor spectrum.
- Measuring a blue tilt would falsify inflation.
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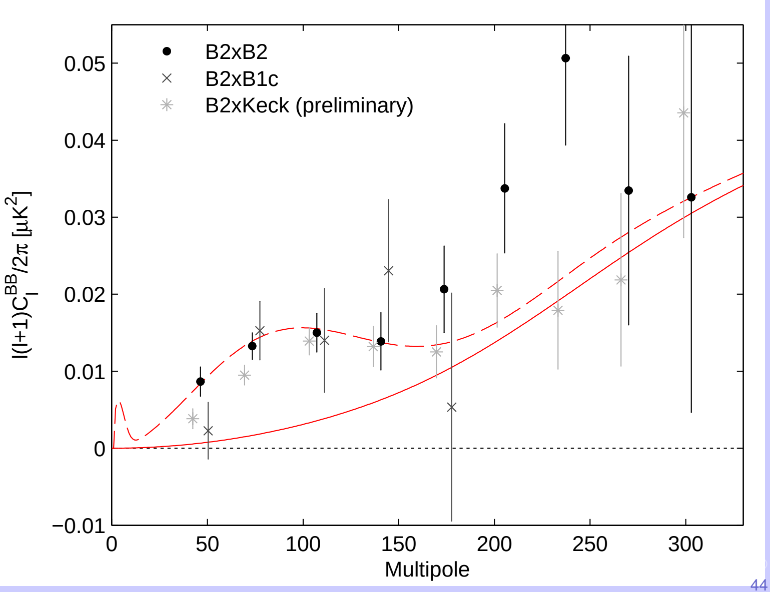
BICEP-2 Results



Is a Large Value of r a Smoking Gun for inflation?

R.B., arXiv:1104.3581

- If the equation of state of the background cosmology does not change in time then tensor and scalar modes follow the same equation → we should expect a large value of r.
- Matter bounce cosmology generically predicts large r.
- String gas cosmology predicts a value of r which is observable.
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- Example: cosmic string wakes produce rectangles in the sky with uniform polarization direction with equal strength of E-mode and B-mode (R. Danos, R.B. and G. Holder, arXiv:1003.0905).

Polarization Signal of a Cosmic String Wake

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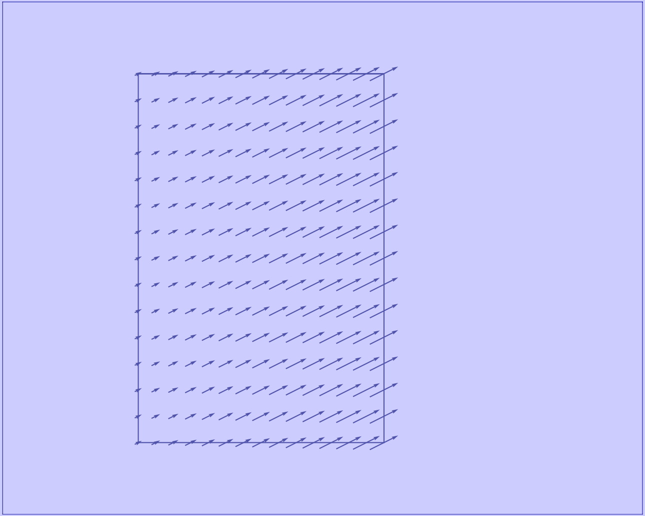

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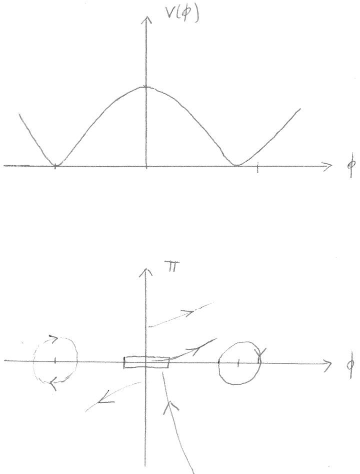
Plan

Initial Conditions for Inflation

D. Goldwirth and T. Piran, Phys. Rept. 214, 223 (1992)

In the case of small field inflation the slow roll trajectory is not an attractor in initial condition space.


$V(\phi)$
ϕ
π
ϕ

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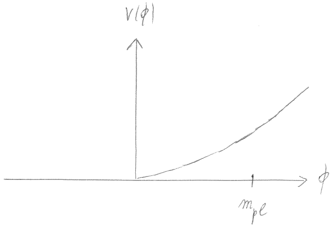
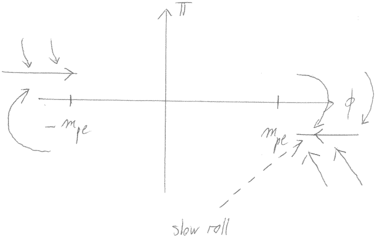
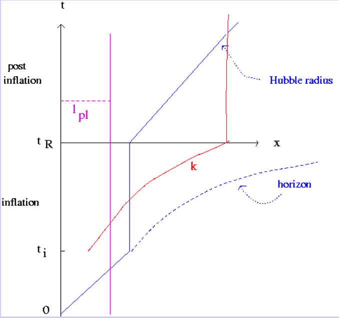

V(φ)
φ
m_pl


Π
φ
- m_pl
m_pl
slow roll

Trans-Planckian Window of Opportunity



- **Success of inflation**: At early times scales are inside the Hubble radius → causal generation mechanism is possible.
- **Problem:** If time period of inflation is more than $70H^{-1}$, then $\lambda_p(t) < l_{pl}$ at the beginning of inflation.
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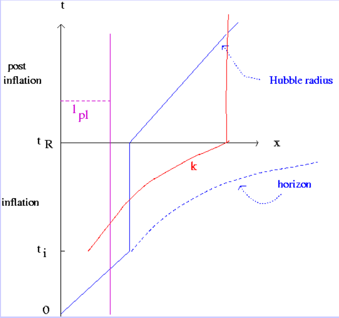

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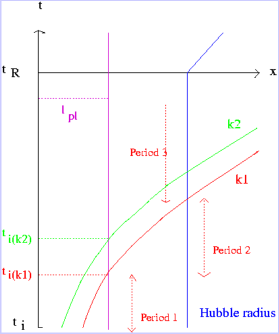
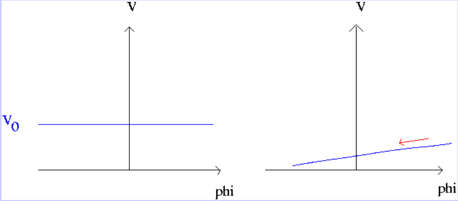


- If evolution in Period I is non-adiabatic, then scale-invariance of the power spectrum might be lost [J. Martin and RB, 2000]
- → Planck scale physics testable with cosmological observations!

Cosmological Constant Problem



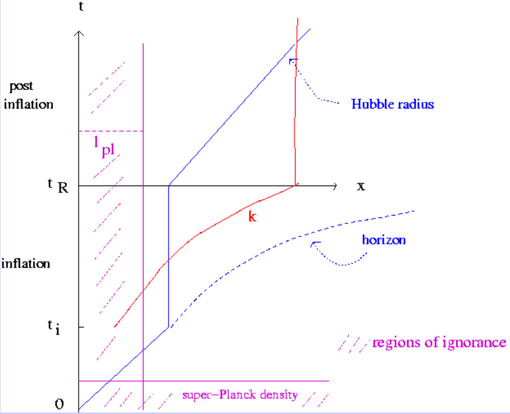
- Quantum vacuum energy does not gravitate.
- Why should the almost constant $V(\varphi)$ gravitate?

$$\frac{V_0}{\Lambda_{obs}} \sim 10^{120}$$

Applicability of GR

- In all approaches to quantum gravity, the Einstein action is only the leading term in a low curvature expansion.
- Correction terms may become dominant at much lower energies than the Planck scale.
- Correction terms will dominate the dynamics at high curvatures.
- The energy scale of inflation models is typically $\eta \sim 10^{16}\mathrm{GeV}$.
- $\rightarrow \eta$ too close to m_{pl} to trust predictions made using GR.

Zones of Ignorance



Plan

Conclusions

- The inflationary paradigm has been very successful. It made predictions which have been observationally confirmed.
- Inflation is not proven.
- Alternatives to inflation exist.
- Observational challenges: n_t and f_{nl}.
- Theoretical challenges: inflation needs a UV embedding.