Multifield Inflation: Consequences and Possibilities

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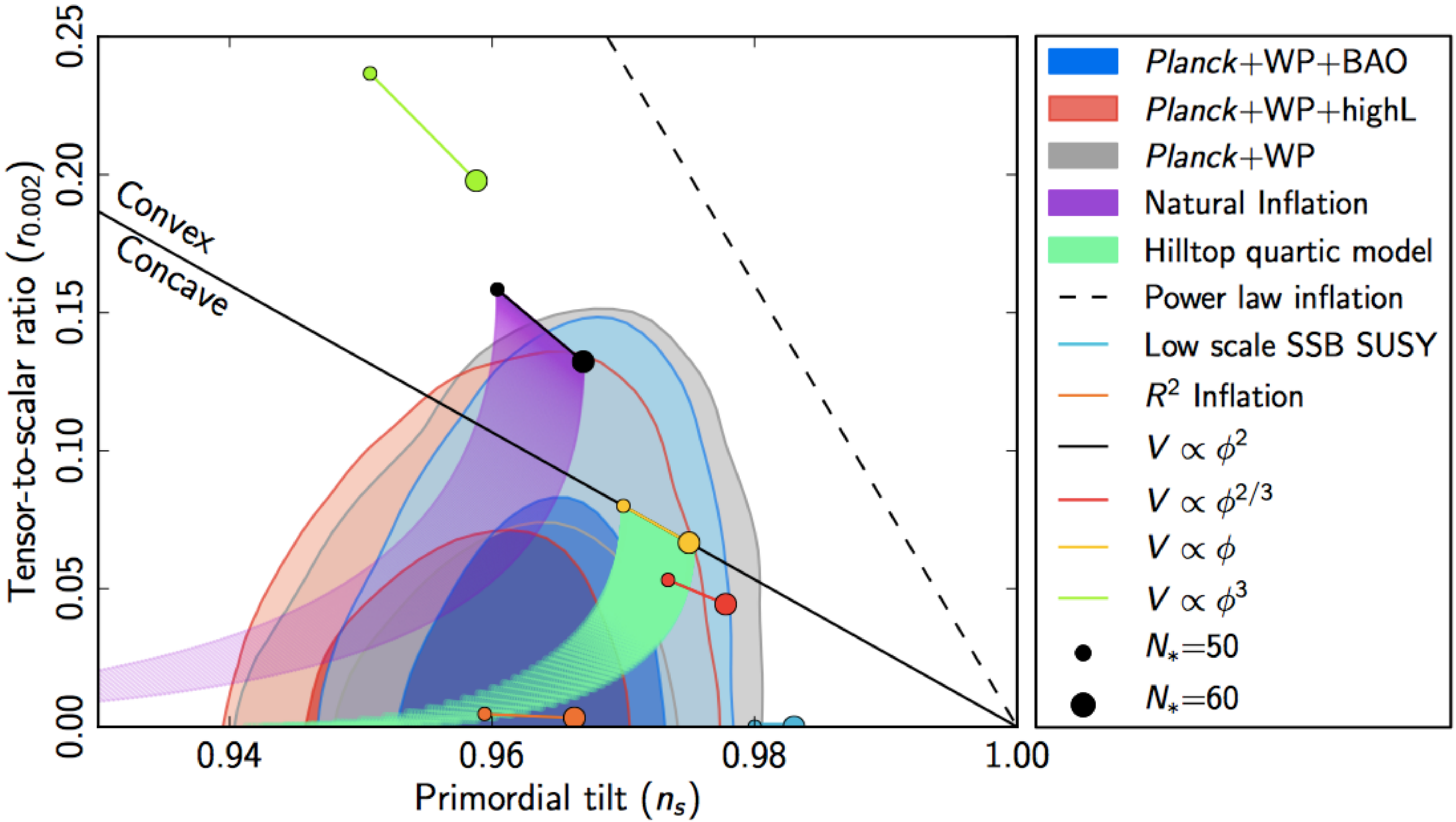
with Jonathan Frazer, Hiranya Peiris, Layne Price, Jonathan White, Jiajun Xu…



Inflationary Constraints From Planck

Planck collaboration

Canonical inflationary model

- Single, minimally coupled scalar field
  - Smallest number of extra degrees of freedom
  - Simplest scenario with negative pressure and accelerated expansion
- Chaotic inflation - “large” $\Delta\phi$
  - Starts for generic (homogeneous) initial configuration
  - Tolerant of initial inhomogeneity / inflation begins at high scales
  - Correlated with observable tensor signal
- Algebraically simple
  - Potentials can contain only tree-level terms

One field is simple but is it natural?

- Field content of particle physics models are (usually) a *choice*
  - e.g. construction of the Standard Model (families matched to observations)
  - Grand Unified Theories — choose number / representations
- Include a scalar field singlet as the "inflaton sector"
  - Must be coupled to other fields (for reheating)
  - But weakly coupled or tuned (to protect $V(\phi)$ from loop corrections)
- Naturalness v. simplicity
  - Many scenarios naturally contain *many* scalar fields
  - Explore generic properties of multifield inflation

Multifield Inflation: few field and many field

- N fields, with Hamiltonian constraint - 2N -1 degrees of freedom
  - N fields in expanding, FRW universe - 2N degrees of freedom
  - $H^2(t)$ is specified if velocities and field values known
- Entropy / isocurvature perturbation
  - Single fluid - only one "clock"
  - Density and metric perturbation well defined (up to gauge choice)
  - Multiple fluids - perturb *mixture* at fixed density
  - Can evolve into density perturbation
- Also, complex valued scalars…

Few Field Inflation

- Two fields are very different from one   RE and Maeda gr-qc/9711035
  - But three fields are not *that* different from two
  - And are four fields very different from three?
- Examples
  - Hybrid inflation (second field provides instability direction)
  - Curvaton models
  - Modular inflation (Kadota and Stewart)
  - Potentials with corners (Langlois et al arXiv:1306.5680)
    - c.f. single field “step” (Adams, Cresswell and RE, astro-ph/0102236)

Few Field Initial Conditions

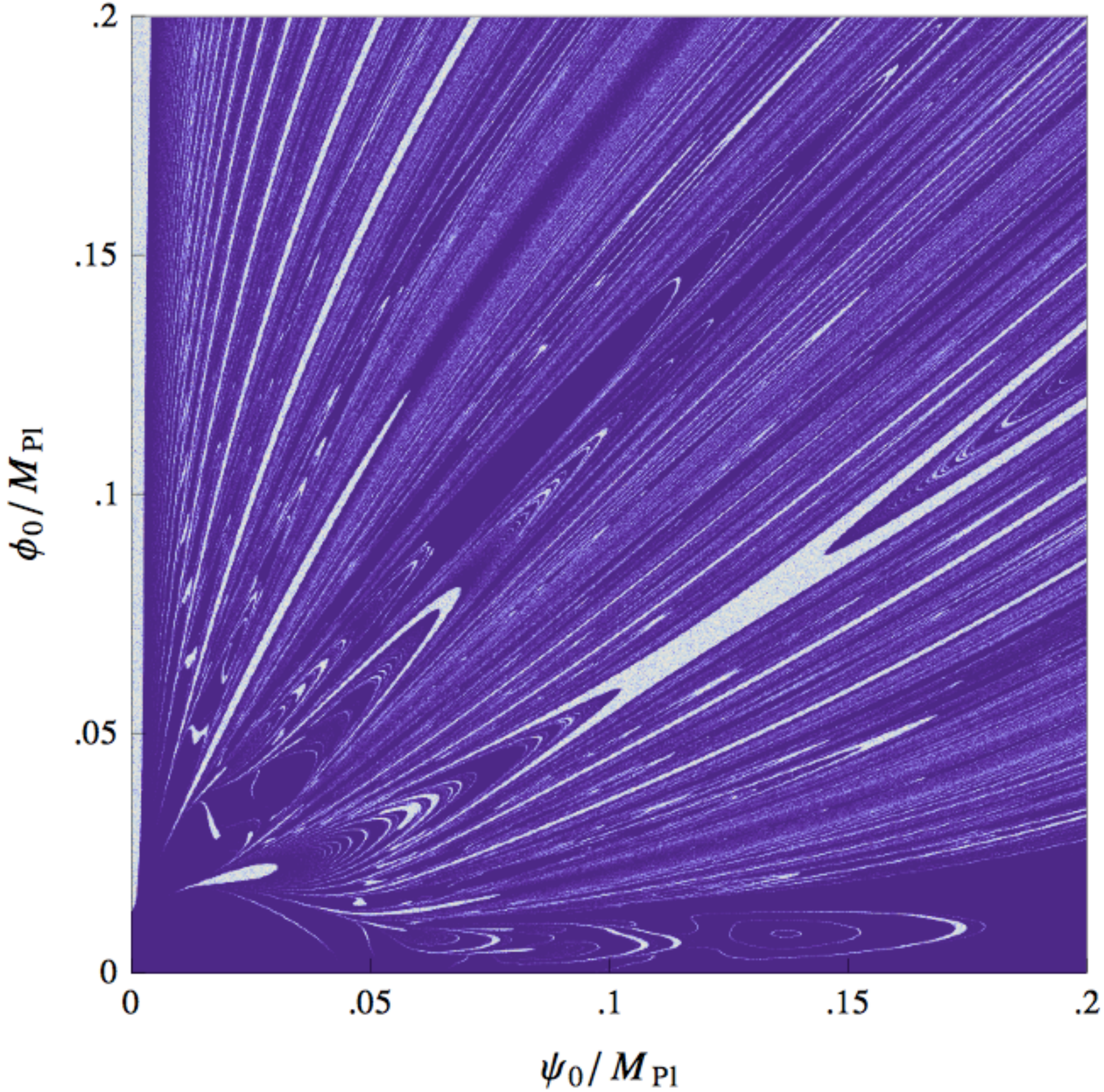
- Initial conditions for inflation
  - Overlaps with trans-Planckian problem; bubble collisions; “just enough” inflation, large scale anomalies…
- Inflation is *supposed* to solve initial conditions problems
  - But if inflation can only start from a special configuration…
- Multifield dynamics are intrinsically *chaotic*
  - Homogeneous limit - count degrees of freedom
  - Trajectories diverge exponentially in phase space
  - Gradients “focus” adjacent points in inhomogeneous solutions

RE, Price and Rasero arXiv:1406.2869
RE and Price arXiv:1304.4244
RE and Maeda gr-qc/9711035

Hybrid Inflation - Toy Model

- Blue perturbation spectrum for reasonable parameters
  - Widely used toy model for 2-field initial conditions
  - But excluded by data (for most parameter values)
- Inflationary valley along ψ=0 direction
  - Inflation ends with instability in ψ direction
- Assume homogeneous, flat (k=0) universe
  - 4 degrees of freedom; 3 at fixed energy
  - Energy monotonically decreasing; trajectories labelled by 3 numbers

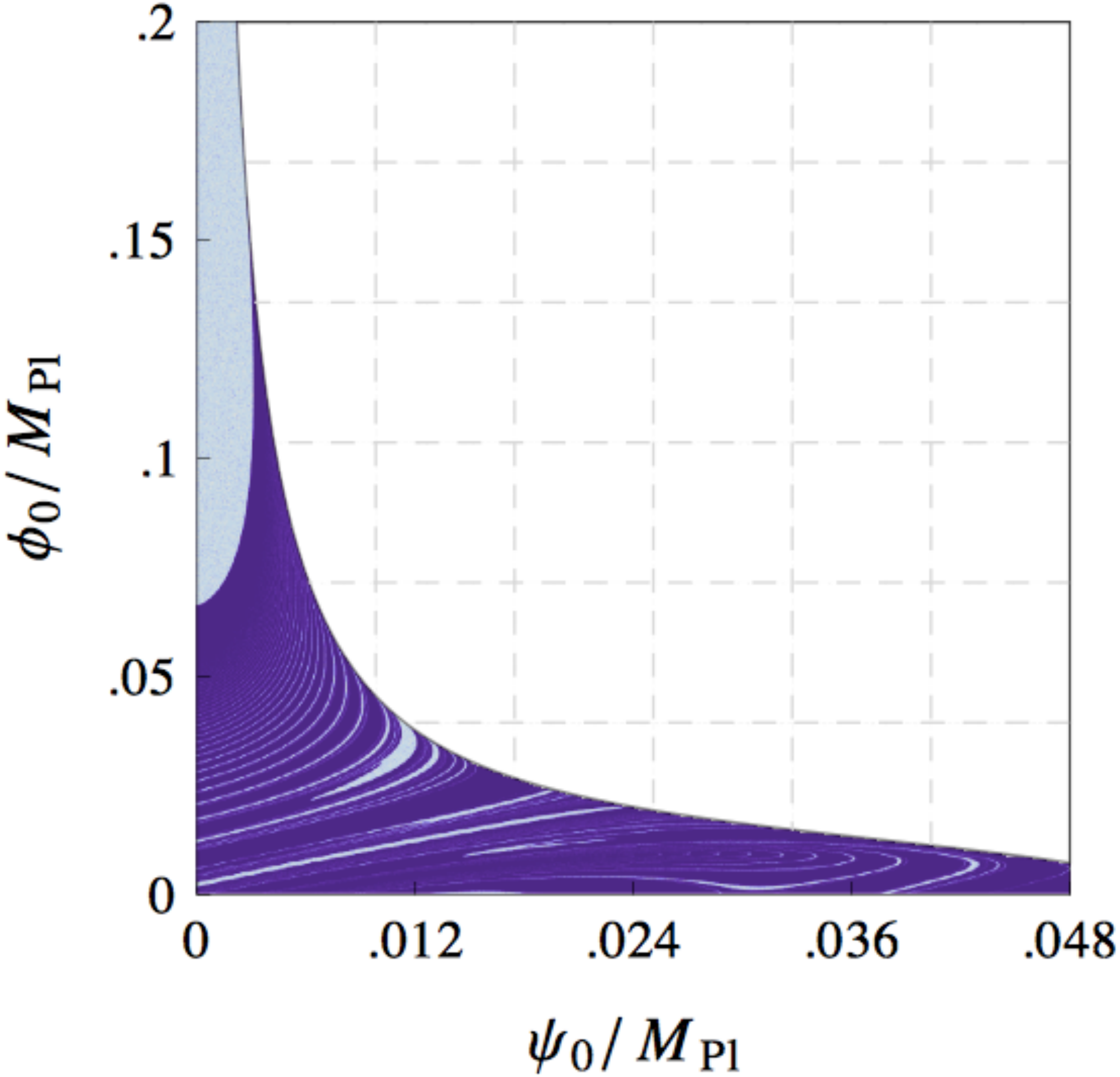
$$V(\psi,\phi) = \Lambda^4 \left[\left(1 - \frac{\psi^2}{M^2}\right)^2 + \frac{\phi^2}{\mu^2} + \frac{\phi^2\psi^2}{\nu^4} \right]$$



Initial Conditions

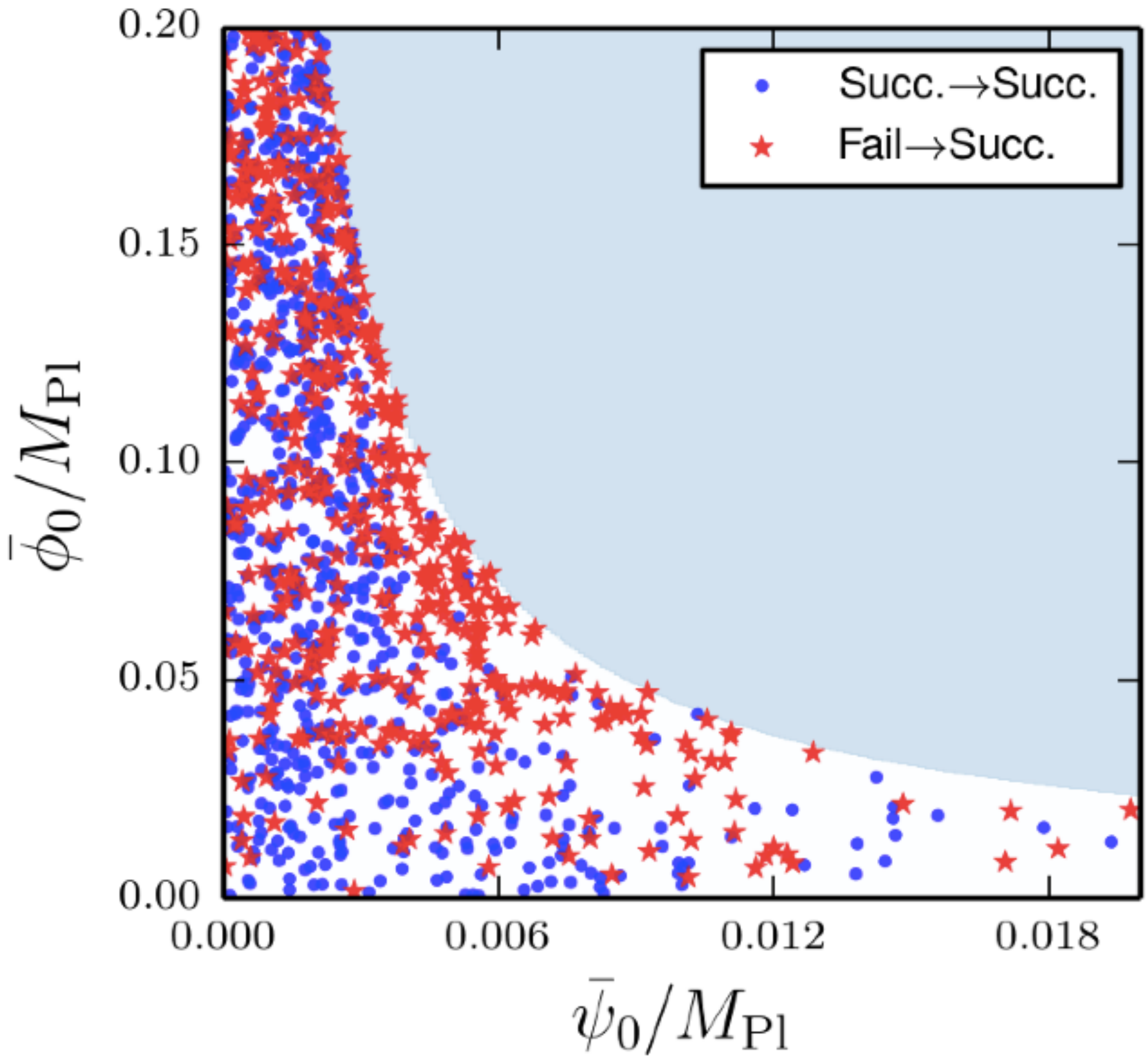
White: Inflationary

RE & Price 1304:4244
See also Clesse, Ringeval and Rocher 0909.0402



Initial Conditions
White - inflationary

RE & Price 1304:4244



Adding Inhomogeneity
(But not local gravity)

RE, Price and Rasero 1406.2869

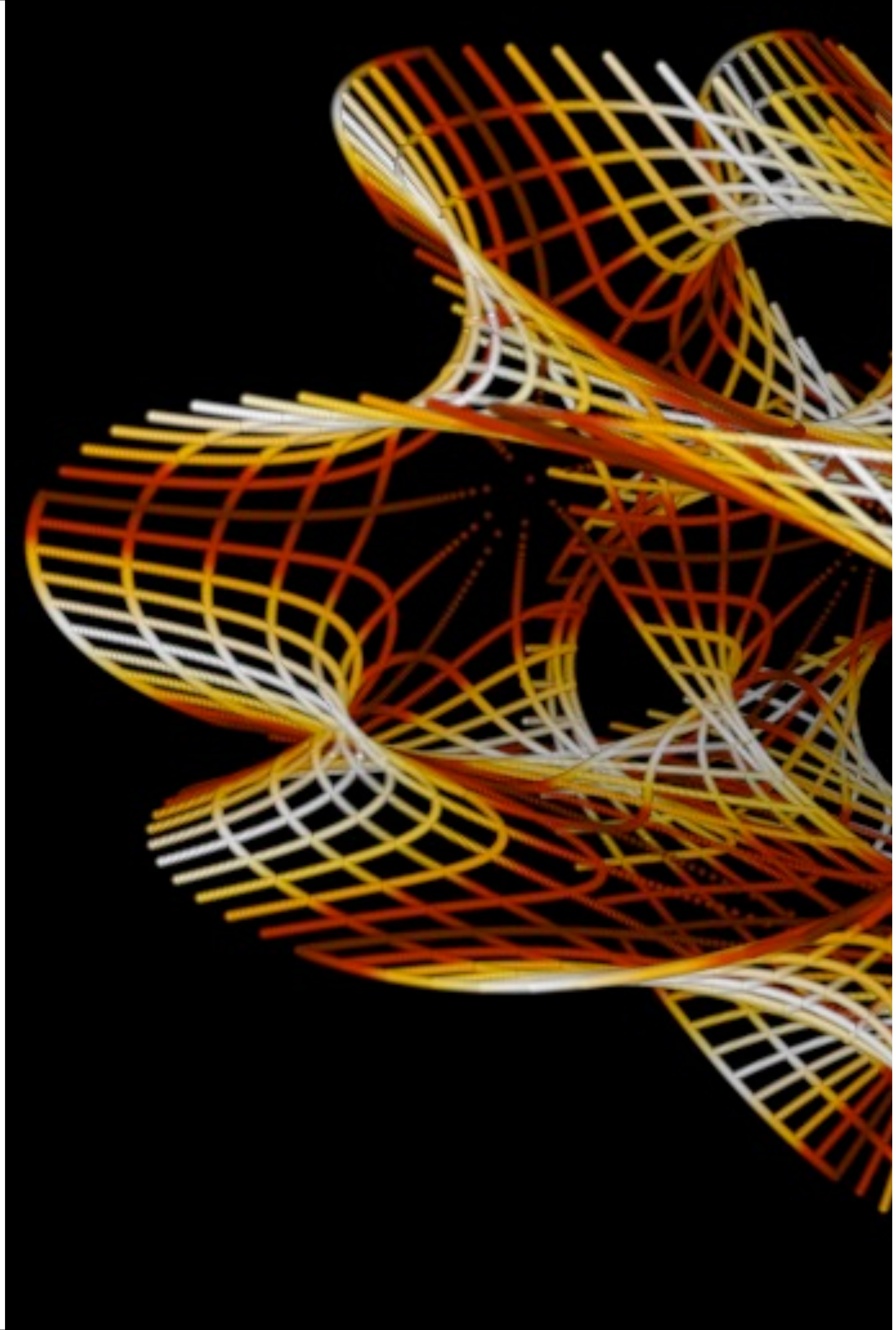
Many Field Inflation

- Assisted inflation (Liddle, Mazumdar and Schunck, astro-ph/9804177)
    - Many identical fields - equivalent to single field inflation
    - Composite inflaton, couplings and VEVs reduced by N^x
- N-flation
    - Multiple stringy axions (Dimopoulos et al. hep-th/0507205)
    - Special case of assisted inflation; avoids Lyth bound
    - Mass spectrum: eigenvalues of NxN random matrix
        - RE and McAllister (hep-th/0512102)
- String Landscape

String Landscape and Multifield Potentials

- What about a general potential?
  - String theory landscape
  - Fluxes on cycles of Calabi-Yau
- Potential with many (100s) scalars
  - Complicated (unknown) form
  - Many cross couplings
  - Many minima
- But does “large N” help?
  - Random Matrix Theory

Aazami and Easther hep-th/0512050



Extrema of a Function

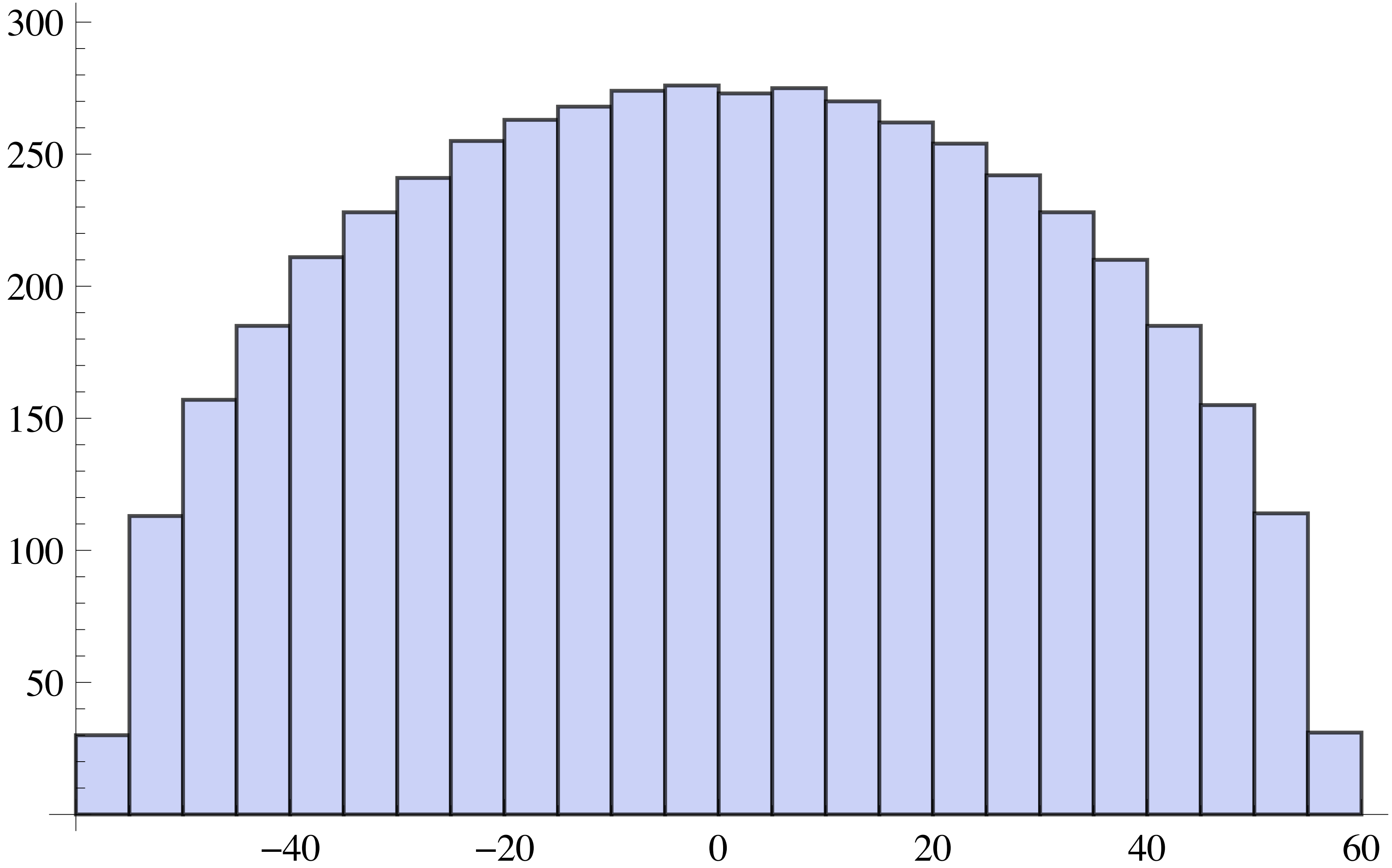
- Maxima, minima and saddles
  - Field point trapped in minima, escape by tunnelling
- Inflation from saddles and maxima (or “near” saddles)
- For successful inflation $\frac{1}{\rho}\frac{d\rho}{dN_e} \ll 1$ [GENERALIZED SLOW ROLL]
  - With many “downhill” directions inflation will be difficult
- Characterize extrema via Hessian matrix $\frac{\partial^2 V}{\partial\phi_i \partial\phi_j}$
  - Diagonalize Hessian: locally orthogonal co-ordinates
  - Maxima: all eigenvalues negative, minima all positive, saddles: all others
  - Intuitively, saddles should be more common than maxima / minima

Uncoupled Fields, Random Functions

- Uncoupled fields (mixed partial derivatives of V all vanish)
- Hessian Matrix: diagonal
  - Assume: eigenvalues uncorrelated
  - Assume: sign of each eigenvalue is random
  - P(maxima) = P(minima) ~ 2^{-N} for N fields
- If we have c > 2 extrema in each direction, we have c^N extrema
  - And a large number of maxima/minima

Coupled Fields, Random Functions

- Now assume cross-couplings with the same magnitude as the mass
  - Hessian matrix naturally symmetric (since $V_{,ab} = V_{,ba}$)
  - Eigenvalues real (as we expect for a real-valued potential)
- Assume elements of Hessian matrix are independent and uncorrelated
  - How are the N eigenvalues distributed?
- Guesses
  - Uniform
  - Normal
  - Something else?

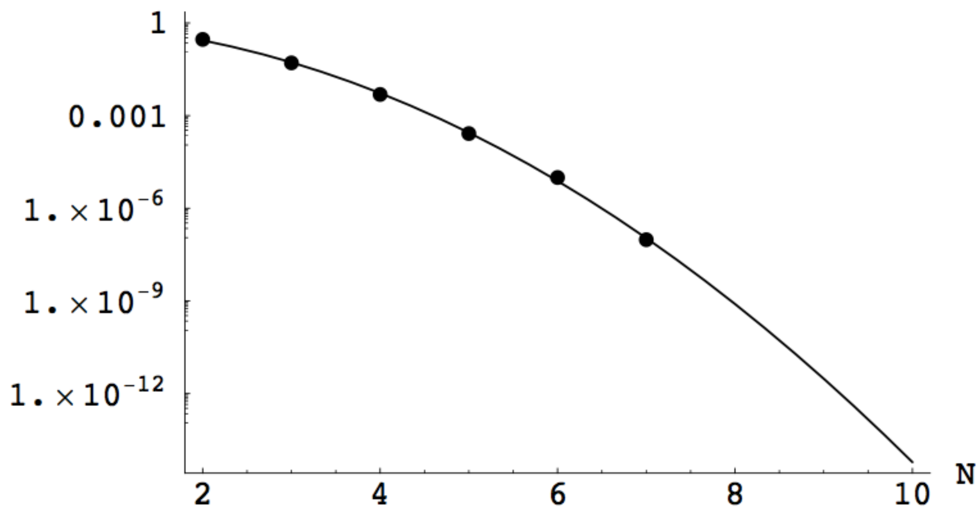


Random Matrix Eigenvalues

5000^2 random matrix

Fraction of Positive Eigenvalues

- Eigenvalues "repel"
- Aazami and Easther
  - Numerical fit; twiddle calculation
  - $p(x_i > 0, \forall i) \sim \frac{1}{2} \exp\left(-\frac{N^2}{4}\right) \frac{\sqrt{\pi}}{N}$
- Number of extrema
  - c "# of extrema in one direction"
  - N-dimensions, # of extrema ~ c^N
- If cross-couplings not suppressed
  - Almost no extrema minima/maxima



Exact result: Dean and Majumdar Phys. Rev. Lett. **97** 160201

Consequences

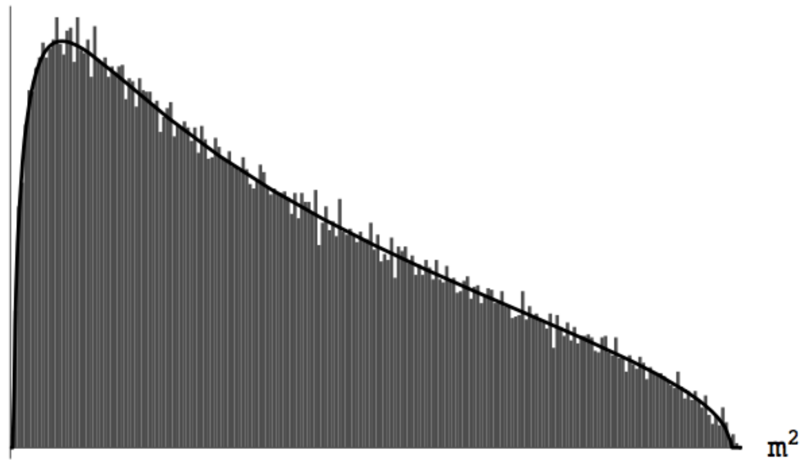
- Random landscape with cross-couplings…
  - Almost no minima/maxima - just saddles
  - Always find a downhill direction: different from string landscape
- Q: Is the Hessian for a landscape with multiple vacua drawn from GOE?
  - Apparently not...
- Second model: diagonal matrix + ε (mixing matrix)
  - Separation of scales ensures existence of minima

Newer Work

- Marsh, McAllister and Wrase 1112.3034
    - $\mathcal{N}$=1 SUGRA, N>>1 scalars - very few minima (cf. Denef and Douglas)
- Tye, Xu and Zhang 0812.1944v3 - Inflationary dynamics & a random potential
- Chen, Shiu, Sumitomo and Tye 1112.3338 - Vacuum counting in Type IIa
- Dynamics of inflation in a “bounded” potential
    - Frazer and Liddle 1101.1619, 1111.6646
    - Battefeld, Battefeld and Schulz 1203.3941
- Marsh, McAllister, Pajer and Wrase 1307.3559
    - Reconstruct trajectories via Dyson Brownian Motion

N-flation

- Multiple axions
  - Cosine potential, but assume quadratic (i.e. small excursion)
  - Diagonalize $Y = XX^T$, where X in an MxN matrix
  - Positive eigenvalues (and m^2)
- Find observables for N-flation
  - Without full stringy calculation
  - Masses: Marcenko-Pastur distribution
- Key lesson: finding large-N limits
  - Is many-field inflation simpler than few-field inflation??



Marcenko-Pastur

Multifield Perturbations

Peiris and Frazer, next week

- What about *perturbations*?
  - Perturbations can evolve *outside* horizon
  - Perturbation equations of motion: computational complexity ~ N^2
- MultiModeCode - Frazer, Peiris, Price, and Xu
  - Generalises ModeCode (Easther and Peiris) Mukhanov-Sasaki solver
  - Getting ready for release; scales to 100s of fields…

$$V = \frac{1}{2} \sum_i m_i^2 \phi_i^2$$

Easther, Frazer, Peiris and Price: 1312.4035
+ in preparation

Numerics…

- Have to be careful with stability
- Tested for fields with cross-couplings, non-zero mixed derivatives in V
- Measure spectrum at N=55, normalized to Planck best fit.

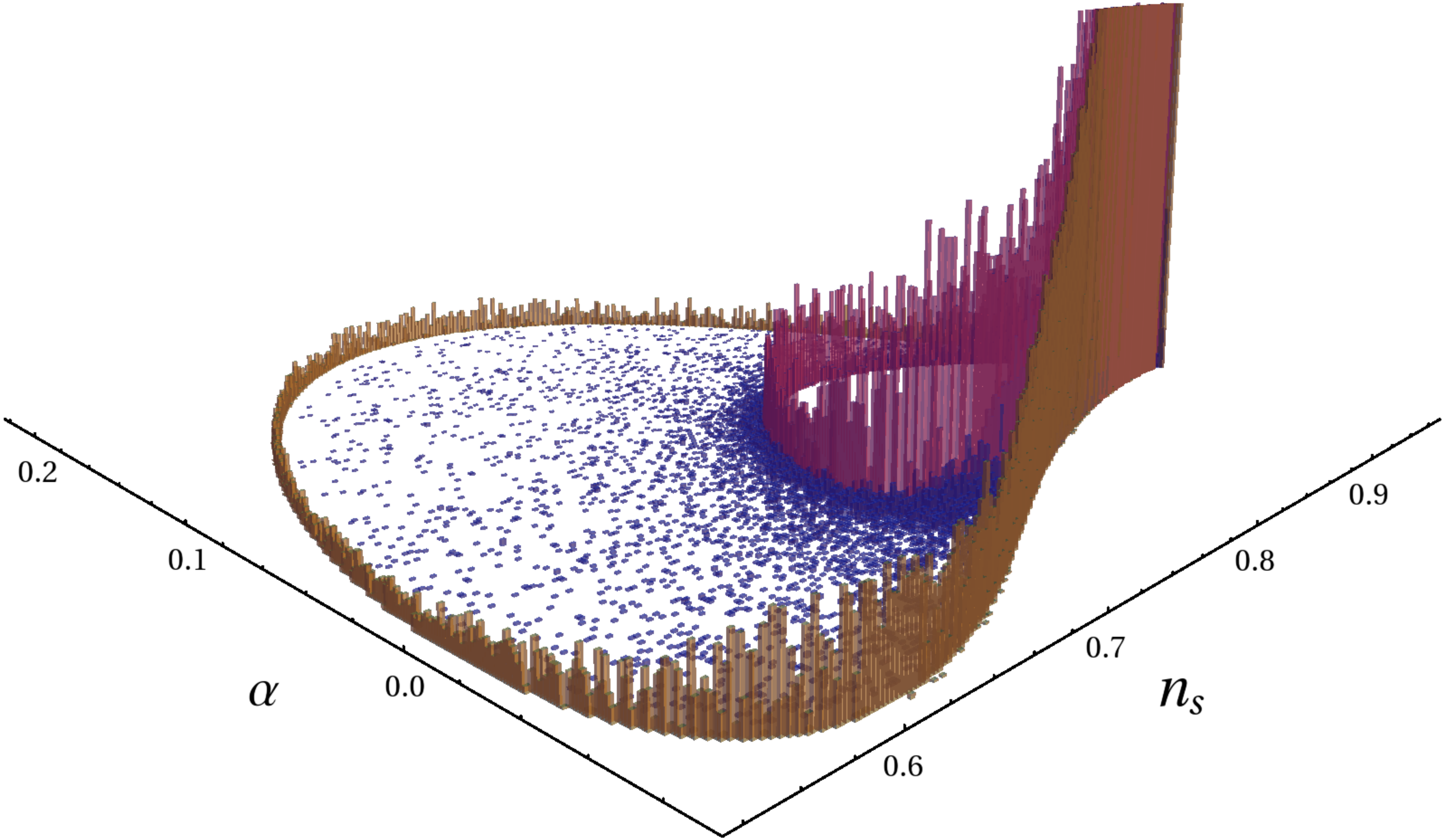
$$\psi''_{\alpha\beta} + (1-\epsilon)\psi'_{\alpha\beta} + \left(\frac{k^2}{a^2H^2} - 2 + \epsilon\right)\psi_{\alpha\beta} + \mathcal{M}_{\alpha\gamma}\psi_{\gamma\beta} = 0$$

$$\mathcal{M}_{\alpha\beta} = \frac{\partial_\alpha\partial_\beta V}{H^2} + \frac{\left(\phi'_\alpha\partial_\beta V + \phi'_\beta\partial_\alpha V\right)}{H^2} + (3-\epsilon)\phi'_\alpha\phi'_\beta$$

$$\epsilon \equiv -\frac{\dot{H}}{H^2}\,; \qquad u_\alpha(\mathbf{k}, N) = \psi_{\alpha\beta}(\mathbf{k}, N)\hat{a}_\beta(\mathbf{k})$$

Few Field Dynamics

- Consider two and three fields
  - Have to choose masses and initial values
- Heavier fields evolve more quickly toward the origin
  - When VEVs get small enough fields oscillate around origin
  - Sharp change in the potential - isolated features in spectrum at low-N
- Three classes of initial condition
  - Iso E_0 - initial values set on a surface of fixed energy
  - Iso N - initial values fixed a given number of e-folds before inflation ends
  - Slow roll - uniform distribution of VEVs, velocities from slow roll



Two and Three field models
m_2/m_1=7,9 (red,gold)

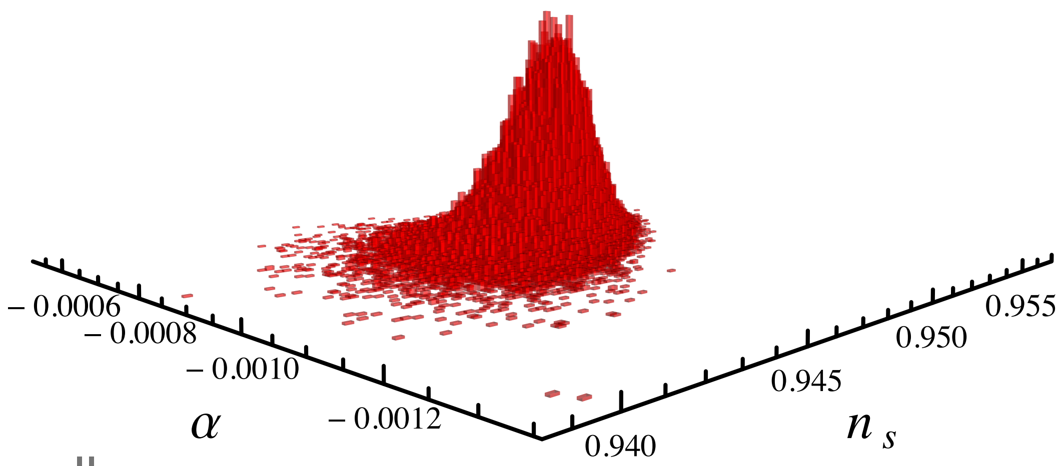
1312.4035

100 fields
Marchenko-Pastur

1312.4035

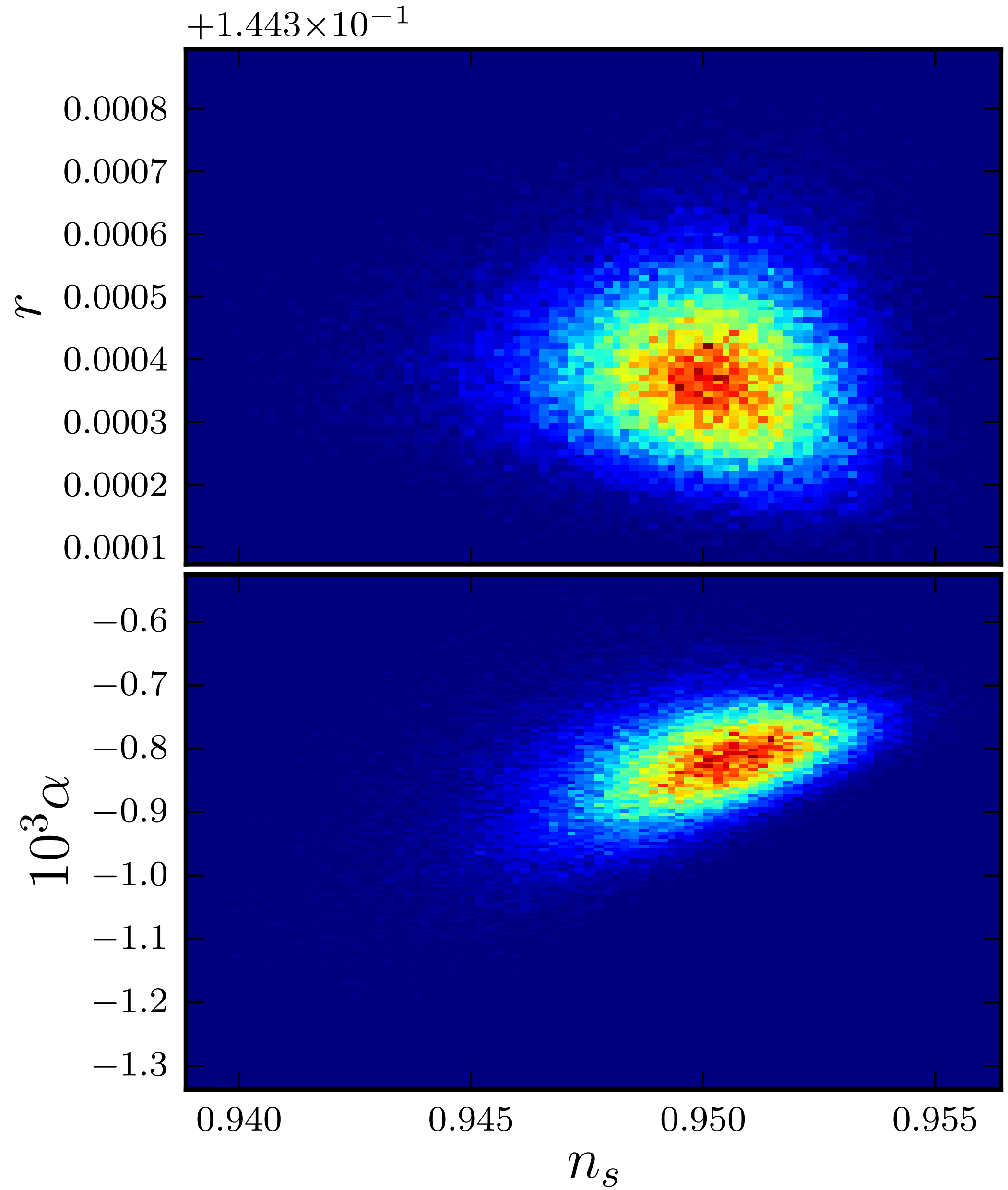
What Do We See?

- Closer to single field limit
  - But with nontrivial spread
  - Limits on predictability for N-field?
- This is effectively N-flation
  - Marginalised over initial conditions
- With 100 fields and 55 e-folds
  - ~1 field *always* close to end of slow roll
  - Depends of overall duration of inflation



Tensor Modes and Running

- Same N=100 case
  - Tensor modes close to N=1 limit
  - Running unobservable in Planck
- Significant spread in n_s
  - Relative to Planck precision
  - Bias toward red spectrum



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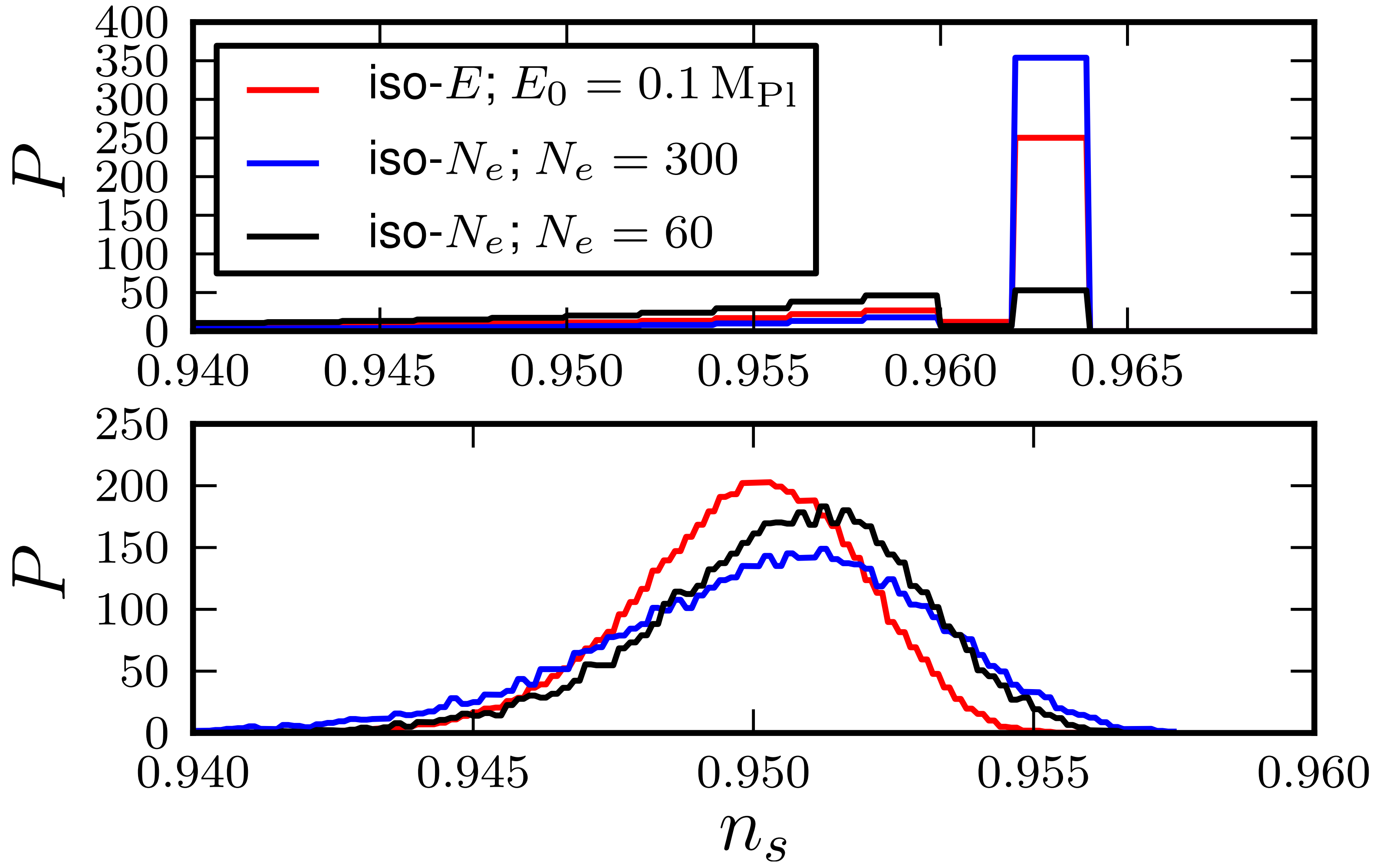


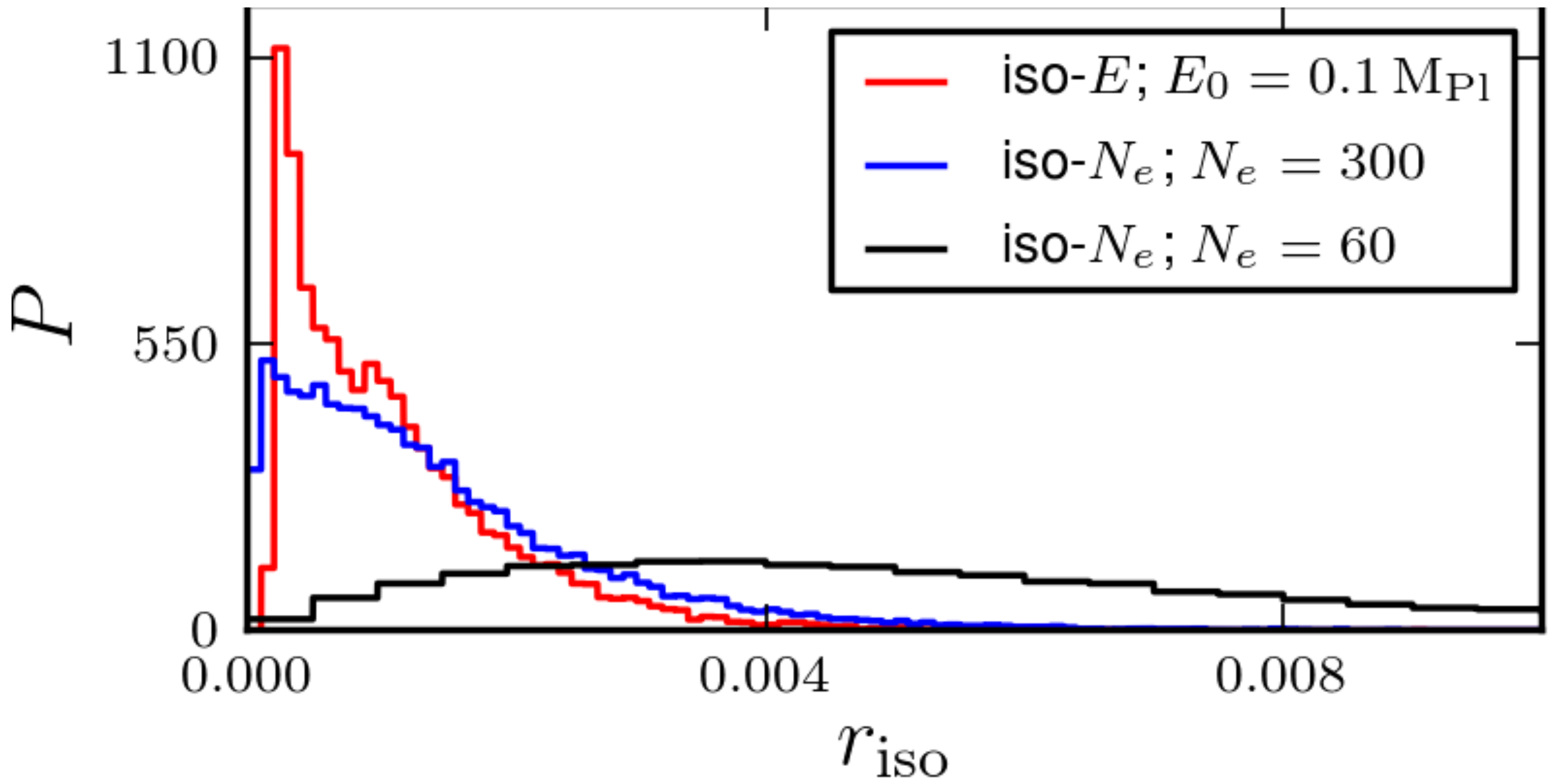
Initial Conditions
Sensitivity

1312.4035

Isocurvature Modes

- Solved for the full perturbation
  - Can compute isocurvature contribution at end of inflation
- Identifying inflationary trajectory
  - Compute N-1 orthogonal perturbations (Gram-Schmidt)
  - Define power spectrum and r_{iso}

$$\mathcal{P}_{\mathcal{S}}(k) = \frac{1}{2\epsilon} \sum_{I,J}^{N-1} s_{I\alpha} s_{J\beta} \mathcal{P}_{\alpha\beta}(k)$$

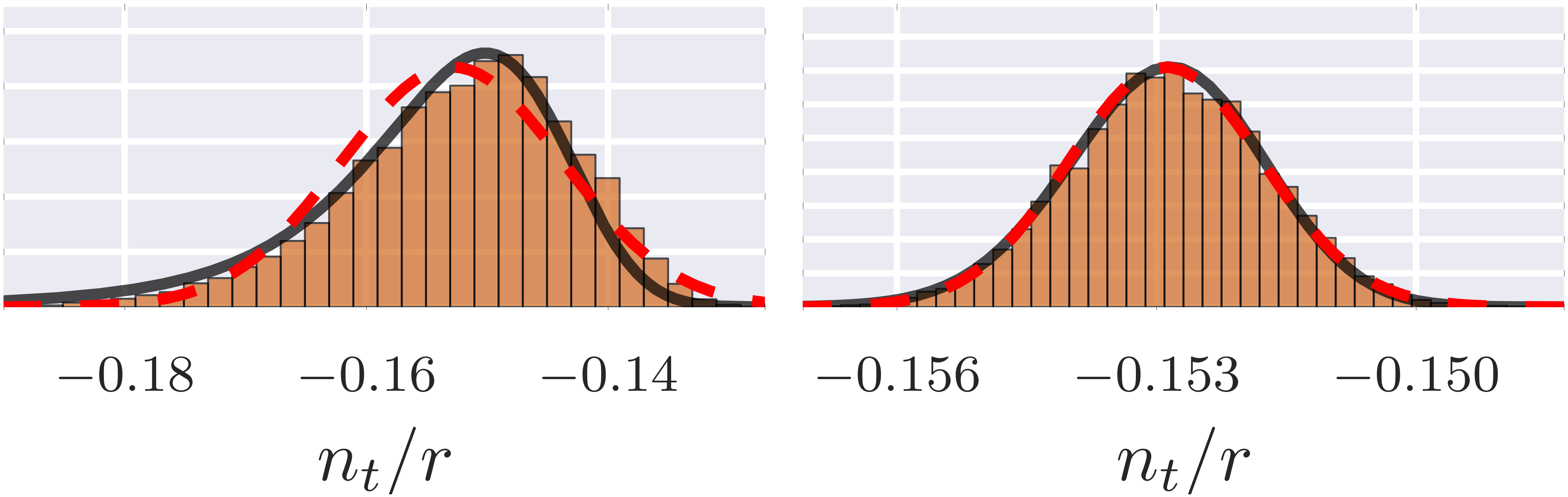


r_{iso} with N=100
Evaluated at end of inflation

Consistency Conditions and Multifield Inflation

- Work in progress — Price, Frazer and Peiris (see Cosmo talks)
  - Marginalize over couplings/masses λ_i and initial field values ϕ_i
  - Consider multiple distributions / priors
- Compute ration of r and n_t
  - Single field, slow roll: $r=-8n_t$
- Multifield result
  - Depends on N (number of fields)
  - Lowest moments of λ_i distribution (1, 2 and 4)
  - Marginalise over ϕ_i (uniform distribution)
  - A job for the central limit theorem..

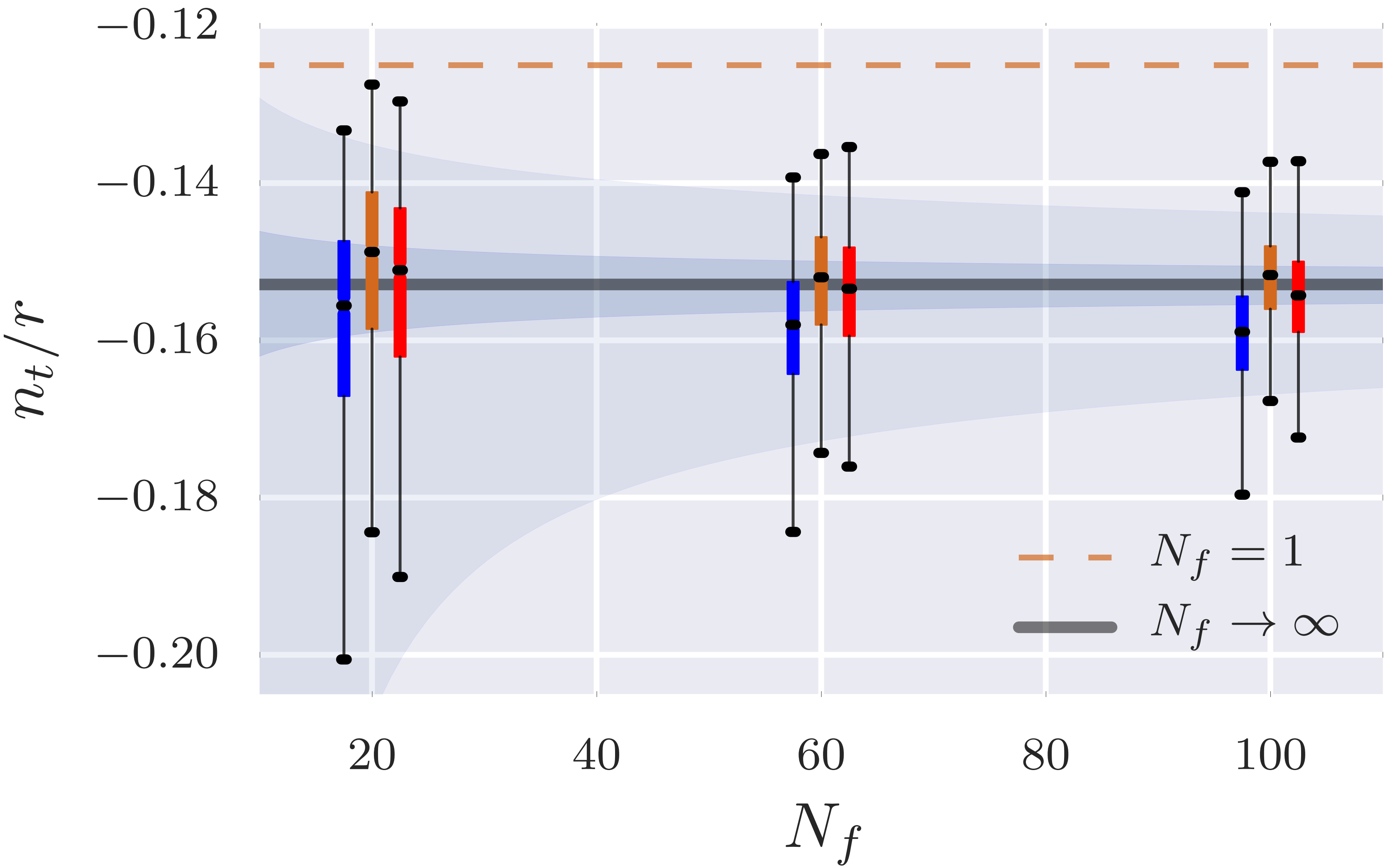
$$V = \frac{1}{p}\sum_i \lambda_i |\phi_i|^p,$$



For reference: 1/8 = 0.125

N=15 [L] and 1000 [R]

In progress…



Consistency condition…

In progress

Summary…

- Strong motivations for considering multifield models
- Phenomenology of multifield inflation complicated (and interesting)
  - Complicated spectra and features in few-field limit
  - Possibility of attractors in large-N limit
- Overlap with random matrix theory / random functions
  - Rich and exciting branch of mathematics
  - Is the landscape of inflationary solutions *simpler* than we might imagine
- Looked at quadratic assisted inflation / N-flation
  - Role of isocurvature modes unknown…
  - Consistency condition…

CosPA 2014 - Early Invitation