EFT Beyond the Horizon: Stochastic Inflation and How Primordial Quantum Fluctuations Go Classical

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Irksome Questions

What is the EFT for super-Hubble modes?

What is the systematic way of integrating out short distance modes when horizons and redshifting are present?

Why do quantum fluctuations re-enter the Hubble scale a
classical distributions?

or

how does

$$\langle \Phi(x_1) \ldots \Phi(x_n) \rangle \to \int \mathcal{D}\phi \ \left[\phi(x_1) \ldots \phi(x_) \right] P[\phi(\cdot)]$$

happen?

Standard answer 1:

STOCHASTIC OE SITTER (INFLATIONARY) STAGE
IN THE EARLY UNIVERSE

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Super horizon modes evolve independently in different Hubble patches; looks like a random walk, so concentrate on the probability distribution for coarse-grained field configurations.

Standard answer 2: Decoherence without decoherence!

Squeezing of BD state during inflation drives it to a state with canonical momenta getting highly suppressed.

Expectation values only see diagonal part of the density matrix in the field basis ("pointer basis") and so *look* classical

What do we do?

- Give a field theoretical derivation of the Fokker-Planck equation of SI,
- Argue that it's not so much that the system looks classical, it really IS! Decoherence happens on the same time scale that squeezing does.

Our tool for this: Open EFT's

Open EFT’s

The usual EFT story: have heavy fields and light fields and we want to construct an effective action only using the light fields

$$e^{iS_{\rm eff}[\phi_L]} \equiv \int \mathcal{D}\psi_H \; e^{iS[\phi_L,\psi_H]}$$

Why does this work?

Need to be sure heavy fields don't get generated by light ones; energy-momentum conservation takes care of that!

But what if… There’s no way to systematically stop the cross-talk between sectors?

This could happen for a particle moving through a medium it interacts with.

Or…if “heavy” modes keep getting converted to “light” modes.

Recall:

$$\langle \mathcal{O} \rangle(t) \equiv \mathrm{Tr}\left(\rho(t)\mathcal{O}\right), \quad i\frac{\partial \rho(t)}{\partial t} = [H, \rho(t)] \Rightarrow \rho(t) = U(t,t_0)\rho(t_0)U^\dagger(t,t_0)$$

Now consider an operator defined on the total Hilbert space for heavy and light fields acting only on the light fields:

$$\mathcal{H}_{\rm tot} = \mathcal{H}_L \otimes \mathcal{H}_H, \quad \mathcal{O} = \mathcal{O}_L \otimes \mathbb{I}_H$$

Then: $$\mathrm{Tr}_{\mathcal{H}}(\rho\mathcal{O})) = \mathrm{Tr}_{\mathcal{H}_L}(\rho_{\rm red}\mathcal{O}_L), \quad \rho_{\rm red} \equiv \mathrm{Tr}_{\mathcal{H}_H}\rho$$

How does $\rho_{\rm red}$ evolve in time?

IF: $\rho_{\rm red}(t) = U_{\rm red}(t,t_0)\rho_{\rm red}(t_0)U^\dagger_{\rm red}(t,t_0)$

THEN: We could define an effective Hamiltonian that would generate light field correlators via

$$i\frac{\partial U_{\rm red}(t,t_0)}{\partial t} = H_I^{\rm Eff}(t)U_{\rm red}(t,t_0)$$

BUT: This is not true in general!

The propagator for the reduced density matrix is the influence functional and it knows about the open channels the light field could lose energy into.

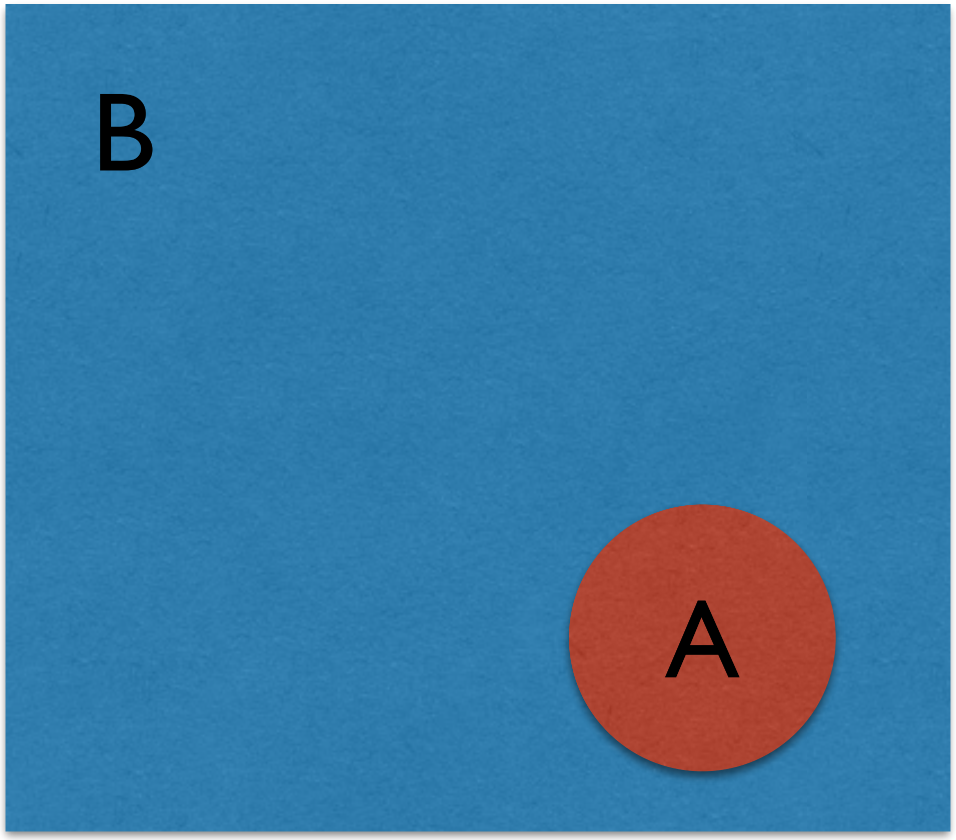
The upshot:

There's no point in looking for an effective action formulation for open systems.

BUT

This does NOT mean there is no EFT! It's just that the systematics and the goal are different.

$$H = H_A + H_B + V_{AB}$$



S'pose we want to focus on the physics of subsystem A.

We can set up something EFT-like, IF there is a hierarchy of time scales and IF we can assume that B does not change appreciably due to the presence of system A.

What time scales do we compare? We'd like

$$\tau_c \ll \tau_p$$

time scale for appreciable autocorrelations of V in subsystem B

time scale for perturbation theory in V to break down

Then:

- Evolution of A is Markovian for times greater than the correlation time scale,

- Evolution is perturbative until the breakdown time scale.

It looks like we only have control of the evolution of A for time intervals

$$\tau_c \ll t \ll \tau_p$$

But it's better than this!

We can coarse grain the time derivative of the reduced density matrix of A

$$\left(\frac{\partial \rho_A}{\partial t}\right)_{\substack{\text{coarse}\\\text{grained}}} = \frac{\Delta \rho_A}{\Delta t} = F[V, \rho_A(t)]$$

$$\tau_c \ll \Delta t \ll \tau_p$$

Markovian assumption

This is valid for ANY time as long as we stay within the allowed window around it!

Applications to Inflation

System A: Super-Hubble modes with $\frac{k}{a(t)} \ll H$

System B: Sub-Hubble modes with $\frac{k}{a(t)} \gg H$

The question is how to integrate out B. Redshifting makes this an open EFT question.

Natural correlation scale for B: $t_c \sim H^{-1}$

Stochastic Inflation

Want to get at Fokker-Planck equation:

$$\frac{\partial P\left[\phi(\cdot);t\right]}{\partial t} = \left(\frac{H^3}{8\pi^2}\right)\frac{\partial^2 P}{\partial \phi^2} + \frac{\partial}{\partial \phi}\left(\frac{V'(\phi)}{3H}P\right)$$

prob. dist.

noise term

drift term

We'll show how to get the noise.

Consider a free scalar field in de Sitter:

$$L = -\frac{1}{2}\int d^3x\sqrt{-g}\,\left[(\partial\chi)^2 + m^2\,\chi^2\right]$$
$$= \sum_k a^3\left(\dot{\chi}_k^*\dot{\chi}_k - M_k^2\,\chi_k^*\chi_k\right)$$

Use Schrodinger picture field theory to quantize this.
We can do this mode by mode.

$$i\frac{\partial}{\partial t}\Psi[\chi] = \mathcal{H}\,\Psi[\chi] = \left[-\frac{1}{a^3}\frac{\partial^2}{\partial\chi\,\partial\chi^*} + a^3\,M^2\chi^*\chi\right]\Psi[\chi]$$

Gaussian ansatz:

$$\Psi[\chi] := N(t)\,\exp\Big(-a(t)^3\,\omega(t)\,\chi^*\chi\Big)$$

$$\dot{\omega} + 3H\omega = -i\omega^2 + iM^2$$

$$\dot{N} = -i\omega\,N$$

$$\omega = -i\frac{\dot{w}}{w} \Rightarrow \ddot{w} + M^2 w = 0,\ \ \frac{\dot{N}}{N} = -\frac{\dot{w}}{w}$$

Prob. dist. are the diagonal matrix elements of the density matrix

$$\langle \xi | \rho | \chi \rangle = \Psi[\xi] \, \Psi^*[\chi]$$

$$\begin{aligned} \frac{\partial}{\partial t} \langle \xi | \rho | \chi \rangle &= \frac{\partial \Psi[\xi]}{\partial t} \, \Psi^*[\chi] + \Psi[\xi] \, \frac{\partial \Psi^*[\chi]}{\partial t} \\ &= \left(-i\mathcal{H}\Psi[\xi] \right) \Psi^*[\chi] + \Psi[\xi] \left(-i\mathcal{H}\Psi[\chi] \right)^* \end{aligned}$$

$$= i \left[\frac{1}{a^3} \left(\frac{\partial^2}{\partial \xi \partial \xi^*} - \frac{\partial^2}{\partial \chi \partial \chi^*} \right) + a^3 M^2 (\chi^* \chi - \xi^* \xi) \right] \langle \xi | \rho | \chi \rangle$$

From Gaussian form of ansatz we can show:

$$\frac{\partial}{\partial t}\langle\chi|\rho|\chi\rangle = \mathcal{N}\,\frac{\partial^2}{\partial\chi\partial\chi^*}\,\langle\chi|\rho|\chi\rangle$$

$$\mathcal{N} \equiv \frac{i}{a^3}\left(\frac{\omega-\omega^*}{\omega+\omega^*}\right)$$

$$a^3\mathcal{N} = -\left(\frac{aH}{k}\right)$$

Looks FP'ish

This is a problem!

Minus sign comes from freezing out of modes when they cross the horizon

But…it's not game over yet!

Have to combine modes to ask correct question: what is the prob. dist for super-horizon configurations?

Coarse graining over Hubble volumes leads to

$$\mathcal{P} = \left(\prod_k \frac{a^3(\omega + \omega^*)}{\pi S_\Omega(k)} \right) \exp\left[-a^3 \sum_k \left(\frac{\omega + \omega^*}{S_\Omega(k)} \right) \chi_k^* \chi_k \right]$$

window function for super-Hubble modes; new time dependence!

$$\frac{\partial}{\partial t}\,\mathcal{P}[\chi_\Omega] = \mathcal{N}_\Omega\,\frac{\partial^2\,\mathcal{P}[\chi_\Omega]}{\partial\chi_\Omega^2}$$

$$\begin{aligned}
\mathcal{N}_\Omega &= \partial_t \int \frac{d^3q}{(2\pi)^3}\; S_\Omega(q)\langle\chi_q^*\,\chi_q\rangle \\
&= \int \frac{d^3q}{(2\pi)^3}\partial_t\left[\frac{S_\Omega(q)}{a^3(\omega+\omega^*)}\right] \\
&= \int \frac{d^3q}{(2\pi)^3}\left[S_\Omega(q)\frac{i(\omega-\omega^*)}{a^3(\omega+\omega^*)} + \frac{\partial_t S_\Omega(q)}{a^3(\omega+\omega^*)}\right] \\
&\simeq \frac{1}{4\pi^2}\left\{i\int_0^{aH}\frac{dq\,q^2}{a^3}\left(\frac{\omega-\omega^*}{\omega+\omega^*}\right) + \left[\left(\frac{q^2}{a^3}\right)\frac{aH^2}{\omega+\omega^*}\right]_{q=aH}\right\}
\end{aligned}$$

Put everything in and we get $\mathcal{N}_\Omega = \dfrac{H^3}{8\pi^2}$!

Lindbland Equation and Decoherence

Let's look at off-diagonal density matrix elements

$$V = \int d^3x \; \mathcal{A}_i(x,t) B^i(x,t)$$

$$\langle \delta B^i(x,t) \, \delta B^j(x',t') \rangle_B = \mathcal{W}^{ij}(t) \, \delta^3(x - x') \, \delta(t - t')$$

$$\left(\frac{\partial \rho_A}{\partial t}\right)_{\rm c\,g} = i \; [\rho_A, \mathcal{A}_j] \; \langle \mathcal{B}^j \rangle_B - \frac{1}{2} \, \mathcal{W}^{jk} \, [\mathcal{A}_j \mathcal{A}_k \rho_A + \rho_A \mathcal{A}_j \mathcal{A}_k - 2 \mathcal{A}_k \rho_A \mathcal{A}_j]$$

This is the Lindblad equation

Squeezing of super-Hubble modes implies

$$\mathcal{A}_i(\Phi, \Pi)|\phi\rangle \to \mathcal{A}_i(\Phi, 0)|\phi\rangle = \alpha_i(\phi)|\phi\rangle$$

This means we can integrate the Lindblad equation in the field basis

$$\langle\phi|\rho_A|\tilde{\phi}\rangle = \langle\phi|\rho_{A0}|\tilde{\phi}\rangle e^{-\Gamma}$$

$$\Gamma = \int d^3x\ dt\ \left[\alpha_i - \tilde{\alpha}_i\right]\left[\alpha_j - \tilde{\alpha}_j\right]\mathcal{W}^{ij}$$

Off-diagonal elements get driven to zero on time scales comparable to the squeezing time scale.

Conclusions

Evolution of inflationary modes is described by an open EFT

This allows us to understand to develop a systematic approach to corrections to Stochastic inflation picture.

I FINALLY understand how to get Stochastic inflation from a field theory! But only in the Gaussian approximation; how can we go beyond that?

It’s not decoherence without decoherence; it’s just decoherence and it’s fast!