How Quantum are the cosmological perturbations?

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KICP Chicago Inflation Workshop 2014

Why do we care?

Primordial perturbations have a quantum origin : correlations are quantum.

Our observations are classical : we got a set of classical *probability distribution functions* pdf

$$P_{cl} = \langle a_{lm} a^*_{lm} \rangle$$

Is there a way to test for the quantum origin of perturbations?

Why do we care?

Example : 2 entangled qubits store 2 bits of info.

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David Tong’s Nightmare : The CMB has quantum correlations telling us the Secret of M-Theory but humanity stupidly built Planck and made measurements which loses this information.

Why do we care?

All is not lost! If a system has quantum correlations, then it doesn't obey classical correlation statistics -- we can check! (e.g. Bell's Inequality.)

General theorem (CHSH inequality) :

Quantum information cannot be represented by a local joint prob. distribution function.

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General theorem (CHSH inequality) :

Quantum information cannot be represented by a local joint prob. distribution function.

Entanglement is **not** *the only measure of quantumness!*

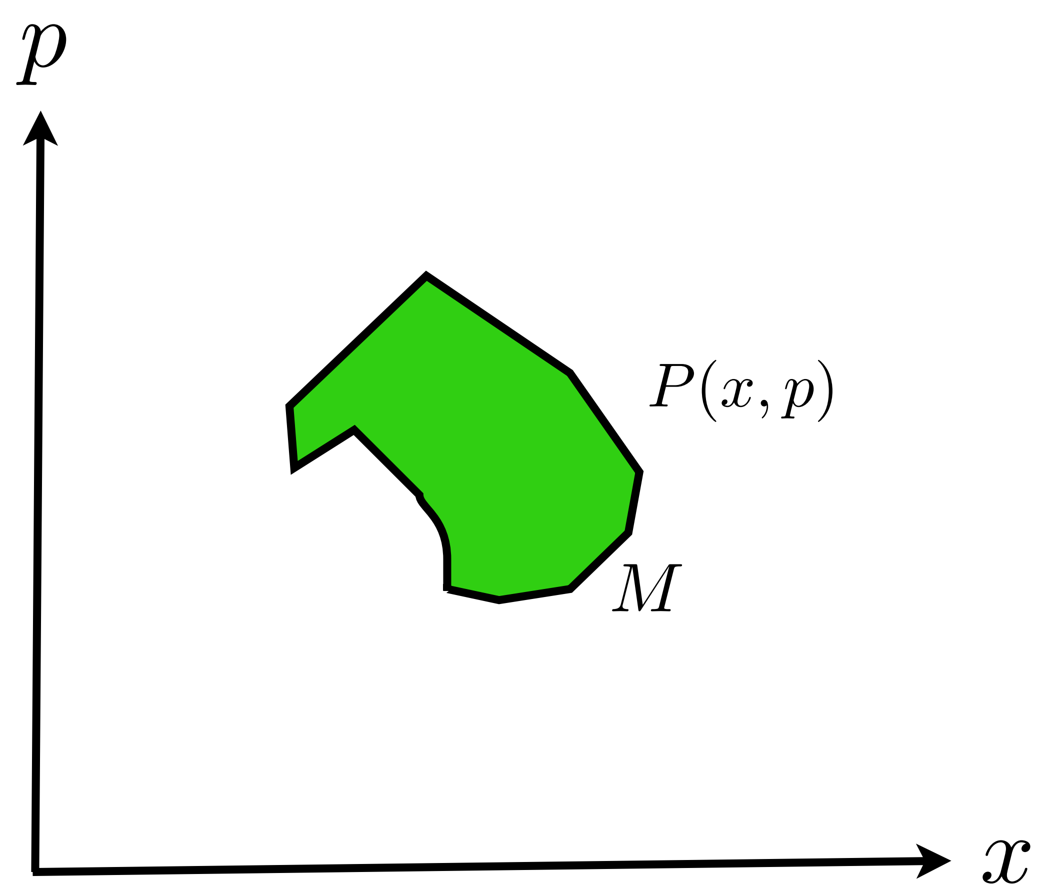
To construct such a statistic, we need to know the nature of the quantum correlations.

Classical vs Quantum States

Classical States : described by *joint* probability distribution functions (pdf) of observables $P(x,p)$

Probability of finding particle in region M

$$\text{Prob} = \int_M dx\ dp\ P(x,p)$$



Phase space for single particle state (x,p)

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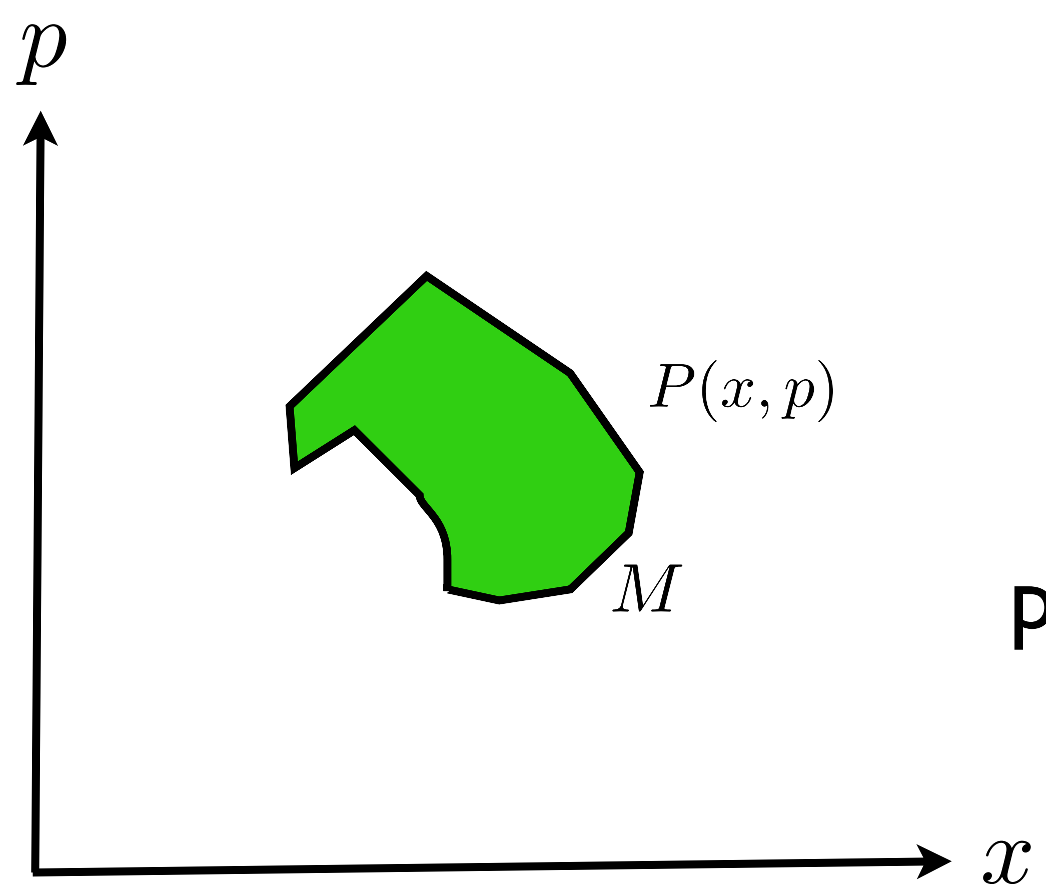

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$$\text{Prob} = \int_M dx\ dp\ P(x,p)$$

Probability = not sure where the particle is = "ambiguity" = entropy

Boltzmann/Shannon Entropy

$$H(P) = -\int dx\ P(x) \log P(x)$$

Classical vs Quantum States

Quantum States : described by *density matrices* ρ

A state vector $|u_i\rangle$ describes a *pure state.*

$\rho = \sum_i p_i |u_i\rangle\langle u_i|$ $\quad \sum_i p_i = 1$ A *mixture* of pure states $|u_i\rangle$

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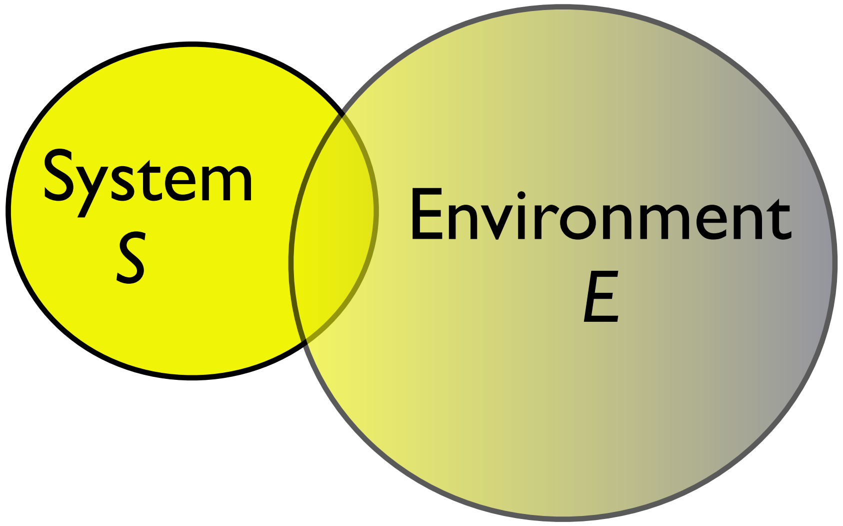
Pure states $\rho = |u\rangle\langle u|$ can evolve into *mixed states* under non-unitary operations in Open systems.



Combined "Bipartite" $\rho_{S\mathcal{E}}$

Access only to S : $\rho_S = \mathrm{Tr}_{\mathcal{E}}\, \rho_{S\mathcal{E}}$

"Ambiguity" = Von Neumann Entropy $S(\rho_S) = -\mathrm{Tr}(\rho_S \log \rho_S)$

Classical vs Quantum States

Quantum States : described by *density matrices* ρ

Given bipartite system, it is *separable* if

$$\rho = \sum_i p_i |u_i\rangle_S |e_i\rangle_{E\,E}\langle e_i|_S\langle u_i|$$

example $\rho = \frac{1}{2}|0_S\rangle|0_E\rangle\langle 0_S|\langle 0_E| + \frac{1}{2}|1_S\rangle|1_E\rangle\langle 1_S|\langle 1_E|$

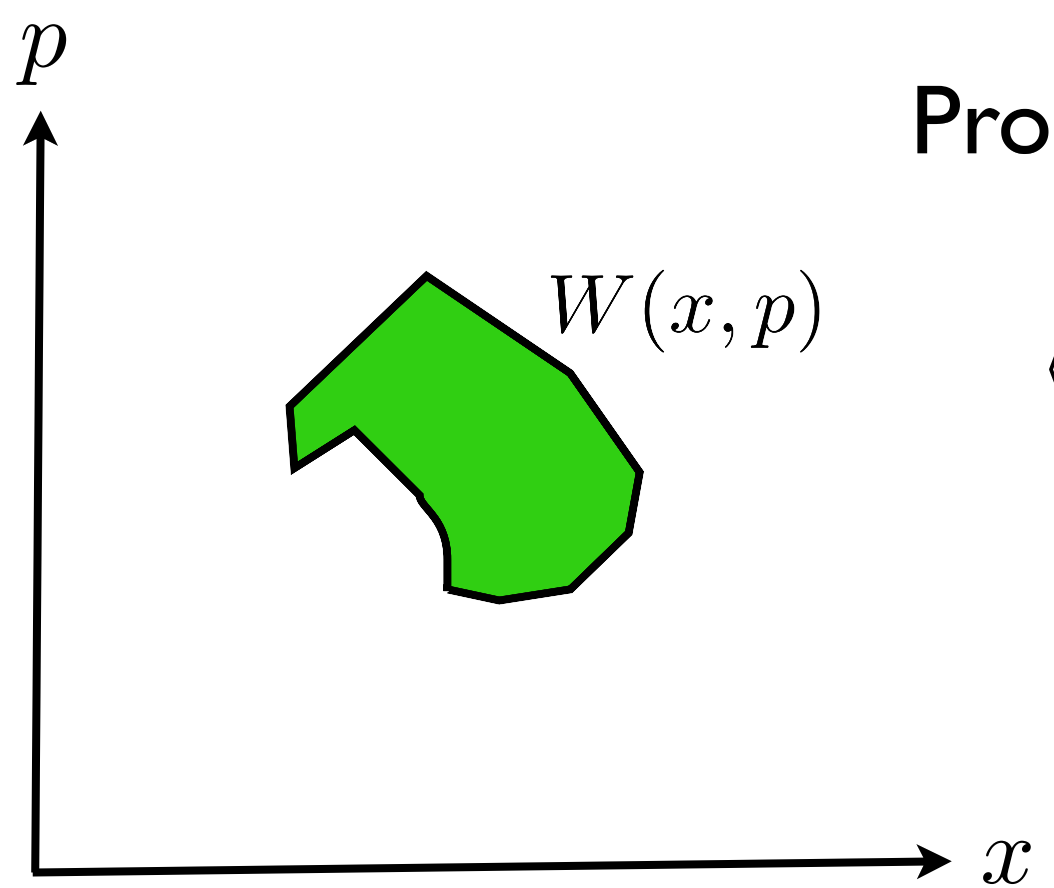
Pure states : *separability* = non-entanglement = classical pdf.

Mixed states : *separability* = non-entanglement $\neq$ classical pdf (*quantum discord)*

Classical vs Quantum States

Equivalent "quasi-pdf" picture : Wigner distribution

$$W(x,p) = \frac{1}{\pi}\int_{-\infty}^{\infty} e^{2ipy}\langle x - \frac{y}{2}|\rho|x + \frac{y}{2}\rangle$$



Phase space for single particle state (x,p)

Prob density of x is then

$$\langle x|\rho|x\rangle = \frac{1}{2\pi}\int_{-\infty}^{\infty} dp\, W(x,p)$$

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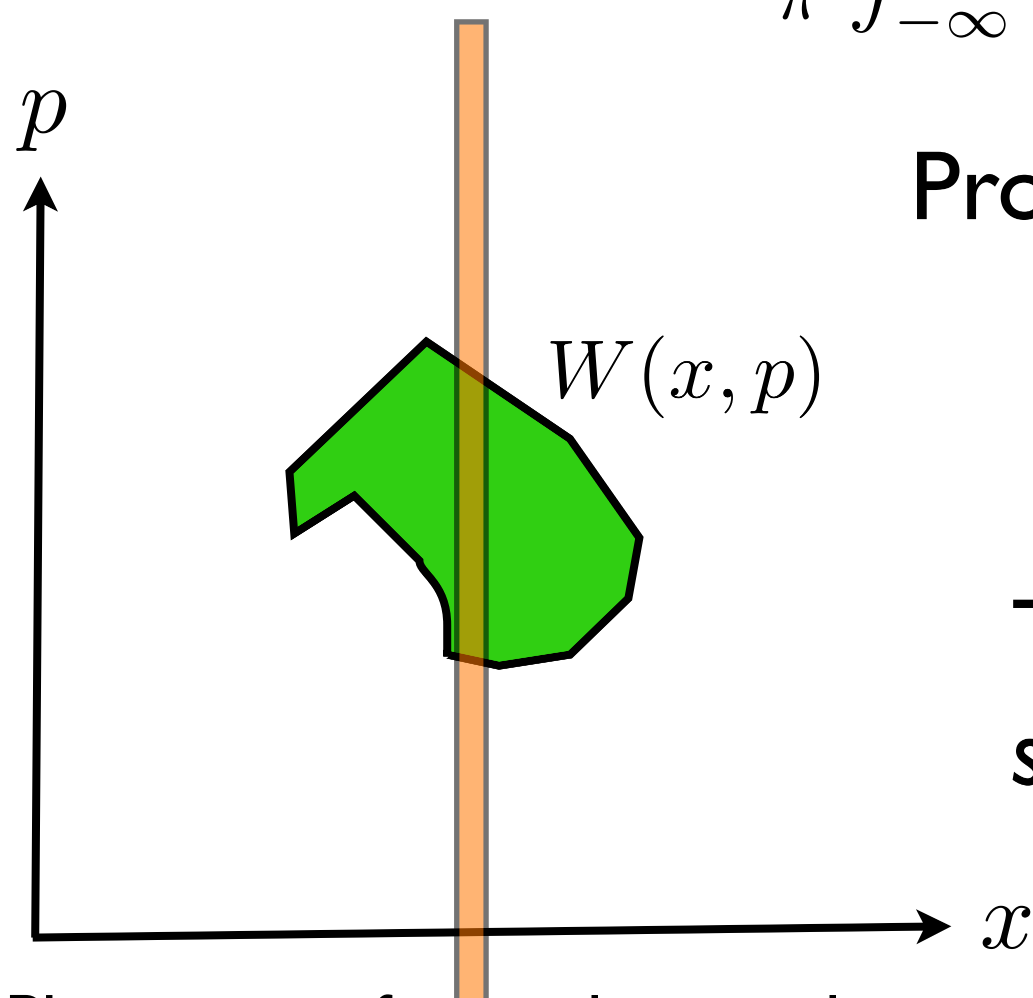

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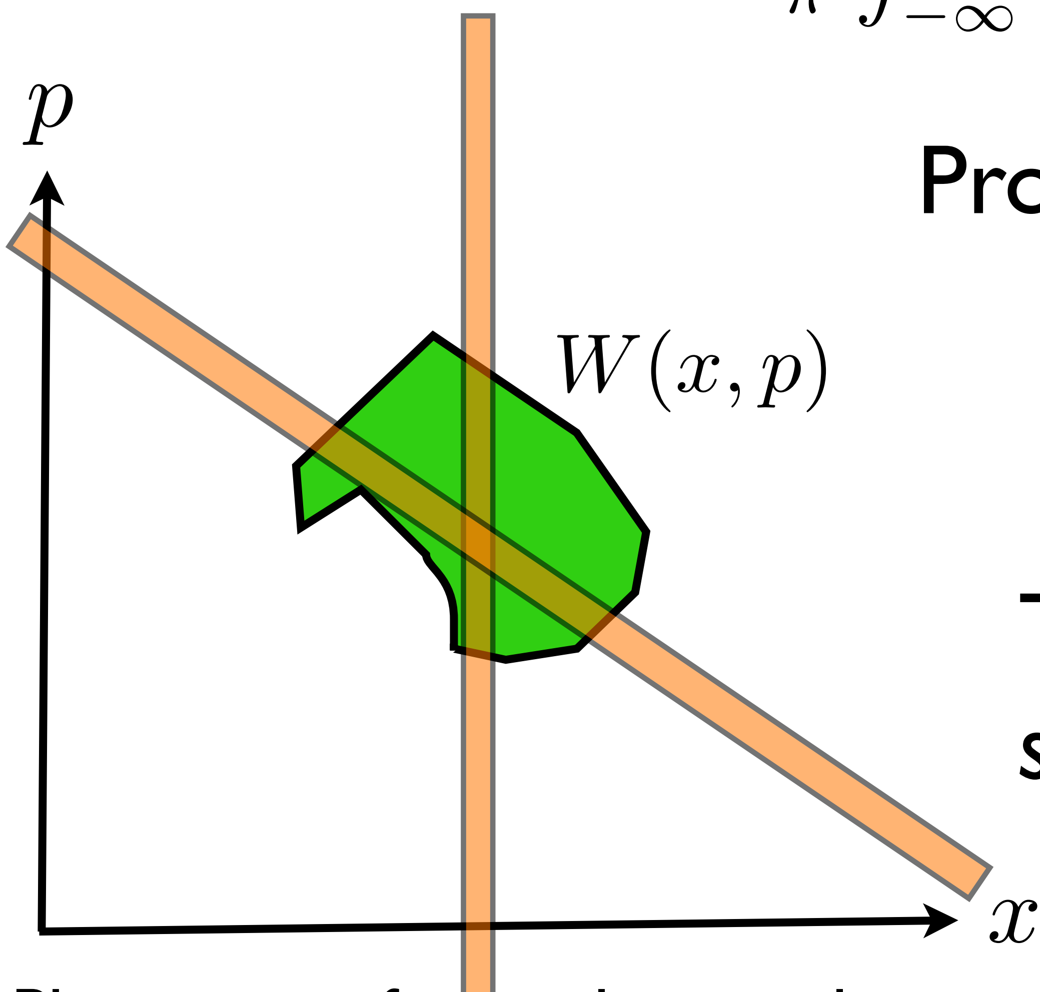

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Bertrand and Bertrand's theorem :
any infinite strip would do.

Decoherence in a nutshell

Consider pure state $|S\rangle = \alpha|0\rangle + \beta|1\rangle$

Coherence = quantum phase of α and β preserved.

$$\rho = |S\rangle\langle S| = |\alpha|^2|0\rangle\langle 0| + |\beta|^2|1\rangle\langle 1| + \alpha\beta^*|0\rangle\langle 1| + \alpha^*\beta|1\rangle\langle 0|$$

$$= \begin{pmatrix} |\alpha|^2 & \alpha\beta^* \\ \alpha^*\beta & |\beta|^2 \end{pmatrix}$$

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Decoherence : couple *S* to environment *E*.

$$|S\rangle \otimes |E\rangle = (\alpha|0\rangle + \beta|1\rangle)|E\rangle \stackrel{couplings}{\longrightarrow} \alpha|0\rangle|E(0)\rangle + \beta|1\rangle|E(1)\rangle$$

If we have only access to *S*, then

$$\rho_S = \mathrm{Tr}_E \rho_{SE} = \begin{pmatrix} \rho_{00} & \rho_{01} \to 0 \\ \rho_{10} \to 0 & \rho_{11} \end{pmatrix} \to \begin{pmatrix} |\alpha|^2 & 0 \\ 0 & |\beta|^2 \end{pmatrix}$$

Final matrix is *mixed* and phase info is lost.

Decoherence basis is crucial

Secret assumption : decoherence occurred in $\{|0\rangle, |1\rangle\}$ basis.

If decoherence occurs at rotated basis

$$\{\cos\theta|0\rangle + e^{i\phi}\sin\theta|1\rangle, -e^{-i\phi}\sin\theta|0\rangle - \cos\theta|1\rangle\}$$

classical pdf obtained from decoherence do not recover all the quantum information.

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Quantum nature of cosmological perturbations depends on how they interact with environment and how we measure them.

What about inflation?

Starobinsky, Polarski (1998)

Single mode Hamiltonian for cosmological perturbations

$$\hat{H}_k = \frac{1}{2}\left(p_k^2 + k^2 y_k^2 + \frac{2a'}{a'} y_k p_k\right)$$

$$\delta\phi_k \equiv y_k p_k = \frac{\partial L(y_k, y_k')}{\partial y_k'} = y_k' - a'/a y_k$$

$\hat{H}_k$ is a unitary evolution operator.

Schrodinger's equation of wave function $\psi(y,\eta)$

$$i\hbar \frac{\partial \psi(y,\eta)}{\partial \eta} = \hat{H}_k \psi(y,\eta).$$

with solution $\psi(y,\eta) = \left(\frac{2\Omega_R(\eta)}{\pi}\right)^{1/4} \exp(-(\Omega_R + i\Omega_I)y^2)$

for inflation background $\Omega_R \to k e^{-2r}$, $\Omega_I \to -k e^{-r}$

$$r = \#\text{efolds}$$

Cosmological "Squeezed states"

Construct density matrix

$$\rho_S = |\psi\rangle\langle\psi| = \frac{2\Omega_R}{\pi} \exp\left[-(\frac{\Omega_R}{2}(y-y')^2 - \frac{\Omega_R}{2}(y+y')^2 - i\Omega_I(y^2 - y'^2)\right]$$

As $\Omega_R/k = e^{-2r} \ll 1 \Rightarrow$ off-diagonal terms get killed off

So far unitary evolution : *no decoherence* so still *pure state*.

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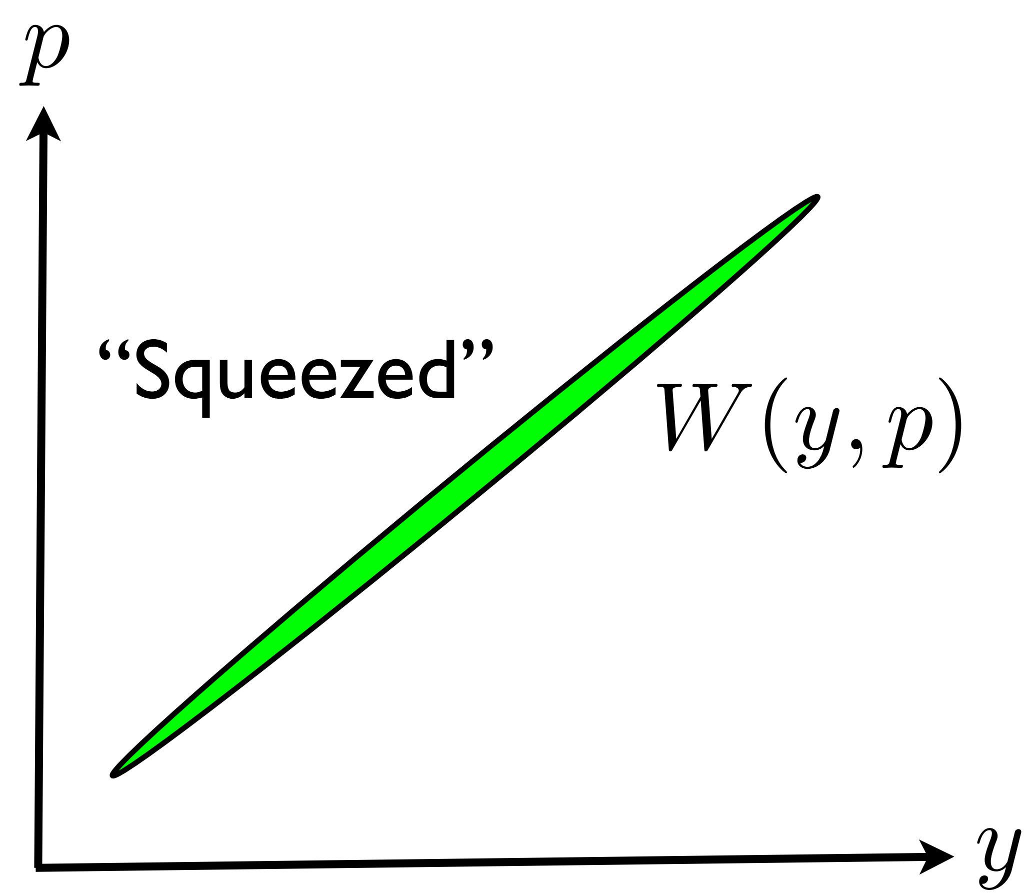

Wigner function is a *gaussian* with ellipsoid axes (e^{-r}, e^{r})

$$W(y,p) = \frac{1}{\pi} \exp\left[-\frac{1}{2} \mathbf{x} \sigma_S^{-1} \mathbf{x}^T \right]$$

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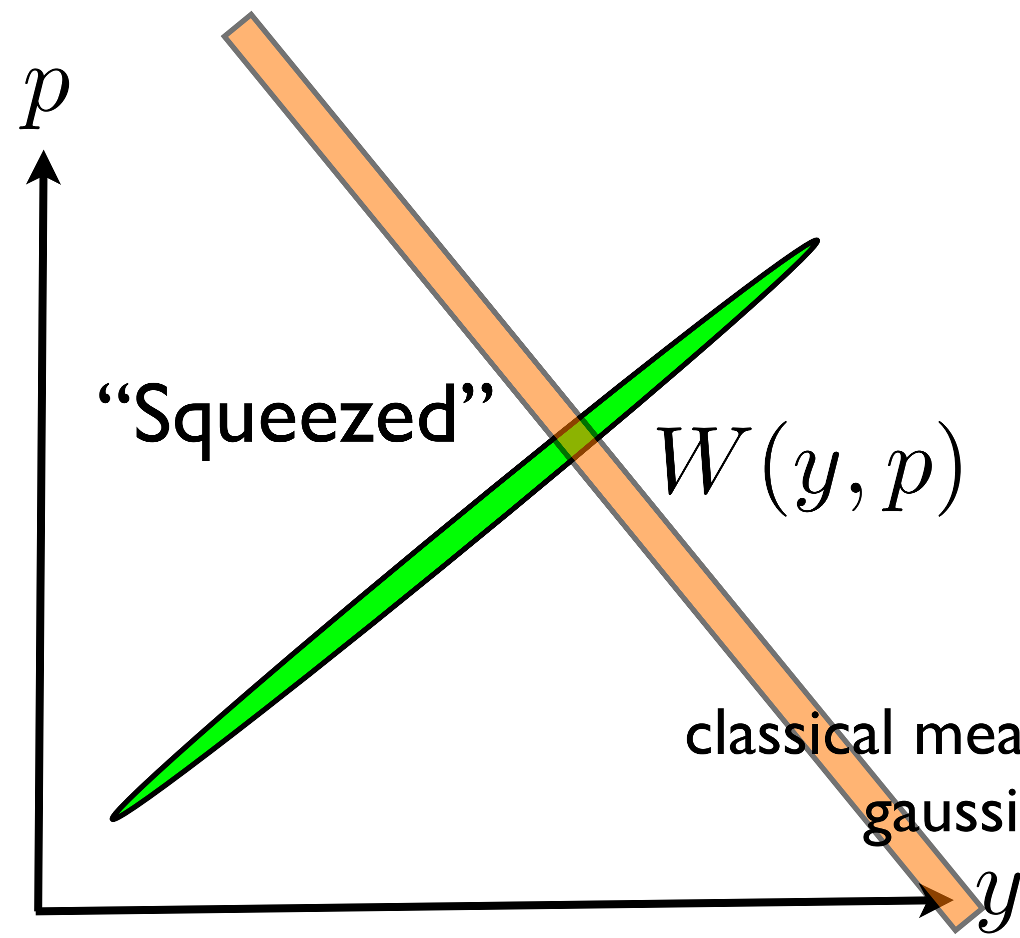

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Cosmological "Squeezed states"

Starobinsky, Kiefer and Polarski's decoherence ansatz (1998)

Couple to environment ρ_{SE} , we add a decoherence term

$$\rho'_S = \mathrm{Tr}_E \rho_{SE} = \rho_S \times \exp[-\frac{\zeta}{2}(y-y')^2] \qquad \zeta \gg \Omega_R$$

New *mixed state* is still a gaussian but with axes (e^r, ζ)

Question 1 : is the decoherence basis parallel to $\{y_k\}$?

Question 2 : how to quantify "quantumness"?

We will model ρ_{SE} and use **quantum discord** to answer both questions.

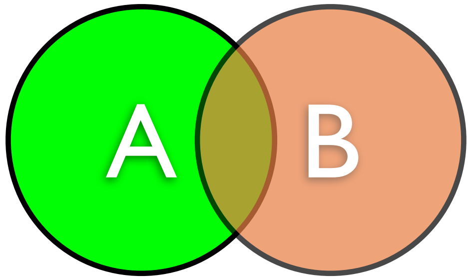
Ollivier and Zurek (2001)
Henderson and Vedral (2001)

Quantum Discord

Ollivier and Zurek (2001)

Classical Mutual Information

$$J(A:B) = H(A) - H(A|B)$$



A and B correlated, mutual info is how much we learn more about A when B is found out.

Classical pdf, Bayes Theorem $H(A|B) = H(A,B) - H(B)$

Get equivalent expression

$$I(A:B) = H(A) + H(B) - H(A,B)$$

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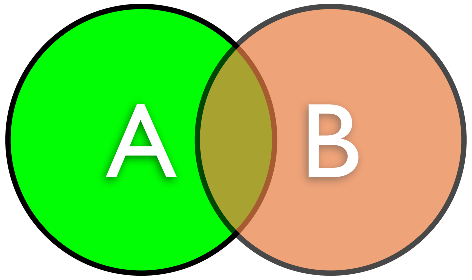

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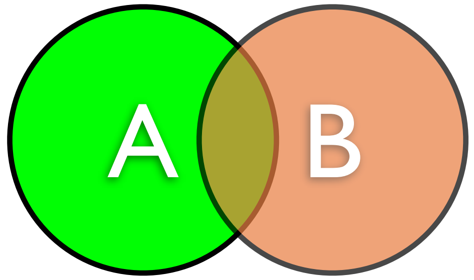
Quantum generalization: replace Shannon with Von Neumann $H(A) \rightarrow S(A)$

$$I(A:B) \rightarrow \mathcal{I}(A:B) = S(A) + S(B) - S(A,B)$$

???

$$J(A:B) \rightarrow \mathcal{J}(A:B) = S(A) - S(A|B)$$

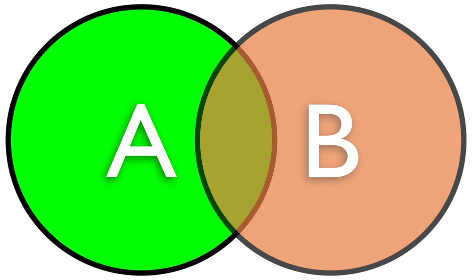
Quantum Discord



What is the quantum version of $S(A|B)$?

"Finding out B" = making measurement on B but quantum mechanically this will *disturb* A!

Quantum Discord



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Ollivier and Zurek (2001) propose the following:

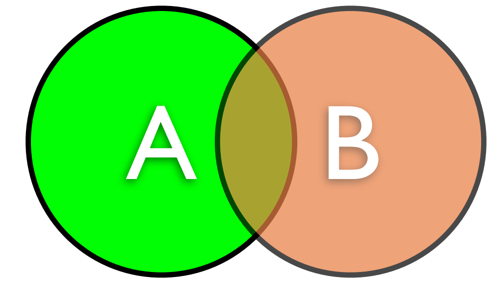
1. Given some basis of measurements on B $\{\Pi_k^B\}$
2. Each Π_k^B measurement occurs with prob. P_k and

$$\rho_{AB} \to \rho_{A|B=\Pi_k^B} = \frac{\rho_{AB}\Pi_k^B}{P_k}$$

3. Define $S(A|B=\{\Pi_k^B\}) = \sum_k P_k S(\rho_{A|B=\Pi_k^B})$
4. Quantum Discord is then

$$\delta(A:B)_{\Pi_k^B} = \mathcal{I}(A:B) - \mathcal{J}(A:B)_{\Pi_k^B} > 0$$

Quantum Discord



Some facts on Discord :

1. Zero discord $\delta(A:B)_{\Pi_k^B} = 0$ means decoherence occurred in "pointer basis" and no entanglement. Can define to be "Classical".

2. Mixed Separable state can have *non-zero* discord. No entanglement $\neq$ no quantum correlations!

3. Basis-independent discord : minimize over all possible decoherence basis.

4. Recently shown separable 2 qubits computers with discord is exponentially faster than classical computers.

Datta, Shaji, Caves (2007)

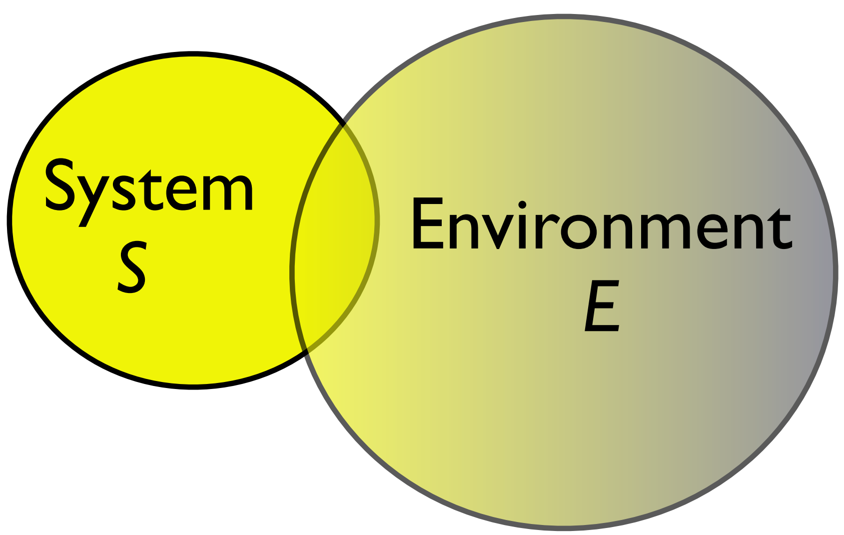
How to model system-environment?

Start with Starobinsky-Polarski-Kiefer Gaussian ansatz

$$\rho'_S \rightarrow \quad W(y,p) = \frac{1}{\pi} \exp\left[-\frac{1}{2} \mathbf{x} \sigma_S^{-1} \mathbf{x}^T \right] \,, \quad \sigma_S(\Omega_R, \Omega_I, \zeta)$$

We want to find a joint density matrix ρ_{SE} such that

$$\rho_S = \mathrm{Tr}_E(\rho_{SE})$$



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$$\rho_S = \mathrm{Tr}_E(\rho_{SE})$$

Not unique and continuous states are HARD. A way forward is to assume that ρ_{SE} is also Gaussian.

Central Limit Theorem

How to model system-environment?

A unique *pure gaussian* ρ_{SE} can be constructed ("gaussian purification" of ρ_S).

$$W(y_1, y_2, p_1, p_2) = \frac{1}{\pi}\exp\left[-\frac{1}{2}\mathbf{x}\sigma_{SE}^{-1}\mathbf{x}^T\right] \qquad \mathbf{x} = (y_1, p_1, y_2, p_2)$$

$$\sigma_{SE} = \begin{pmatrix} \frac{1}{\lambda} & \frac{a}{\lambda} & \frac{\sqrt{\zeta}}{\lambda^{3/4}\sqrt[4]{\zeta+\lambda}} & 0 \\ \frac{a}{\lambda} & \frac{a^2}{\lambda}+\zeta+\lambda & \frac{a\sqrt{\zeta}}{\lambda^{3/4}\sqrt[4]{\zeta+\lambda}} & -\sqrt{\zeta}\sqrt[4]{\frac{\zeta+\lambda}{\lambda}} \\ \frac{\sqrt{\zeta}}{\lambda^{3/4}\sqrt[4]{\zeta+\lambda}} & \frac{a\sqrt{\zeta}}{\lambda^{3/4}\sqrt[4]{\zeta+\lambda}} & \sqrt{\frac{\zeta+\lambda}{\lambda}} & 0 \\ 0 & -\sqrt{\zeta}\sqrt[4]{\frac{\zeta+\lambda}{\lambda}} & 0 & \sqrt{\frac{\zeta+\lambda}{\lambda}} \end{pmatrix}$$

$$a = -\Omega_I/k\ ,\ \lambda = \Omega_R/k\ ,\ \xi = \zeta/k$$

How to model system-environment?

A unique *pure gaussian* ρ_{SE} can be constructed ("gaussian purification" of ρ_S).

Perturb around ρ_{SE} to obtain general mixed states.

Given pure state bipartite ρ_{SE} we can compute the discord (turns out to be basis independent)

$$\delta(A:B) = \frac{1+\sqrt{1+\chi}}{2}\log\left(\frac{1+\sqrt{1+\chi}}{2}\right) - \frac{\sqrt{1+\chi}+1}{2}\log\left(\frac{\sqrt{1+\chi}-1}{2}\right) \ , \ \chi = \frac{\zeta}{\Omega_R} \gg 1$$

Zero when $\zeta = 0$ so the non-decohered perturbations has classical statistics!

How to model system-environment?

This is actually equal to the Von Neumann entropy

$$\delta(A:B) = S(\rho_S)$$

(Kiefer, Starobinsky and Polarski 1999)

Reason : ρ_{SE} is pure, and discord captures *mixed* state quantum correlations beyond entanglement entropy.

Still work in progress : mixed ρ_{SE}

Summary

1. Environment picks out the basis of which we measure quantum cosmological correlations.

2. Even if cosmological perturbations are highly squeezed, measurements in off-basis may retain "quantum" correlations.

3. Propose a gaussian construction of joint ρ_{SE} perturbations-environment bipartite state

4. Propose quantum discord as robust measure of quantum correlations in the joint system.