Quantum gravity and inflation

lessons from 2d toy models

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w/ Wynton Moore. Arxiv: 1401.7681 & to appear
(older work: da Cunha & EJM, hep-th/0303087)

Outline

Quantum gravity is confusing and hard.

- How do quantum gravity effects interact with inflationary dynamics? What is a quantum superposition of cosmological states? When and how do cosmological fluctuations decohere?
- Is there some theory of initial conditions?
- What does the late-time universe look like?
- If there is a landscape, how does the theory explore it?
- etc etc etc ...

Ultimately would like to understand string/M theory in a cosmological setting in its full nonperturbative glory.

Today, a more modest ambition – construct toy models incorporating some of the structures we wish to study:

- Dynamical gravity
- Matter fields, so we can study inflation
- Spatial extent, so we can have horizons, inhomogeneity, etc. *Want to go beyond minisuperspace.*
- A quantum theory of all the above, preferably UV complete; in particular backreaction of gravitational field on inflaton fluctuations on various scales.

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2D Liouville gravity gives us all of the above

The Einstein tensor vanishes identically in 2d
$\mathcal{S}_{\rm EH}$ is topological $\Rightarrow$ no geometrodynamics.
Next best thing: The Liouville equation

$$R = \Lambda$$

which describes 2d (anti)deSitter spacetime.
Classical action:

$$\mathcal{S}_{\rm grav} = \frac{1}{2\pi\gamma^2}\int\sqrt{-g}\left[-(\nabla\chi)^2 - R\chi - \Lambda\right]$$

yields e.o.m.'s

$$\nabla^2\chi = R \quad , \qquad \nabla^2\chi = \Lambda$$

- Add matter; homogeneous FRW background e.o.m.'s

$$H^2 = +\frac{\gamma^2}{4}\left[\dot{X}_0^2 + \mathcal{V}(X_0) + \rho_\perp\right]$$
$$\dot{H} = -\frac{\gamma^2}{4}\left[\dot{X}_0^2 + \rho_\perp + P_\perp\right]$$
$$0 = \ddot{X}_0 + H\dot{X}_0 + \mathcal{V}_{,X}$$

are the usual Friedman equations if we identify $m_p^2 = \frac{\gamma^2}{4}$.

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- Slow-roll parameters

$$\epsilon_H = \frac{m_p^2}{2}\left(\frac{\mathcal{V}_{,X}}{\mathcal{V}}\right)^2$$
$$\eta_H = m_p^2\left[\left(\frac{\mathcal{V}_{,XX}}{\mathcal{V}}\right) - \frac{1}{2}\left(\frac{\mathcal{V}_{,X}}{\mathcal{V}}\right)^2\right]$$

⋆ Parametrize 2d metric

$$g_{ab} = e^{2\phi}\hat{g}_{ab} = e^{2\phi}\begin{pmatrix} -N_t^2 + N_x^2 & N_x \\ N_x & 1 \end{pmatrix}$$

⋆ Degrees of freedom:

- 5 scalars ϕ, N_t, $N_x = \partial_x B$, auxiliary scalar χ, inflaton X
- 2 constraints $\mathcal{H} = 0 = \mathcal{P}$, and 2 gauge freedoms
- **Total:** $5 - 2 - 2 = 1$ physical gauge-invariant scalar

$$v = \boldsymbol{x} + (X_0'/\phi_0')\boldsymbol{\chi}$$

⋆ Features:

- Quadratic action identical to 4d (Mukhanov-Sasaki action)

$$\mathcal{S}_2 = \frac{1}{8\pi}\int\left((v')^2 - (\partial_x v)^2 + \frac{z''}{z}v^2\right)$$

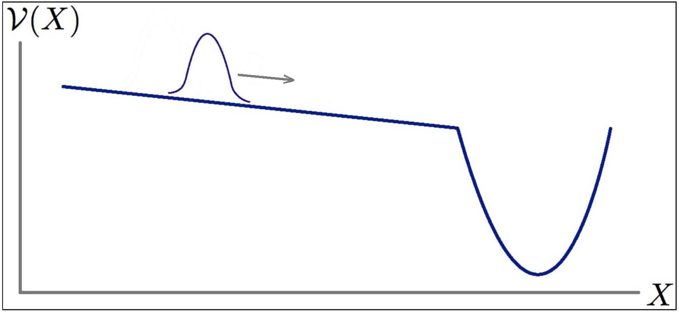
where

$$z = (X_0'/\phi_0')$$

- 1-1 map to scalar sector in 4d
- Cubic action also highly analogous

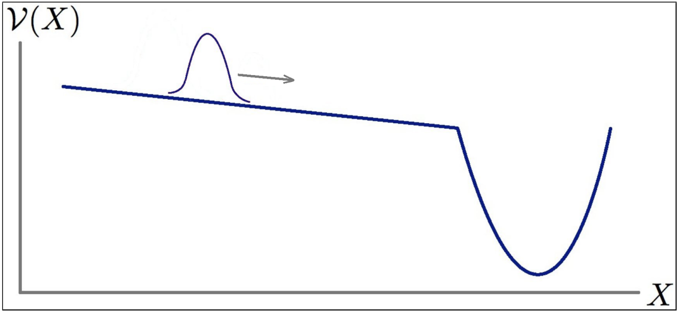
Beyond perturbation theory

Slow-roll eternal inflation – the standard argument



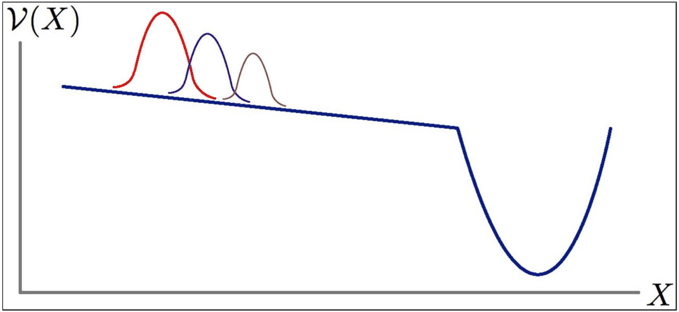
The standard argument...[Linde, Vilenkin, Starobinsky, Guth, *et al*]

- Consider evolution of X in a Hubble volume H^{-n} over a Hubble time H^{-1}



The standard argument...[Linde, Vilenkin, Starobinsky, Guth, *et al*]

- Classical rolling: $\delta X_{\rm cl} \approx \dot{X}_0 H^{-1}$
- Fluctuation of X wavefunction: $|\delta X_{\rm qu}| \lesssim 1$
- New Hubble scale $\sim \mathcal{V}(X_0 + \delta X_{\rm cl} + \delta X_{\rm qu})^{1/2}$



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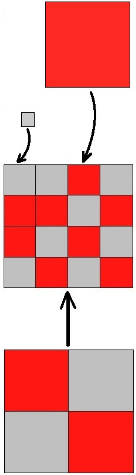
- Stochastic evolution of physical volume distribution $P_c(X,t)$ [Linde *et al*]

$$\frac{\partial P_c}{\partial t} = \frac{\partial}{\partial X}\left(\mathcal{D}^{1/2}\frac{\partial(\mathcal{D}^{1/2}P_c)}{\partial X} + \kappa\mathcal{V}_{,X}(X)P_c\right) + 3H(X)P_c$$



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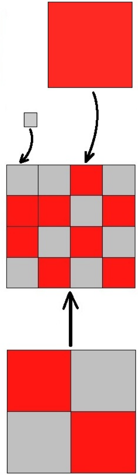
- Phase transition to SREI [Creminelli *et al*]

$$\frac{\pi^{1+n/2}}{n\Gamma(n/2)}\frac{\dot{X}_0^2}{H^{n+1}} \lesssim 1$$



- What is the mechanism for rewarding larger volumes?
- Does it survive a proper incorporation of quantum gravity backreaction?
- Could worry that standard treatment backreacts metric on quantum matter fluctuations classically – decoherence is assumed. What is producing this decoherence?
- Quantum fluctuations of geometry have been ignored. Are these independent of inflaton fluctuations? How do we extract the classical geometry in the presence of such fluctuations? [Bousso *et al*, Susskind]
- In principle one should evolve quantum wavefunctions of inflaton and metric together, then interpret the result.

Consider $\mathcal{V}(X) \to \Lambda \cosh[\beta X]$

- Work in conformal gauge $N_t = 1$, $N_x = 0$; curvature
$$R = -2e^{-2\phi}\nabla^2\phi$$
- Relevant part of action
$$\mathcal{S} = \frac{1}{8\pi}\int\sqrt{-\hat{g}}\left[\frac{4}{\gamma^2}(\hat{\nabla}\phi)^2 - (\hat{\nabla}X)^2 - \Lambda e^{2\alpha\phi}\cosh[\beta X]\right]$$

What the reward mechanism is **not**

The peak of the wavefunction does not run up the potential

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- $SO(1,1)$ symmetry structure of kinetic term

$$\begin{pmatrix} 2\phi/\gamma \\ X \end{pmatrix}, \quad \begin{pmatrix} \gamma\alpha \\ \pm\beta \end{pmatrix}$$

- Parametrize boost in field space

$$\begin{pmatrix} 2\phi/\gamma \\ X \end{pmatrix} = M(-\lambda)\begin{pmatrix} 2\tilde{\phi}/\gamma \\ \tilde{X} \end{pmatrix}, \quad M(\lambda) = \begin{pmatrix} \cosh\lambda & \sinh\lambda \\ \sinh\lambda & \cosh\lambda \end{pmatrix}$$

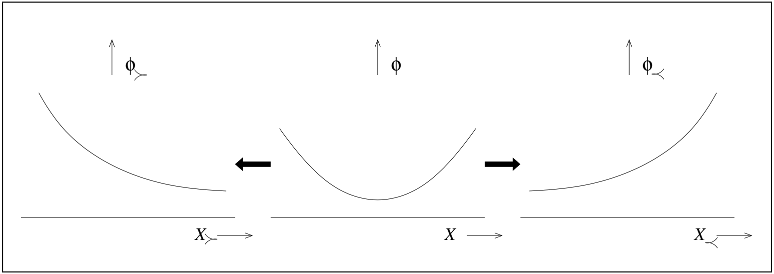
Field redefinitions

Relating the cosh to quintessence

Can choose a boost transformation in either of two ways

$$e^{\alpha\phi+\beta X} \to e^{\alpha_\succ \phi_\succ} \qquad or \qquad e^{\alpha\phi+\beta X} \to e^{\tilde\alpha_\prec \phi_\prec + \tilde\beta X_\prec}$$

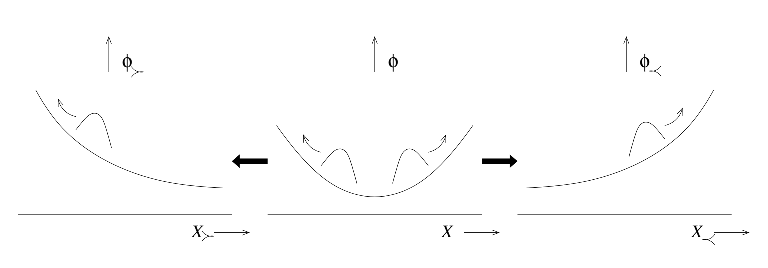
$$e^{\alpha\phi-\beta X} \to e^{\tilde\alpha_\succ \phi_\succ - \tilde\beta X_\succ} \qquad\qquad e^{\alpha\phi-\beta X} \to e^{\alpha_\prec \phi_\prec}$$



Field redefinitions

Relating the cosh to quintessence

If SREI moves the peak of the wavepacket (so the quantum corrected evolution is such that large scale factor is correlated with large values of the inflaton potential), then we predict



These predictions are mutually incompatible.

- Liouville theory is deSitter spacetime; quantum deSitter is of some interest on its own.
- Moreover, the preceding boost field redefinition means that if one can solve the quantum theory of deSitter space, one can solve quintessence in the potential

$$\mathcal{V}(\tilde{X}) = \Lambda \exp[\beta \tilde{X}]$$

using the boost that relates

$$\exp[\alpha\tilde{\phi} + \beta\tilde{X}] = \exp[\gamma\phi]$$

- Criteria for eternal inflation satisfied for $\beta \ll \gamma$.

- *Bäcklund equations* relate Liouville ϕ to free field ψ

$$\partial_t\phi = \partial_x\psi + \sqrt{\Lambda}e^{\phi}\cosh\psi \quad , \quad \partial_x\phi = \partial_t\psi + \sqrt{\Lambda}e^{\phi}\sinh\psi$$

- Relates e.o.m.'s

$$\hat{\nabla}^2\phi = \Lambda e^{2\phi} \quad \Leftrightarrow \quad \hat{\nabla}^2\psi = 0$$

- Canonical transformation generated by

$$W[\phi,\psi] = \int dx\left(\phi\partial_x\psi + \sqrt{\Lambda}e^{\phi}\cosh\psi\right)$$

Classical Liouville theory

the Bäcklund transformation

- Classical solutions characterized by $A(x^+), B(x^-)$

$$\psi = \frac{1}{2}\ln(-\partial_+ A/\partial_- B) \quad \Longleftrightarrow \quad \Lambda e^{2\phi} = -4\frac{\partial_+ A\,\partial_- B}{(A-B)^2}$$

- Example: "hyperbolic" solution of Liouville theory

$$A = e^{\varepsilon x^+}\ , \quad B = e^{-\varepsilon x^-}$$

$$\Lambda e^{2\phi} = \frac{\varepsilon^2}{\sinh^2(\varepsilon t)}\ , \quad \psi = \varepsilon t$$

  - ◇ describes Milne universe in far past $t \to -\infty$ (radiation dominated FRW singularity),
  - ◇ dS asymptotics as conformal time $t \to 0$.

Bäcklund transform

Mapping Liouville to free field theory

- Wavefunctionals $\Psi[\phi]$ and $\tilde{\Psi}[\psi]$ formally satisfy functional Schrödinger equations

$$i\frac{\partial}{\partial t}\Psi[\phi] = H_\phi \Psi[\phi] \ , \quad i\frac{\partial}{\partial t}\tilde{\Psi}[\psi] = H_\psi \tilde{\Psi}[\psi]$$

- Integral transform relates wavefunctionals

$$\Psi[\phi] = \int \mathcal{D}\psi \, e^{(2i/\gamma^2)W[\phi,\psi]}\tilde{\Psi}[\psi]$$

- Ground state (Bunch-Davies) wavefunctional for free field

$$\tilde{\Psi}_0[\psi] = C_0 \, e^{-ik_\psi \psi_0} \prod_{k>0} e^{-\omega_k |\psi_k|^2/\gamma^2}$$

- In the Liouville + free field frame, $\Psi[\phi, X] = \Psi_0[X]\ \Psi_0[\phi]$

$$\Psi_0[X] = C_X \exp\Big[\Big(-ik_X x_0 - \sum_{k>0} \frac{\omega_k}{2} |X_k|^2\Big)\Big]$$

$$\Psi_0[\phi] = \int \mathcal{D}\psi\, e^{(2i/\gamma^2) W[\phi,\psi]}\, \tilde{\Psi}_0[\psi]$$

$$\tilde{\Psi}_0[\psi] = C_0\, e^{-ik_\psi \psi_0} \prod_{k>0} e^{-\omega_k |\psi_k|^2/\gamma^2}$$

- Superpose zero-mode momenta to form localized wavepackets
- In the quintessence frame, the wavefunction is localized in

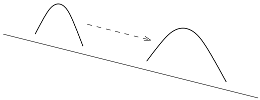
$$\phi = \tilde{\phi} \cosh\lambda - \tilde{X}(\gamma/2) \sinh\lambda$$
$$X = -\tilde{\phi}(2/\gamma) \sinh\lambda + \tilde{X} \cosh\lambda$$

An alternative interpretation of SREI
(thanks to M. Kleban)

- While the center of the wavepacket follows the classical trajectory, there are fluctuations.
- It's all 2d free field theory:

$$\Delta^2 = \langle X(0)X(\sigma)\rangle \sim \log[\sigma]$$

- Qualitatively, the wavepacket locally moves down the potential at a speed v



An alternative interpretation of SREI

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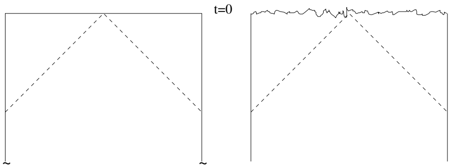
- The probability distribution averaged over a region of fixed comoving size $\sigma = \sigma_0\, e^{-H\tau}$ is looking like

$$\exp[-(X - v\tau)^2/\Delta] \sim \exp[-(X - v\tau)^2/H\tau]$$

- Probability to remain at a particular place on the potential and still inflate at that Hubble rate is decreasing as $e^{-c\tau}$.
- On the other hand this domain is inflating at a rate $e^{H\tau}$.
- If the rules are to compute volume-weighted observables, then this sets up the sort of competition leading to a SREI transition for small enough v.
- Need to explain why physical observables should be weighted this way.

- Bäcklund transform is not innocuous. At the level of classical solutions,

$$\psi = \omega t + \epsilon(x^+) + \tilde{\epsilon}(x^-) \ \Leftrightarrow \ \Lambda e^{2\phi} \sim \frac{\omega^2 + \mathit{stuff}}{[\sinh(\omega t) + \frac{1}{2}(\epsilon + \tilde{\epsilon})]^2}$$



Cautionary comments

where there be dragons...

- Minisuperspace version of Bäcklund transform is Bessel transform. Which Bessel function you get depends on how you route the integration contour in complex ψ plane.
- Sensitive dependence on short distance structure near $t = 0$?
- Does the physics change depending on how we incorporate locations in the potential where inflation ends and reheating takes place?

If a universe falls in the landscape and nobody hears the BAO, did it really happen?

- 2d gravity is rich enough to model many aspects of inflationary dynamics
- Precise analog of scalar sector of inflationary perturbations
- Can now begin to ask questions about fully quantum dynamics, including gravitational backreaction
- Holds the promise to sharpen our understanding of a variety of issues
- Proposal for nonperturbative dS quantum gravity, which also has quintessence interpretation