Leonardo Senatore (Stanford)

Effective Field Theory of Large Scale Structure

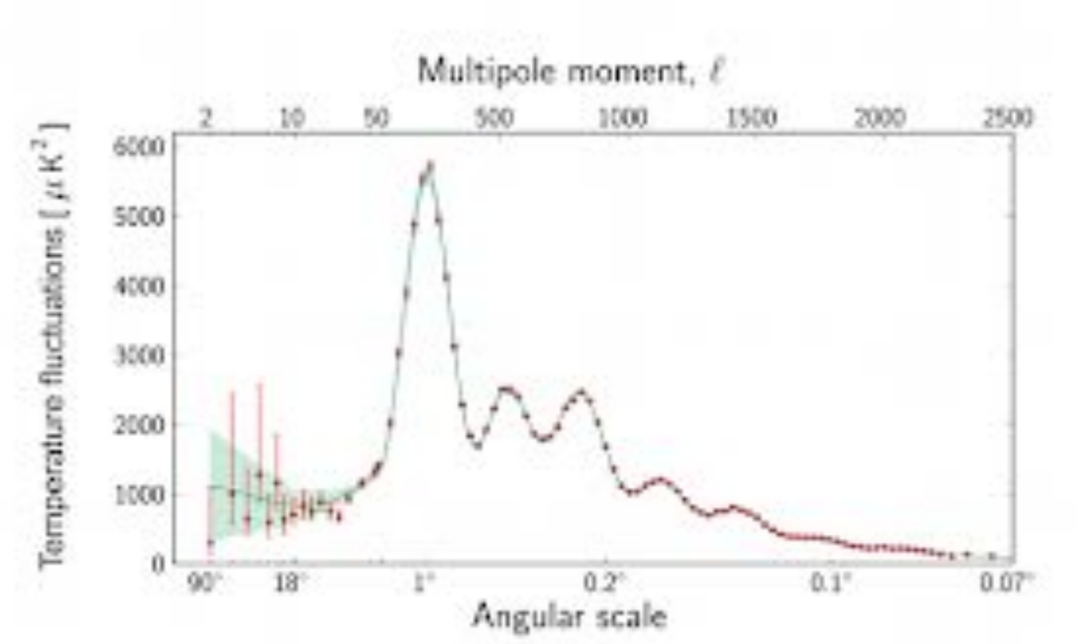
the way to go for inflation

How do we probe inflation

- The only observable we are testing from the background solution is

  $\Omega_K \lesssim 3 \times 10^{-3}$

- All the rest, comes from the fluctuations



- For the fluctuations
  - they are primordial
  - they are scale invariant
  - they have a tilt $n_s - 1 \simeq -0.04 \sim \mathcal{O}\left(\frac{1}{N_e}\right)$
  - they are quite gaussian

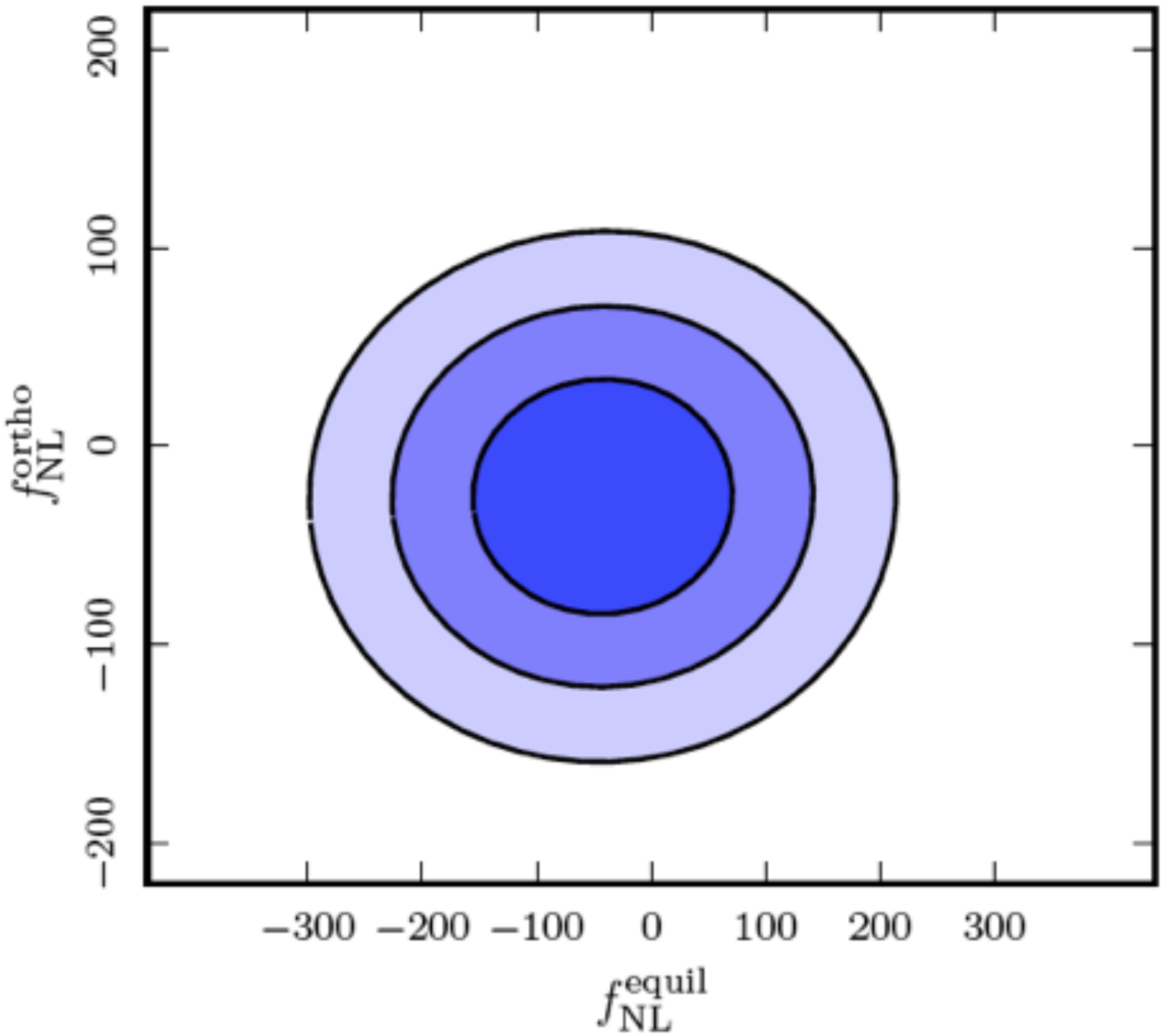
    $$\mathrm{NG} \sim \frac{\langle\zeta^3\rangle}{\langle\zeta^2\rangle^{3/2}} \lesssim 10^{-3}$$

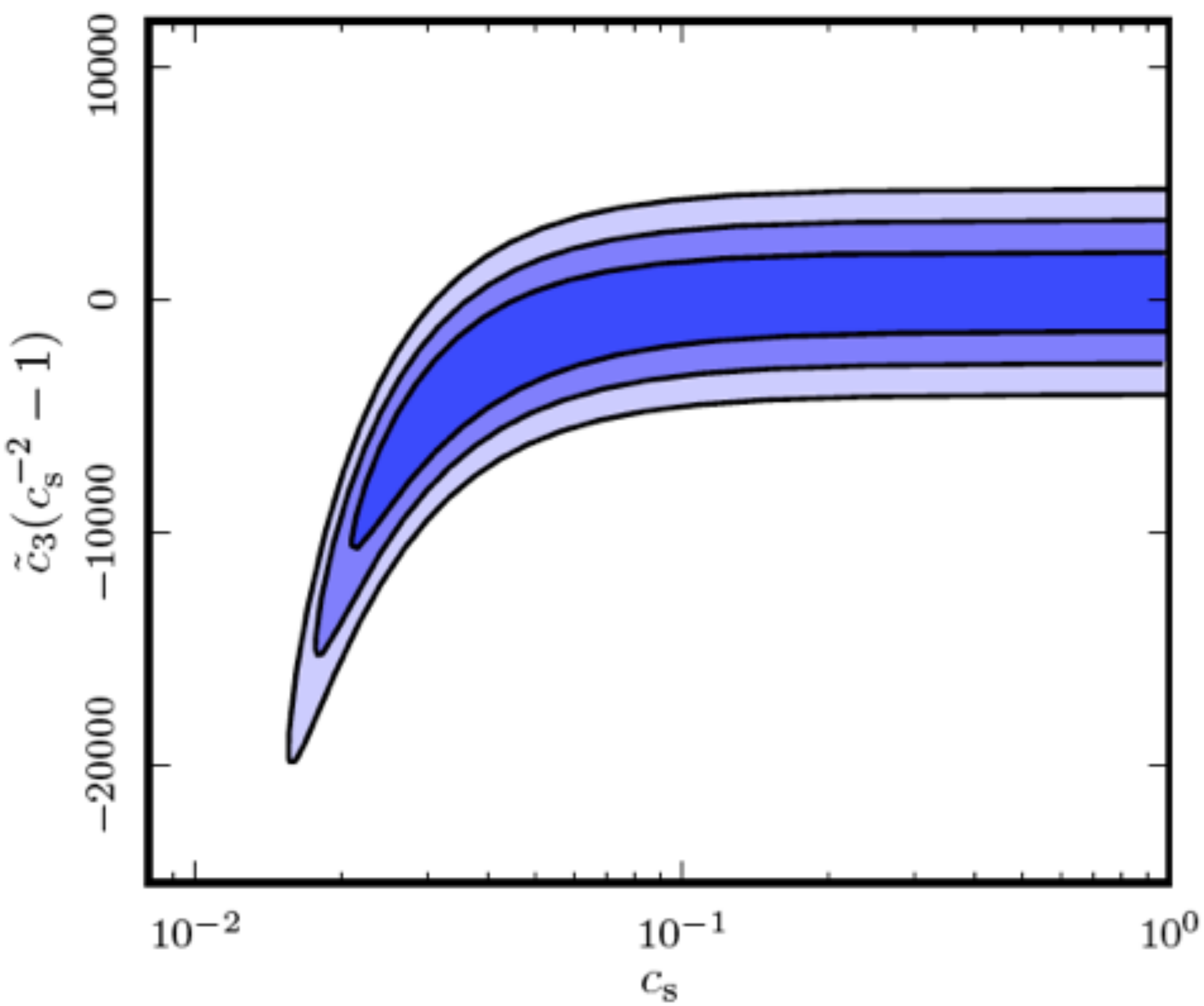
  - both scalar and maybe tensors

Limits in terms of parameters of a Lagrangian

- $$S = \int d^4x\,\sqrt{-g}\left[-\frac{M_{\rm Pl}^2\dot H}{c_s^2}\left(\dot\pi^2 - c_s^2\frac{(\partial_i\pi)^2}{a^2}\right) + (M_{\rm Pl}^2\dot H)\frac{1-c_s^2}{c_s^2}\left(\frac{\dot\pi(\partial_i\pi)^2}{a^2} + \frac{A}{c_s^2}\dot\pi^3\right) + \cdots\right]$$

with C. Cheung, P. Creminelli, L. Fitzpatrick, J. Kaplan **JHEP 2008**





- these are limits on the cutoff of the theory $\sim \frac{\dot\pi^3}{\Lambda^2}$

with Smith and Zaldarriaga, **JCAP2010**
Planck Collaboration **2013**

Cosmology is going to change in a few months

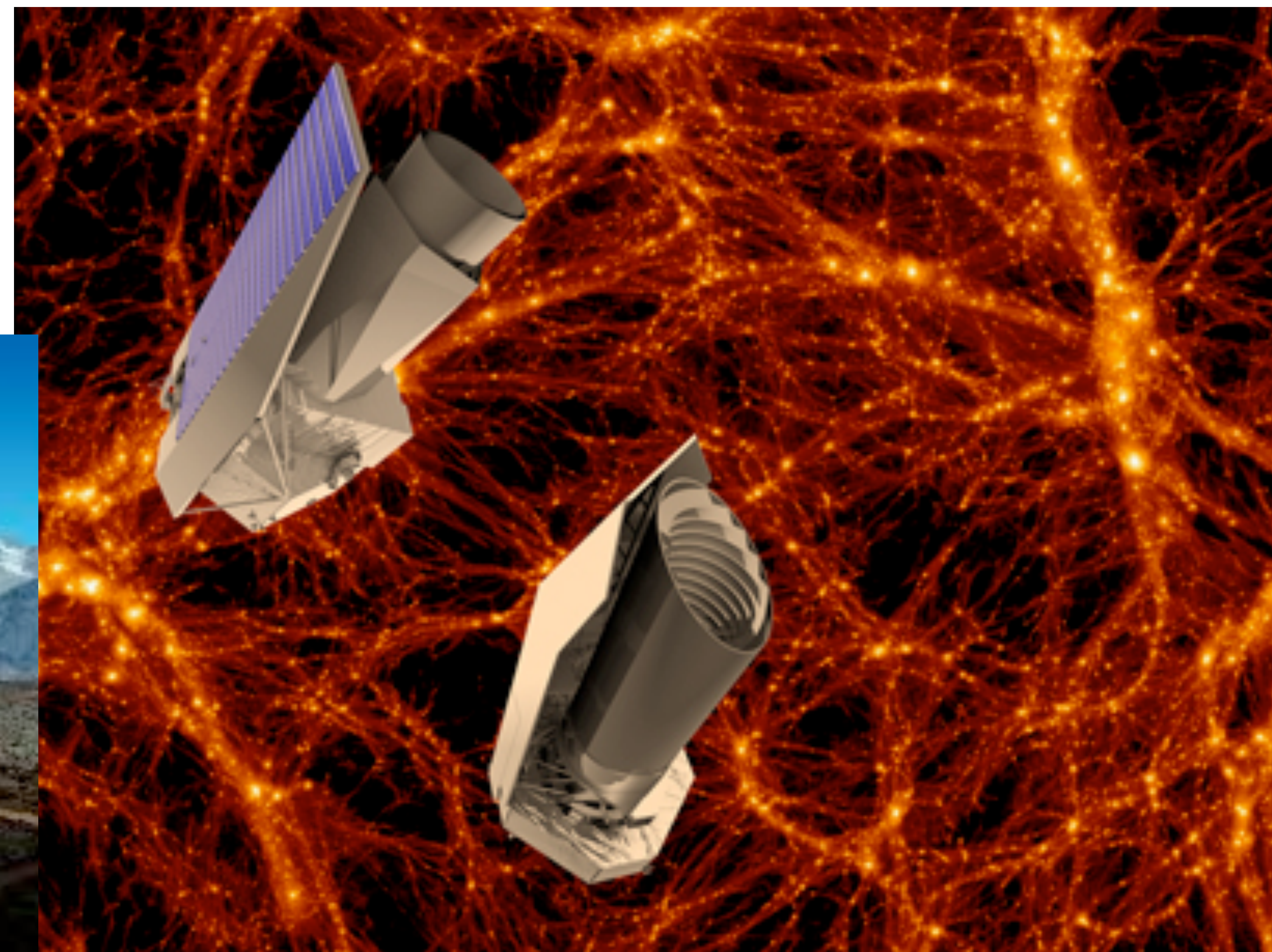
- Tremendous progress has been made through observation of the primordial fluctuations
- In order to increase our knowledge of Inflation, we need more modes
- Planck will soon have observed all the modes from the CMB
- and then what?
- I will assume we are not lucky
  - no B-mode detection
  - no signs from the beginning of inflation
- Unless we find a way to get more modes, the game is over
- Large Scale Structures offer the only medium-term place for hunting for more modes
  - but we are compelled to understand them
    - I do not think, so far, we understand them well enough

What is next?

- Euclid and LSST like: this is our only next chance
  - we need to understand how many modes are available

$$\text{Number of modes} \sim \left(\frac{k_{\max}}{k_{\min}}\right)^3$$

  - Need to understand short distances
  - Similar as from LEP to LHC

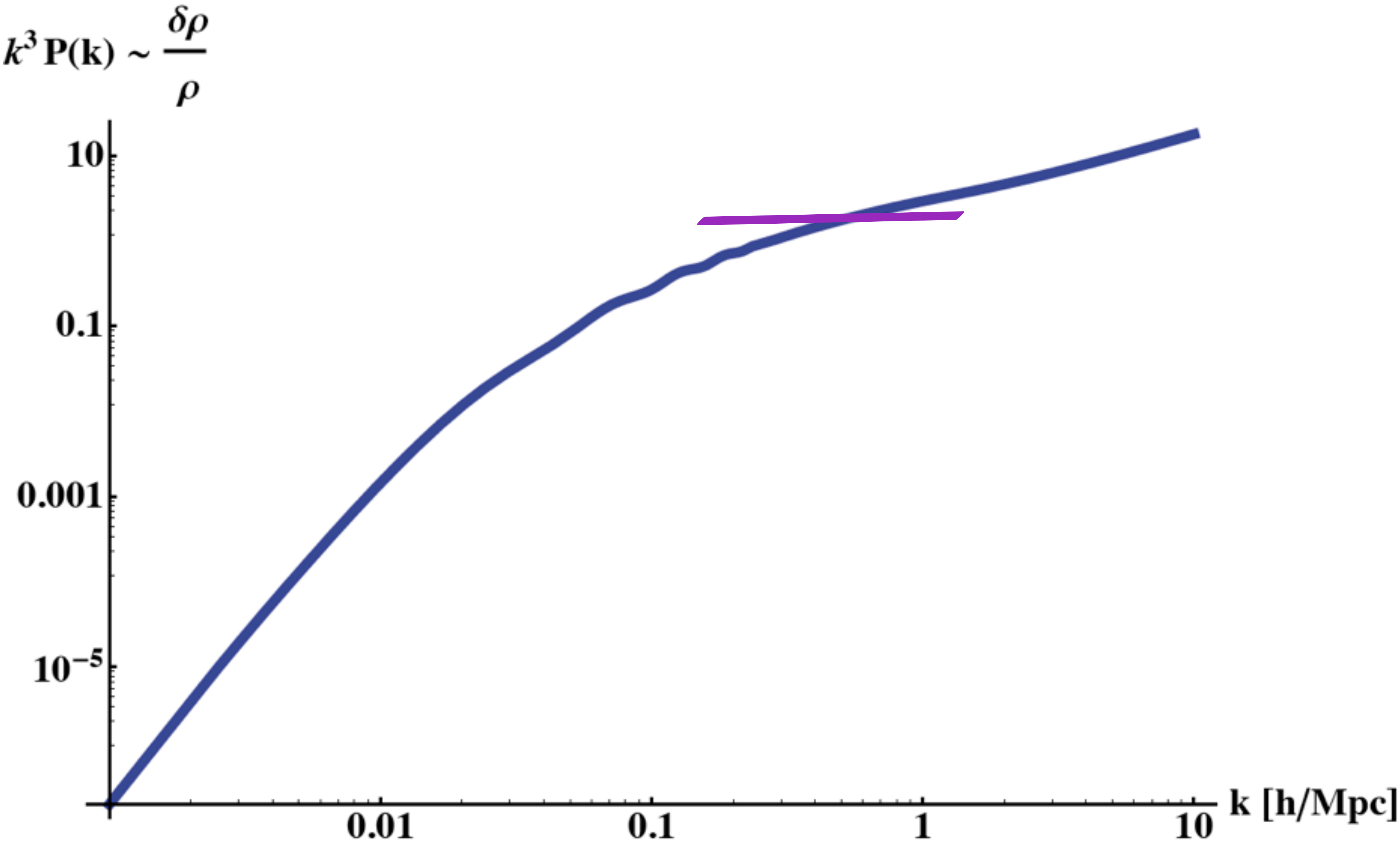


The Effective Field Theory of Cosmological Large Scale Structures

Bias in the EFTofLSS	Senatore **1306**
The one-loop bispectrum in the EFTofLSS	with Angulo, Foreman, Schmittful **1306** see also Baldauf, Mirbabayi, Mercolli,Pajer **1306**
The IR-resummed EFTofLSS	with Zaldarriaga **1304**
The Lagrangian-space EFTofLSS	with Porto and Zaldarriaga **JCAP1405**
The EFTofLSS at 2-loops	with Carrasco, Foreman and Green **JCAP1407**
The 2-loop power spectrum and the IR safe integrand	with Carrasco, Foreman and Green **JCAP1407**
The Effective Theory of Large Scale Structure (EFTofLSS)	with Carrasco and Hertzberg **JHEP 2012**
Cosmological Non-linearities as an Effective Fluid	with Baumann, Nicolis and Zaldarriaga **JCAP 2012**

A well defined perturbation theory

- Non-linearities at short scale



A well defined perturbation theory

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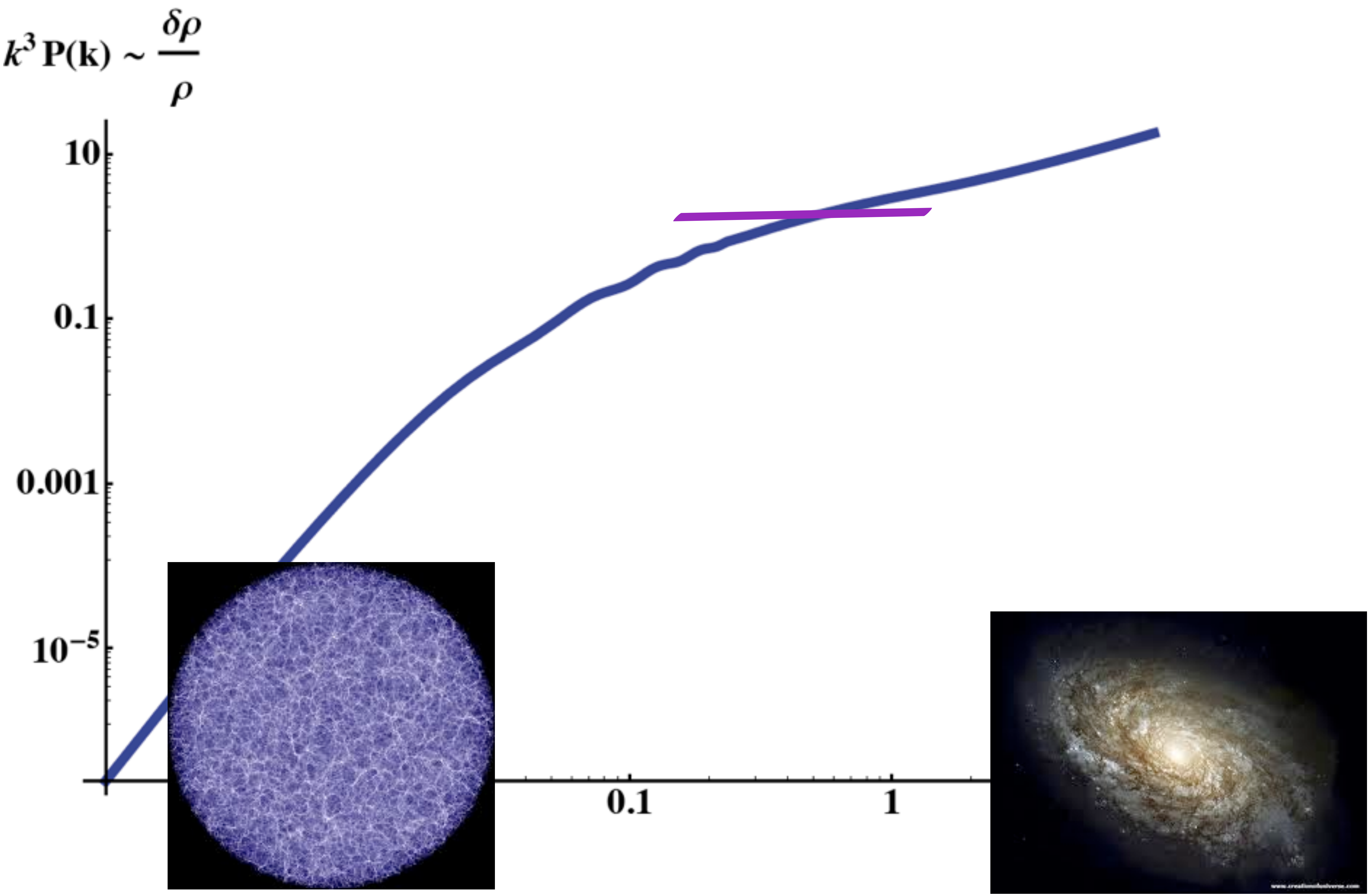


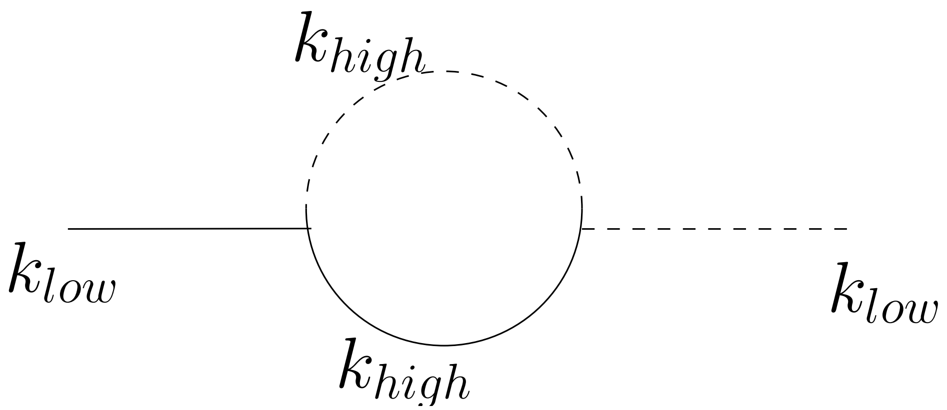
A well defined perturbation theory

- Standard perturbation theory is not well defined
- Standard techniques
  - perfect fluid $\quad \dot{\rho} + \partial_i \left(\rho v^i\right) = 0 \ ,$
  - expand in $\quad \delta \sim \frac{\delta\rho}{\rho}$ and solve iteratively

$$\delta^{(n)} \sim \int \text{GreenFunction} \times \text{Source}^{(n)}\left[\delta^{(1)}, \delta^{(2)}, \ldots, \delta^{(n-1)}\right]$$

$$\Rightarrow \quad \langle \delta_k^{(2)} \delta_k^{(2)} \rangle \sim \int d^3k' \ \langle \delta_{k-k'}^{(1)} \delta_{k-k'}^{(1)} \rangle \ \langle \delta_{k'}^{(1)} \delta_{k'}^{(1)} \rangle$$

- Perturbative equations break in the UV
  - $\delta \sim \frac{k}{k_{NL}} \gg 1 \quad \text{for} \quad k \gg k_{NL}$
  - no perfect fluid if we truncate

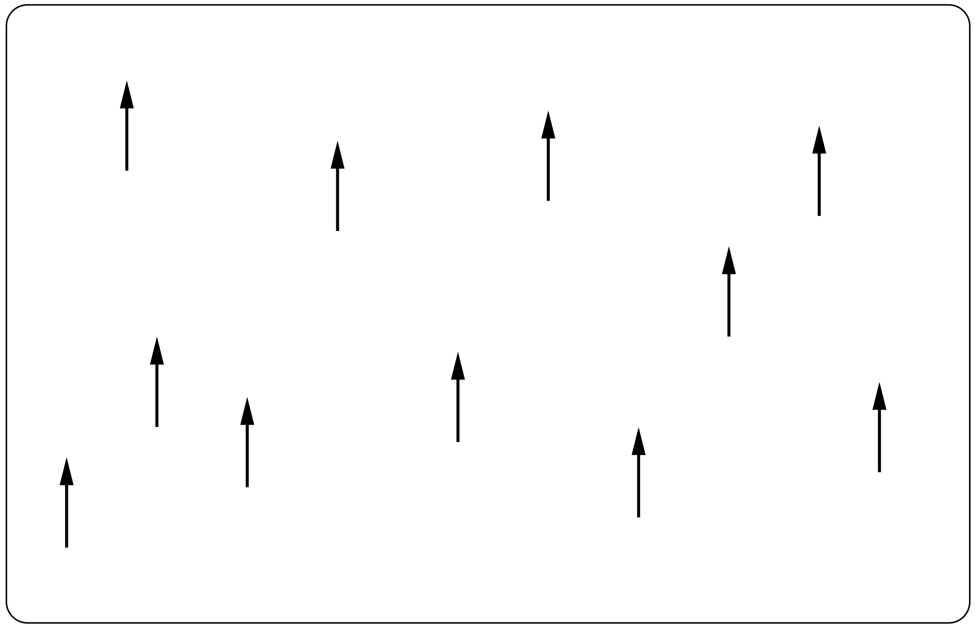


Idea of the
Effective Field Theory

Consider a dielectric material

- Very complicated on atomic scales d_{atomic}
- On long distances $d \gg d_{\text{atomic}}$
  - we can describe atoms with their gross characteristics
    - polarizability $\vec{d}_{\text{dipole}} \sim \alpha \, \vec{E}_{\text{electric}}$: average response to electric field
  - we are led to a uniform, smooth material, with just some macroscopic properties
    - we simply solve Maxwell dielectric equations, we do not solve for each atom.
- The universe looks like a dielectric

Dielectric Fluid



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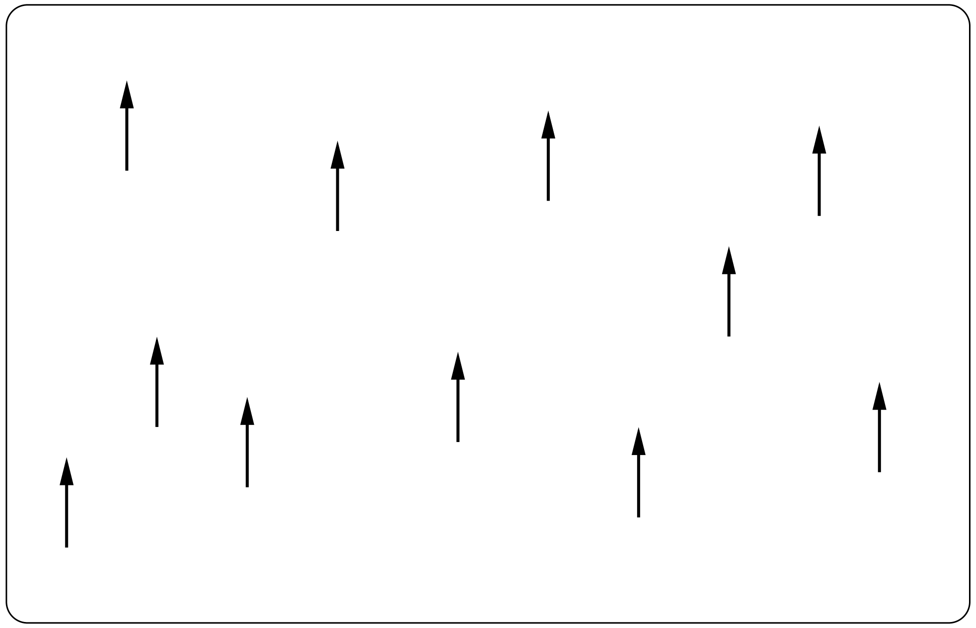
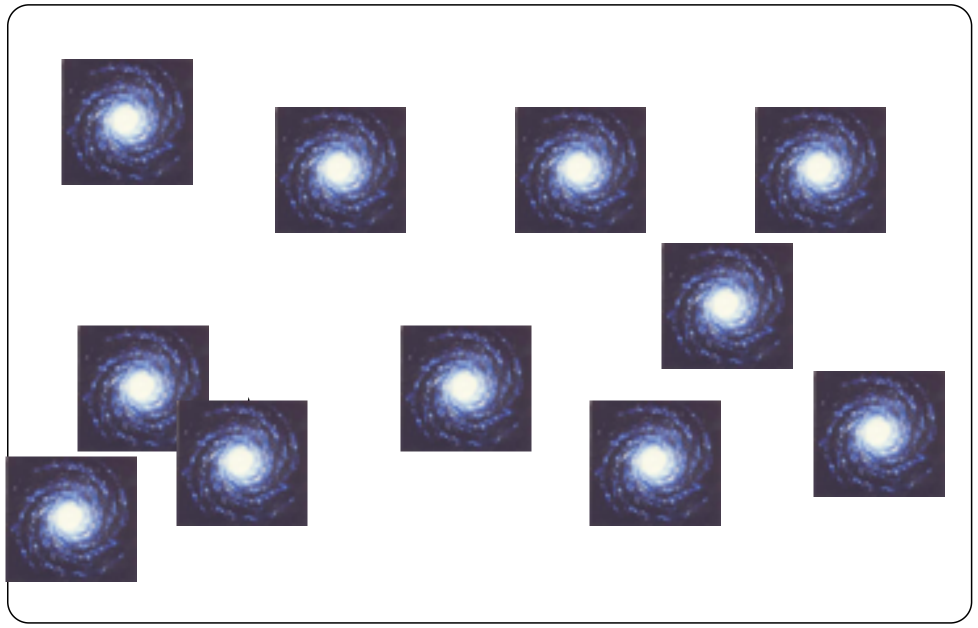
$$\text{EM} \quad \longrightarrow \quad \text{GR}$$

Dielectric Fluid

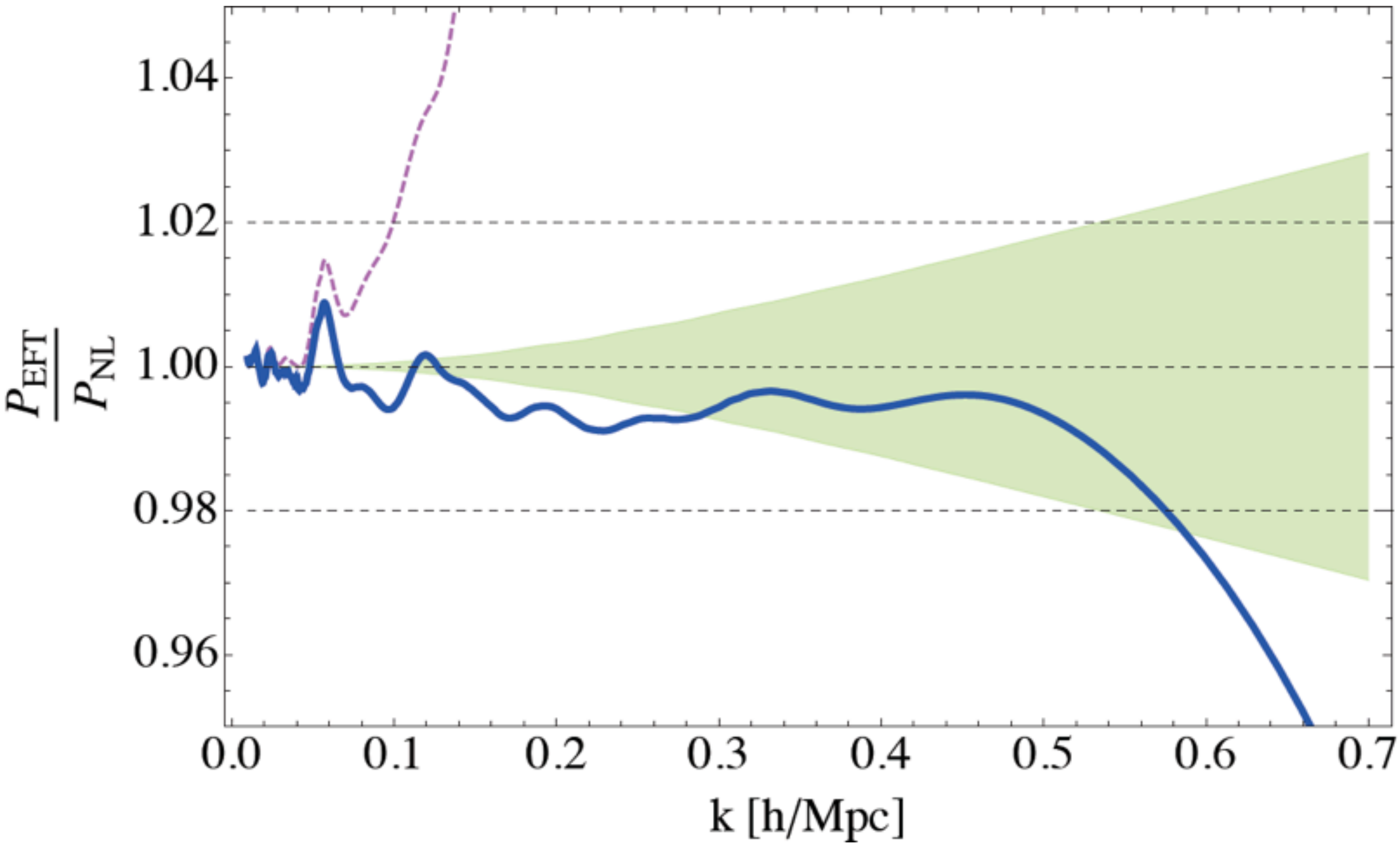


Dielectric Fluid



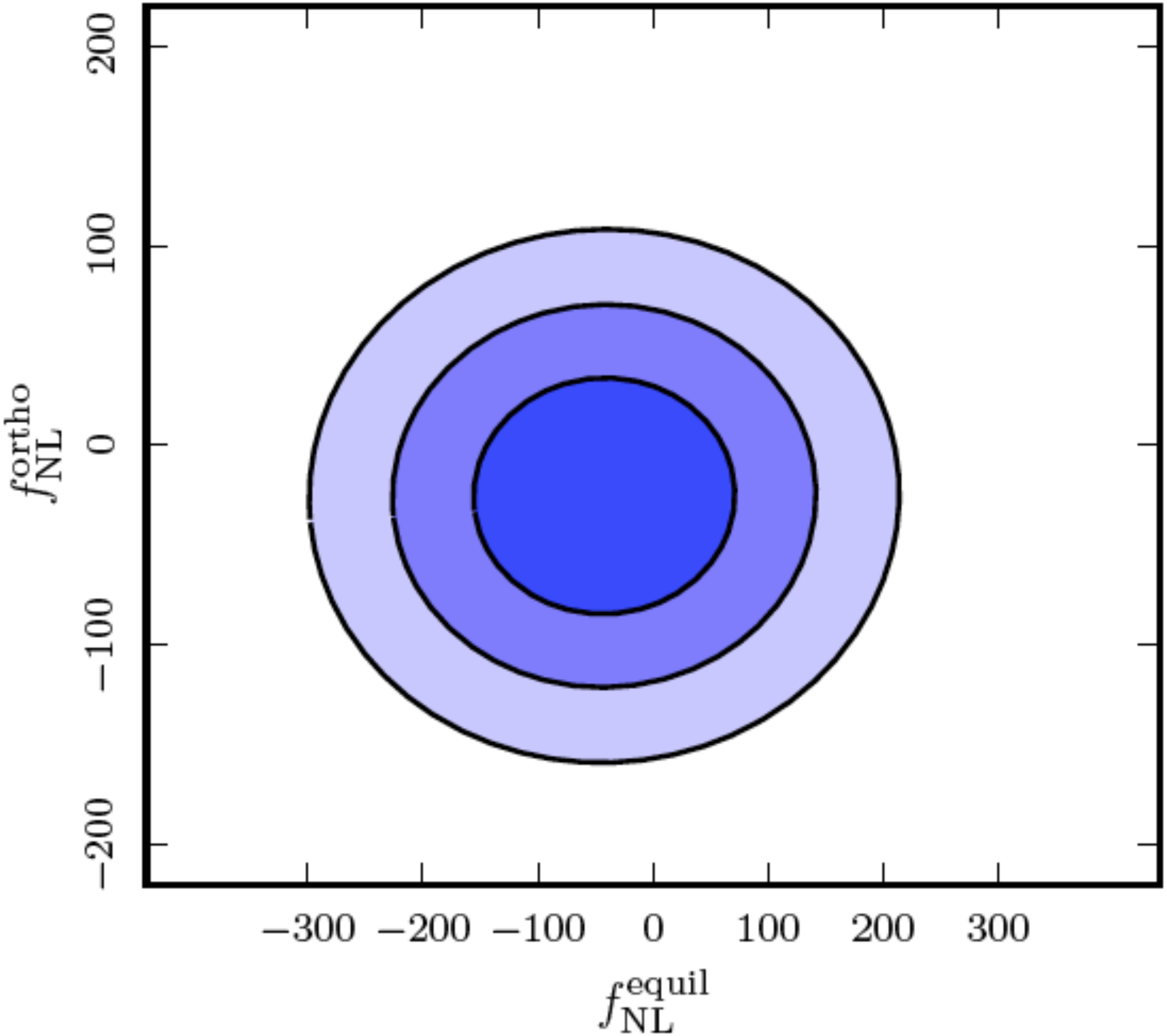
Bottom line result

- A well defined perturbation theory
- 2-loop in the EFT, with IR resummation



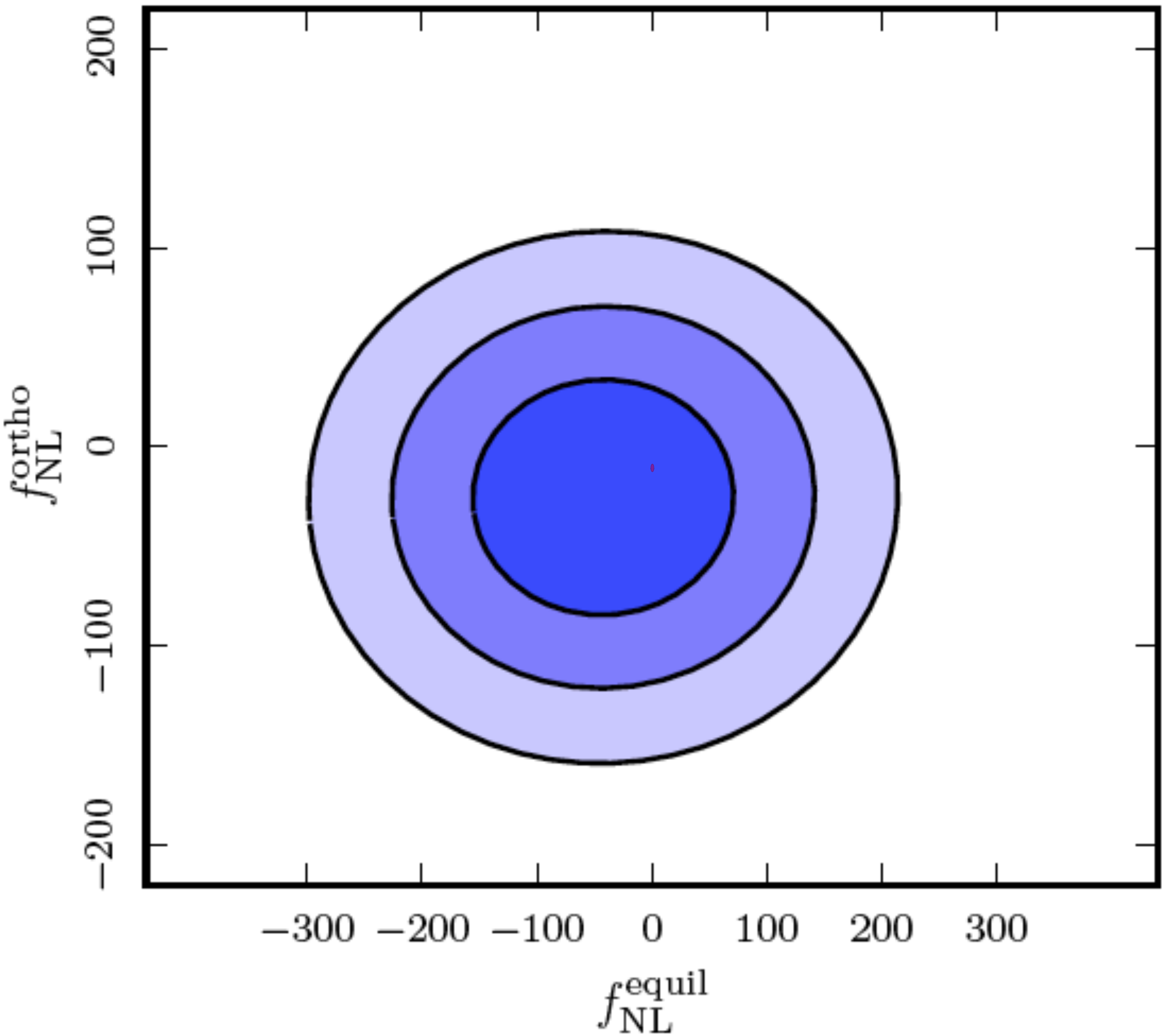
- Data go as $k_{\rm max}^3$: factor of 200 more modes than naive

With this



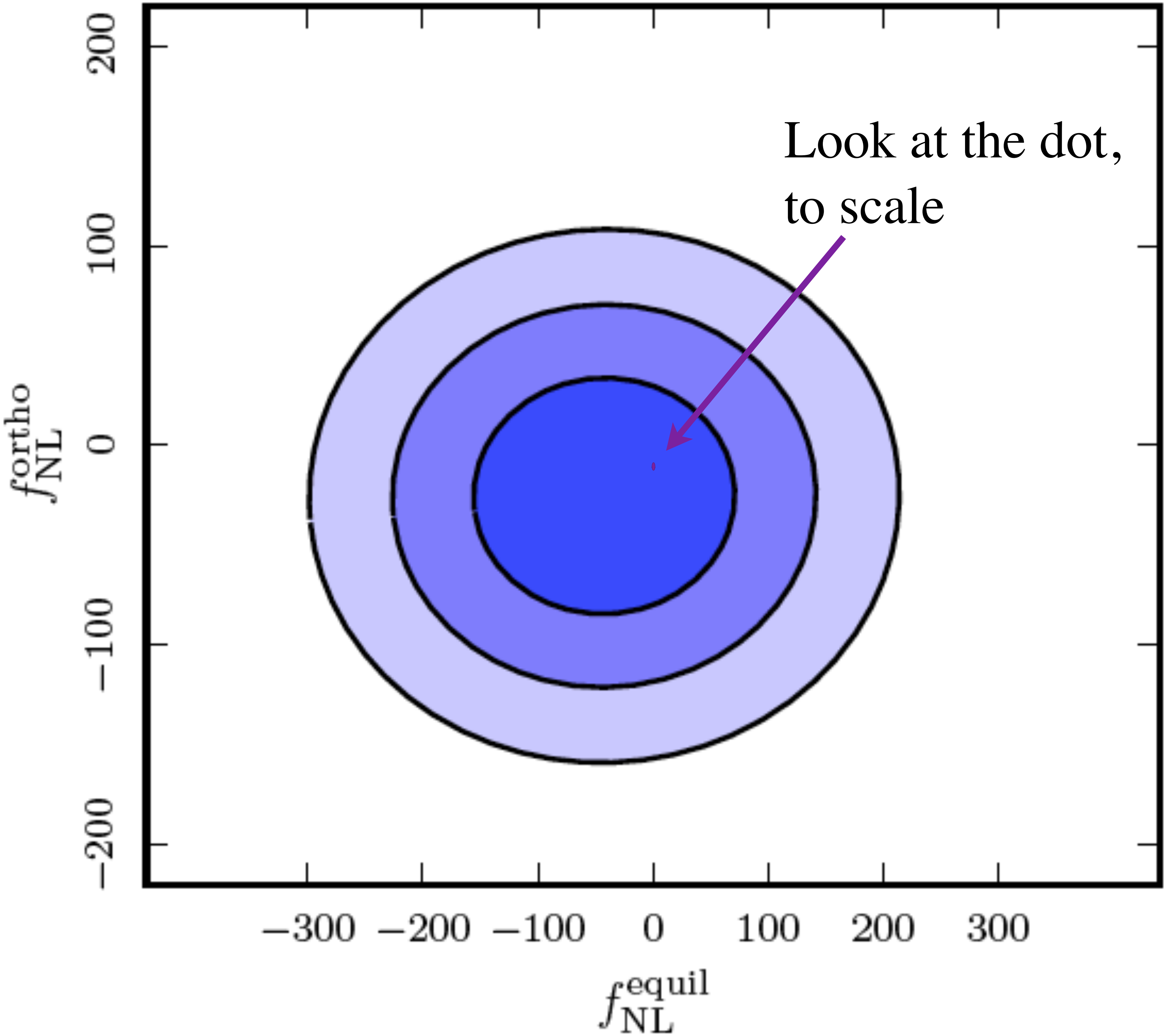
200
100
0
−100
−200
$f_{\mathrm{NL}}^{\mathrm{ortho}}$
−300
−200
−100
0
100
200
300
$f_{\mathrm{NL}}^{\mathrm{equil}}$

With this



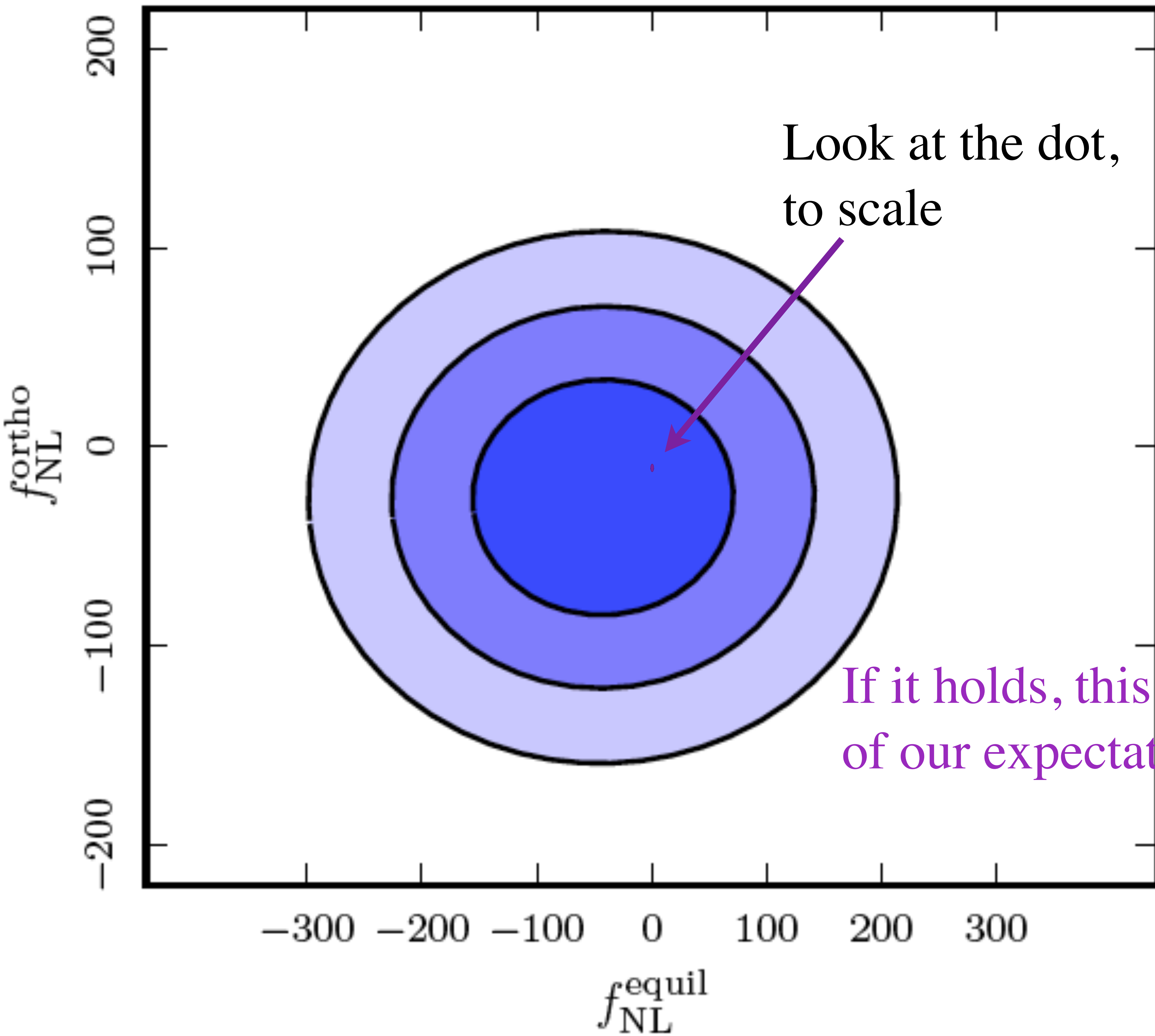
200
100
0
−100
−200
$f_{\rm NL}^{\rm ortho}$
−300
−200
−100
0
100
200
300
$f_{\rm NL}^{\rm equil}$

With this



Look at the dot,
to scale
200
100
0
−100
−200
f_{NL}^{ortho}
−300
−200
−100
0
100
200
300
f_{NL}^{equil}

With this



If it holds, this is a revolution
of our expectations

Construction of the Effective Field Theory

Point-like Particle versus Extended Objects

- On short distances, we have point-like particles
  - they move

$$\frac{d^2\vec{z}(\vec{q},\eta)}{d\eta^2}+\mathcal{H}\frac{d\vec{z}(\vec{q},\eta)}{d\eta}=-\vec{\partial}_x\Phi[\vec{z}(\vec{q},\eta)]$$

  - induce overdensities

$$1+\delta(\vec{x},\eta)=\int d^3q\;\delta^{(3)}(\vec{x}-\vec{z}(\vec{q},\eta))$$

  - Source gravity

$$\partial^2\Phi(\vec{x})=\mathcal{H}^2\delta(\vec{x})$$

Point-like Particle versus Extended Objects

- But we cannot describe point-like particles: we need to focus on long distances.
  - We deal with Extended objects
    - they move differently:

$$\frac{d^2\vec{z}(\vec{q},\eta)}{d\eta^2} + \mathcal{H}\frac{d\vec{z}(\vec{q},\eta)}{d\eta} = -\vec{\partial}_x\Phi[\vec{z}(\vec{q},\eta)]$$

Point-like Particle versus Extended Objects

- But we cannot describe point-like particles: we need to focus on long distances.
  - We deal with Extended objects
    - they move differently:

$$\frac{d^2\vec{z}_L(\vec{q},\eta)}{d\eta^2}+\mathcal{H}\frac{d\vec{z}_L(\vec{q},\eta)}{d\eta} \quad = \quad -\vec{\partial}_x\left[\Phi_L[\vec{z}_L(\vec{q},\eta)]+\frac{1}{2}Q^{ij}(\vec{q},\eta)\partial_i\partial_j\Phi_L[\vec{z}_L(\vec{q},\eta)]+\cdots\right]+\vec{a}_S(\vec{q},\eta)$$

Point-like Particle versus Extended Objects

- they induce number over-densities and real-space multipole moments

$$1+\delta_{n,L}(\vec{x},\eta) \equiv \int d^3\vec{q}\;\delta^3(\vec{x}-\vec{z}_L(\vec{q},\eta))\ ,$$

$$\mathcal{Q}^{i_1\ldots i_p}(\vec{x},\eta) \equiv \int d^3\vec{q}\;Q^{i_1\ldots i_p}(\vec{q},\eta)\delta^3(\vec{x}-\vec{z}_L(\vec{q},\eta))$$

- they source gravity with the `overall' mass

$$\partial_x^2\Phi_L=\frac{3}{2}\mathcal{H}^2\Omega_m\left(\delta_{n,L}(\vec{x},\eta)+\frac{1}{2}\partial_i\partial_j\mathcal{Q}^{ij}(\vec{x},\eta)-\frac{1}{6}\partial_i\partial_j\partial_k\mathcal{Q}^{ijk}(\vec{x},\eta)+\cdots\right)\equiv\frac{3}{2}\mathcal{H}^2\Omega_m\delta_{m,L}(\vec{x},\eta)$$

$$\sim \text{Energy}_{\text{electrostatic}}=q\,V+\vec{d}\cdot\vec{E}+\ldots$$

- These equations can be derived from smoothing the point-particle equations
  - but actually these are the assumption-less equations

How do we treat the new terms?

- Similar to treatment of material polarizability: $\vec{d}_{\rm dipole} \sim \vec{d}_{\rm intrinsic} + \alpha \, \vec{E}$
- Take moments:

$$Q^{ij} = \langle Q^{ij} \rangle_S + Q^{ij}_{\mathcal{S}} + Q^{ij}_{\mathcal{R}}$$

- Expectation value $\langle Q^{ij} \rangle_S = l_S^2(\eta)\delta_{ij}$
- Response (non-local in time) $Q_{ij,\mathcal{R}} \sim l_1(\eta)^2 \, \partial_i \partial_j \Phi_L(\vec{z}_L(\vec{q},\eta))$
- Stochastic noise

$$\langle Q_{\mathcal{S}} \rangle = 0 \qquad \langle Q_{\mathcal{S}} Q_{\mathcal{S}} \ldots \rangle \neq 0$$

- Overall

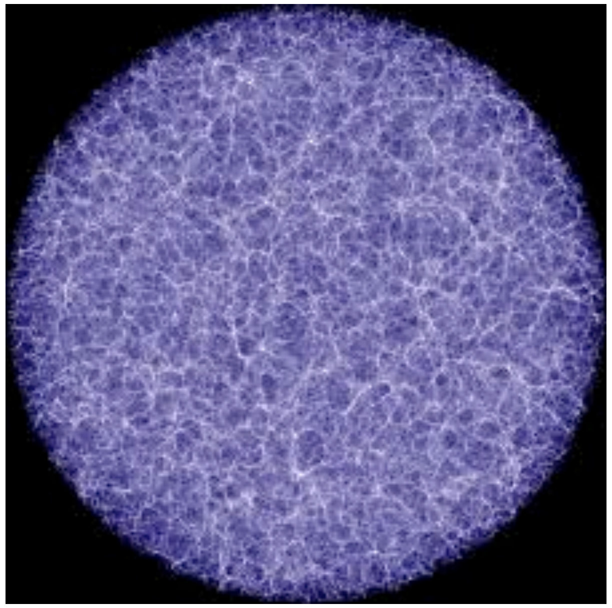
$$Q_{ij}(\vec{x},t) = l_0^2(t)\, \delta_{ij} + l_1^2(t)\, \partial_i \partial_j \Phi(\vec{x},t) + \ldots$$

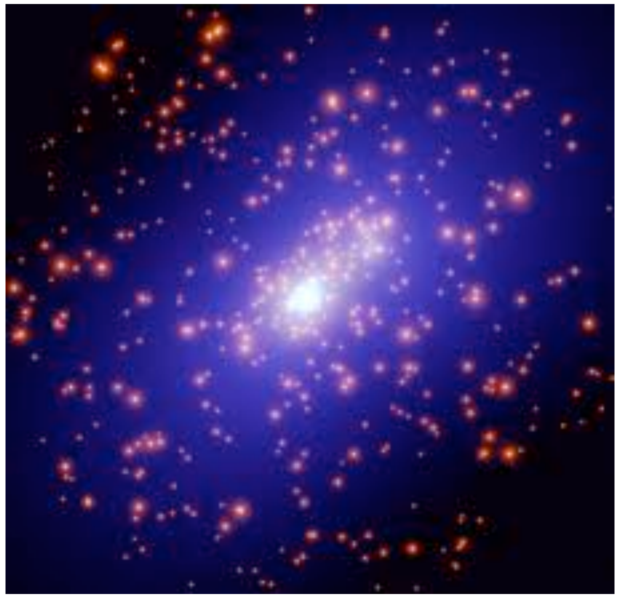
- In summary: we obtain an expression just in terms of long-wavelength variables

$$\frac{\partial^2}{H^2} \Phi(\vec{x},t) = \delta(\vec{x},t) + \partial_i \partial_j Q_{ij} \left(\delta(\vec{x},t), \ldots \right) + \ldots$$

This EFT is non-local in time

- For local EFT, we need hierarchy of scales.
  - In space we are ok





  - In time we are not ok: all modes evolve with time-scale of order Hubble

with Carrasco, Foreman and Green **1310**

Carroll, Leichenauer, Pollak **1310**

- $\Longrightarrow$ The EFT is local in space, non-local in time
  - Technically it does not affect much because the linear propagator is local in space

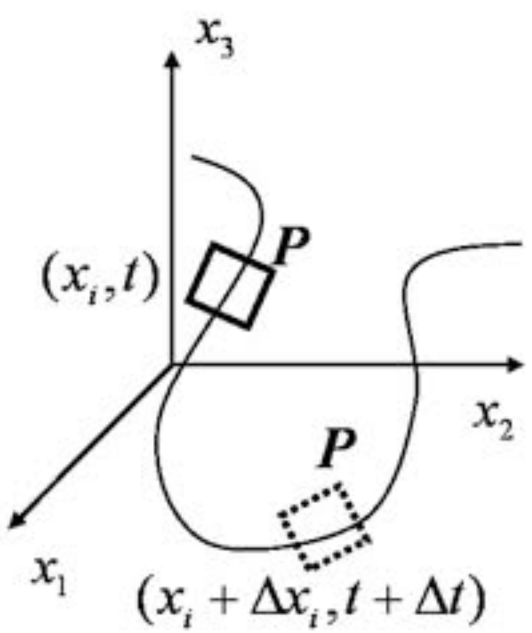
When do we stop?

- Similar to treatment for material polarizability: $\vec{d}_{\text{dipole}} \sim \alpha\, \vec{E}_{\text{electric}}$, $Q_{ij}^{\text{electric}} = c\, E_i E_j$, ...
- Short distance physics is taken into account by expectation value, response, and noise
- Poisson equation breaks when $\delta_{n,L}(\vec{x},\eta) \sim \partial_i \partial_j \mathcal{Q}^{ij}(\vec{x},\eta)$
  - gravitational potential from quadrupole moment ~ the one from center of mass
- By dimensional analysis, this happens for distances shorter than a critical length
  - the non-linear scale $k \gtrsim k_{\text{NL}}$
  - on long distances, $k \ll k_{\text{NL}}$, write as many terms as precision requires.
    - Manifestly convergent expansion in $\left(\frac{k}{k_{\text{NL}}}\right) \ll 1$

Connecting with the Eulerian Treatment

- In the universe, finite-size particles move

$$\vec{z}(\vec{q},t) = \vec{q} + \vec{s}(\vec{q},t)$$



- In Lagrangian space, we do not expand in $\vec{s}(\vec{q},t)$
- In Eulerian, we do: we describe particles from a fixed position
  - Expand in $k\,s \ll 1$

Connecting with the Eulerian Treatment

- The resulting equations are equivalent to Eulerian fluid-like equations

$$\nabla^2\phi = H^2\frac{\delta\rho}{\rho}$$

$$\partial_t\rho + H\rho + \partial_i(\rho v^i) = 0$$

$$\dot{v}^i + Hv^i + v^j\partial_j v^i = \frac{1}{\rho}\partial_j\tau^{ij}$$

  – here it appears a non trivial stress tensor for the long-distance fluid

$$\tau_{ij} = p_0\,\delta_{ij} + c_s^2\,\delta_{ij}\,\partial^2\delta\rho + \ldots$$

Perturbation Theory with the EFT

Perturbation Theory within the EFT

- In the EFT we can solve iteratively (loop expansion) $\delta_\ell, v_\ell, \Phi_\ell \ll 1$

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$$\partial_t\rho + H\rho + \partial_i(\rho v^i) = 0$$

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Perturbation Theory within the EFT

- Regularization and renormalization of loops (scaling universe)
  - evaluate with cutoff. By dim analysis:

$$P_{1-\mathrm{loop}} = c_0^\Lambda \left(\frac{\Lambda}{k_{\mathrm{NL}}}\right)^2 \left(\frac{k}{k_{\mathrm{NL}}}\right) P_{11} + c_1^\Lambda \left(\frac{\Lambda}{k_{\mathrm{NL}}}\right) \left(\frac{k}{k_{\mathrm{NL}}}\right)^2 P_{11}$$
$$+ c_2^\Lambda \log\left(\frac{\Lambda}{k_{\mathrm{NL}}}\right) \left(\frac{k}{k_{\mathrm{NL}}}\right)^3 P_{11} + c_1^{\mathrm{finite}} \left(\frac{k}{k_{\mathrm{NL}}}\right)^3 P_{11} + \text{subleading in } \frac{k}{k_{\mathrm{NL}}}$$

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  - absence of counterterm $\quad \tau_{ij} = p_0\, \delta_{ij} + c_s^2\, \delta_{ij}\, \partial^2 \delta\rho$

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  - absence of counterterm $\quad \tau_{ij} = p_0\, \delta_{ij} + c_s^2\, \delta_{ij}\, \partial^2 \delta\rho$

$$\Rightarrow \qquad P_{1-\text{loop, counter}} = c_{\rm counter}^\Lambda \left(\frac{k}{k_{\rm NL}}\right)^2 P_{11}$$

$$\Rightarrow \qquad c_{\rm counter}^\Lambda = -c_1^\Lambda + \delta c_{\rm counter} \left(\frac{k_{\rm NL}}{\Lambda}\right)$$

$$\Rightarrow \quad P_{1-\text{loop}} + P_{1-\text{loop, counter}} = \delta c_{\rm counter} \left(\frac{k}{k_{\rm NL}}\right)^2 P_{11} + c_1^{\rm finite} \left(\frac{k}{k_{\rm NL}}\right)^3 P_{11}$$

Calculable terms in the EFT

- Has everything being lost?

$$P_{1-\text{loop}} + P_{1-\text{loop, counter}} = \delta c_{\text{counter}} \left(\frac{k}{k_{\text{NL}}}\right)^2 P_{11} + c_1^{\text{finite}} \left(\frac{k}{k_{\text{NL}}}\right)^3 P_{11}$$

  – to make result finite, we need to add a counterterm with finite part

    - need to fit to data (like a coupling constant), but cannot fit the k-shape

Calculable terms in the EFT

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  – to make result finite, we need to add a counterterm with finite part
    - need to fit to data (like a coupling constant), but cannot fit the k-shape
  – the subleading finite term is not degenerate with a counterterm.
    - it cannot be changed
    - it is calculable by the EFT
      – so it predicts an observation $c_1^{\text{finite}} = 0.044$

Lesson from Renormalization

- Each loop-order L contributed a finite, calculable term of order

$$P_{\rm L-loops} \sim \left(\frac{k}{k_{\rm NL}}\right)^L$$

  – each higher-loop is smaller and smaller

- This happens after canceling the divergencies with counterterms

$$P_{\rm L-loops;\ without\ counterterms} = \left(\frac{\Lambda}{k_{\rm NL}}\right)^L \frac{k^2}{k_{\rm NL}^2} P(k)$$

  - each loop contributes the same

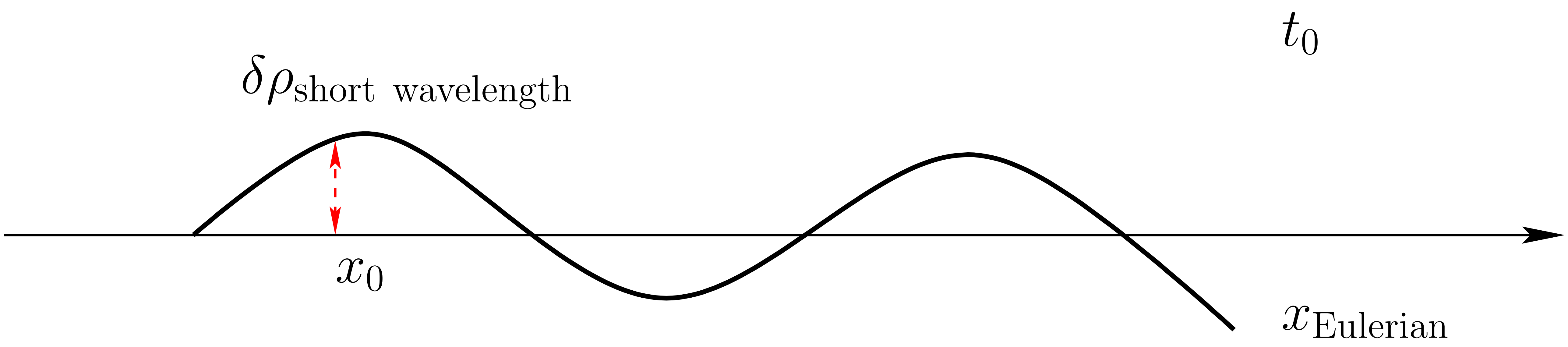
- Up to 2-loops, we need only the 1-loop counterterm

IR-effects

with Zaldarriaga **1404**

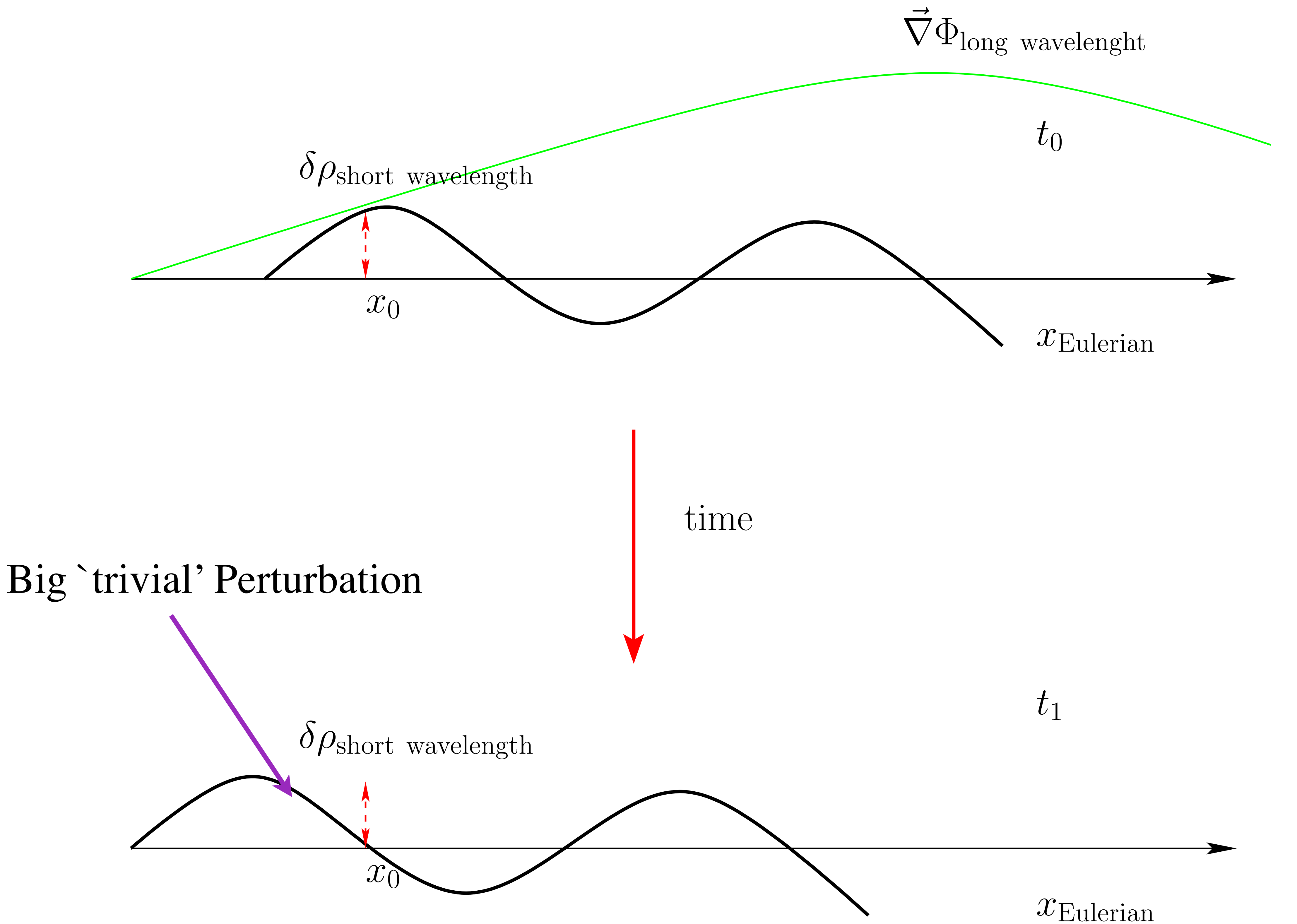
The Effect of Long-modes on Shorter ones

- In Eulerian treatment



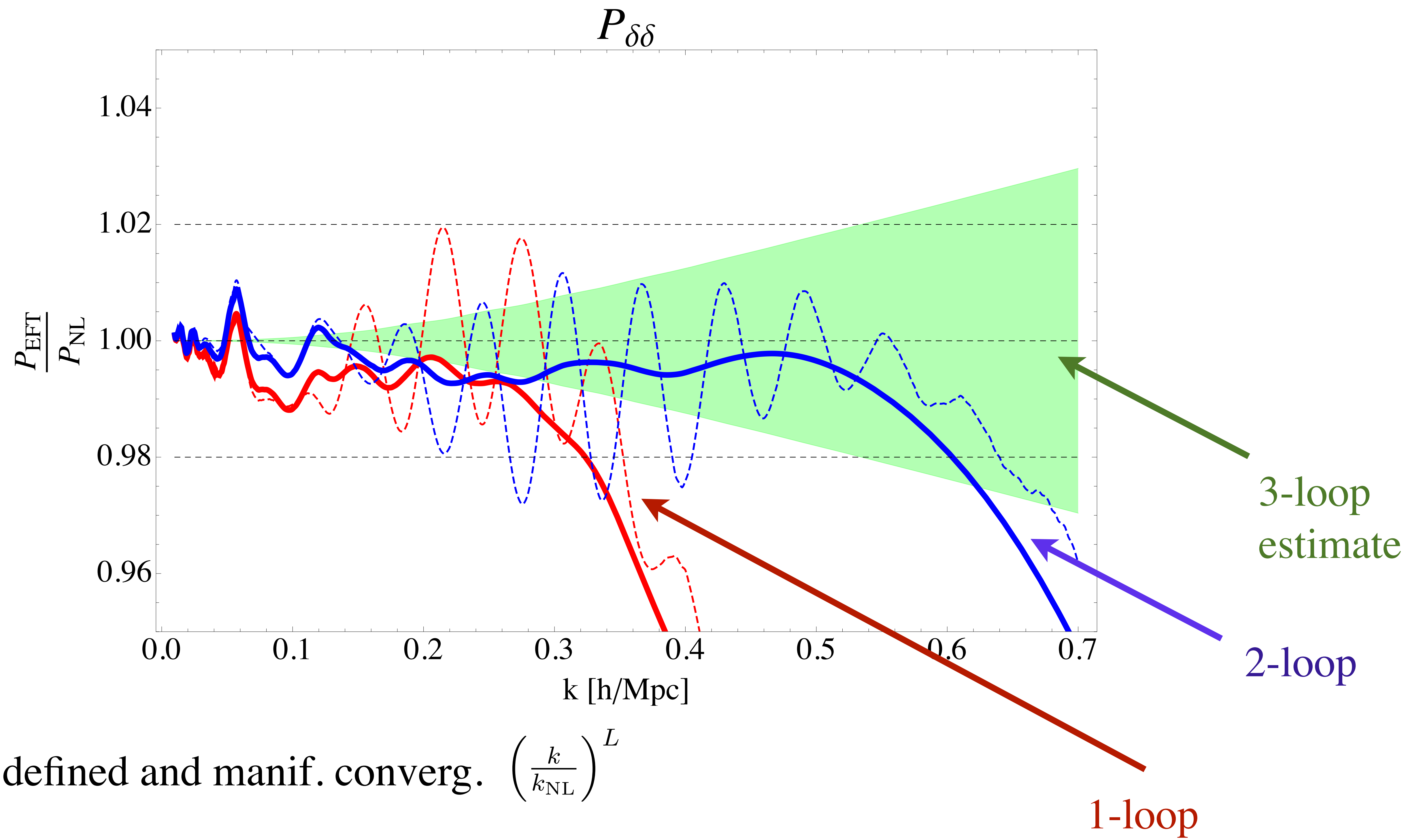
The Effect of Long-modes

- Add a long `trivial' force (trivial by GR)
- This tells you that one can resum the IR modes: this is the Lagrangian treatment



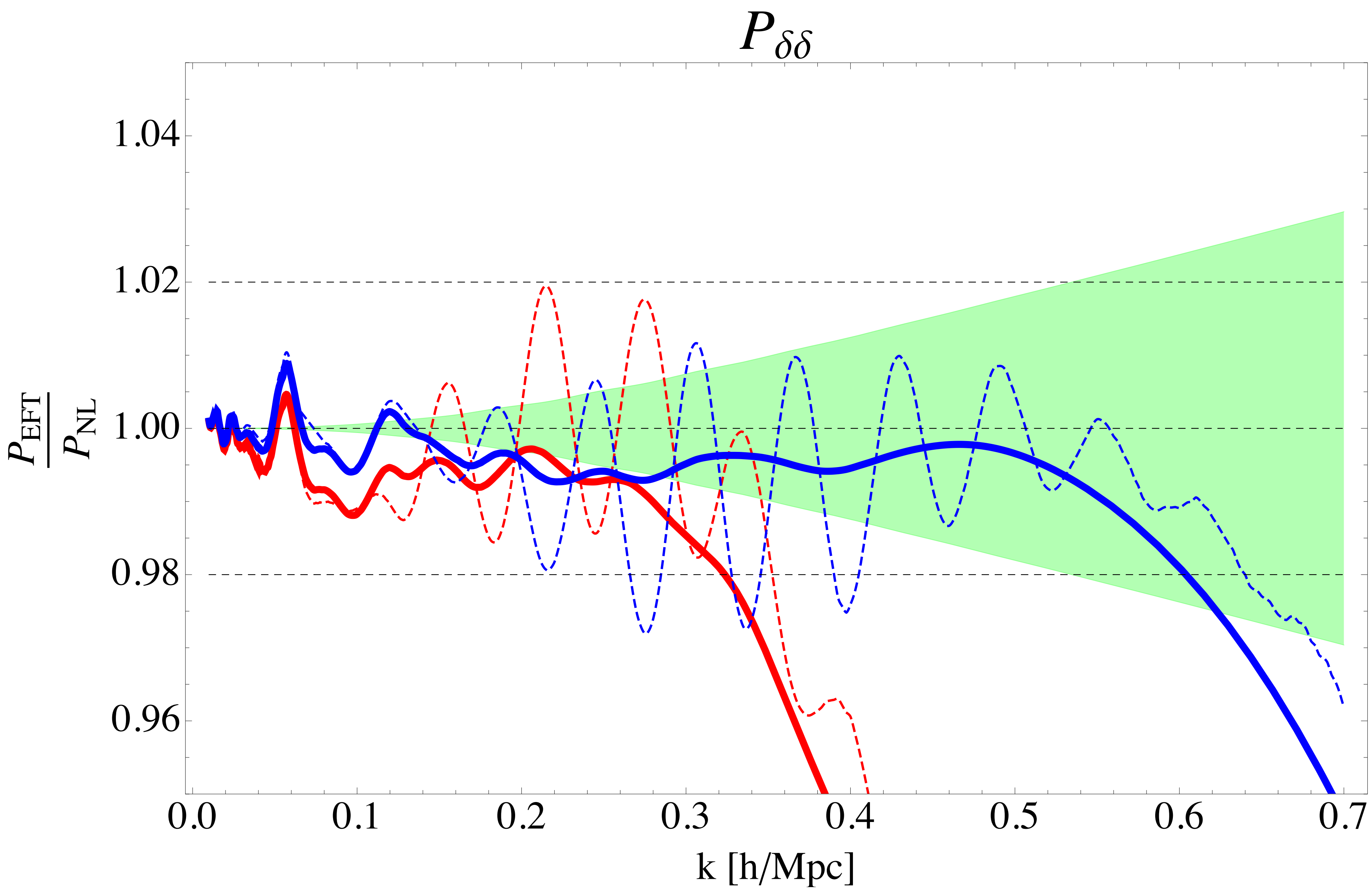
Results

EFT of Large Scale Structures



- Well defined and manif. converg. $\left(\frac{k}{k_{\rm NL}}\right)^L$
- Every perturbative order improves the agreement as it should
- We know when we should fail, and we fail when we should

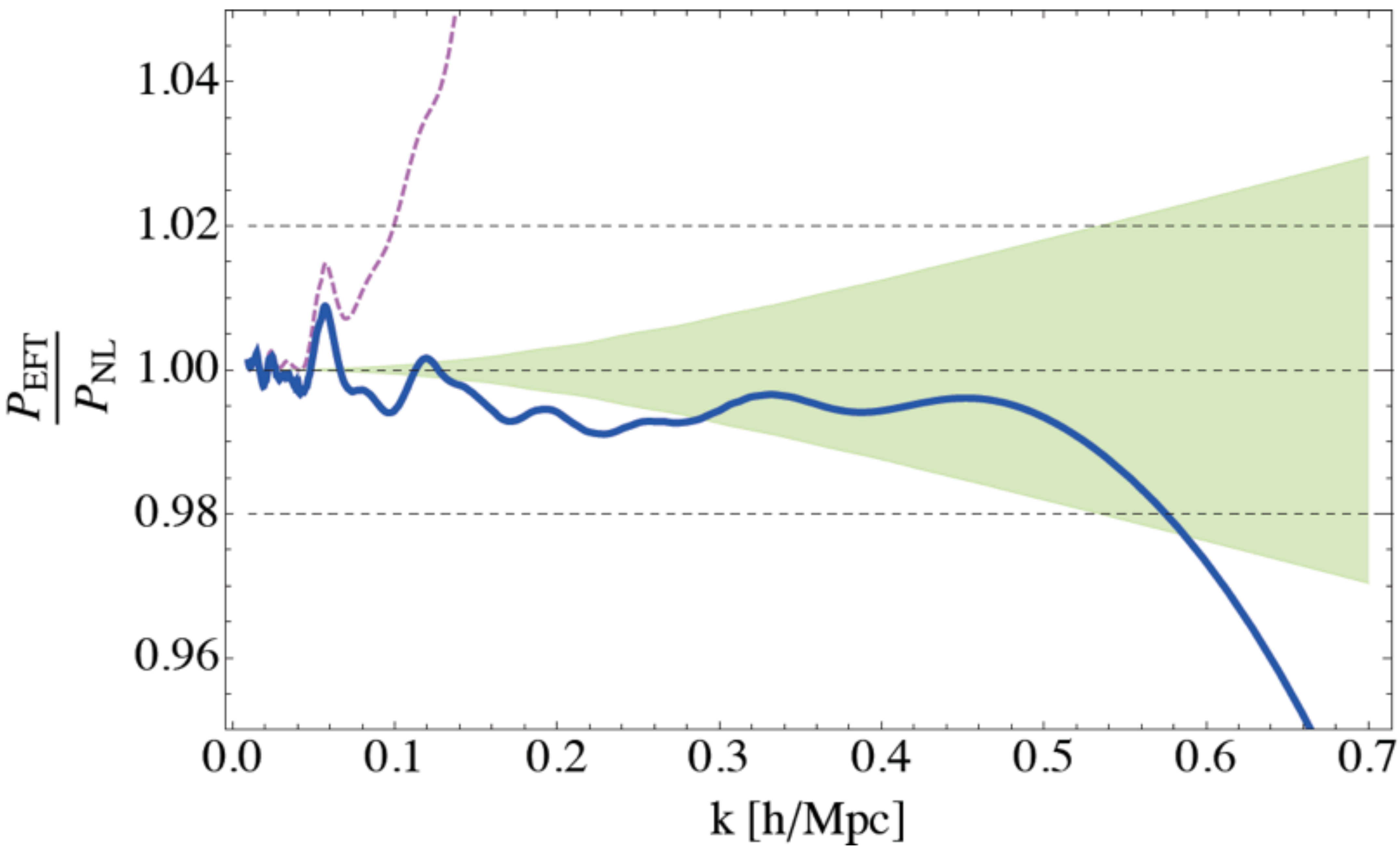
EFT of Large Scale Structures



- The lines with oscillations are obtained without resummation in the IR

with Carrasco, Foreman and Green **1310**

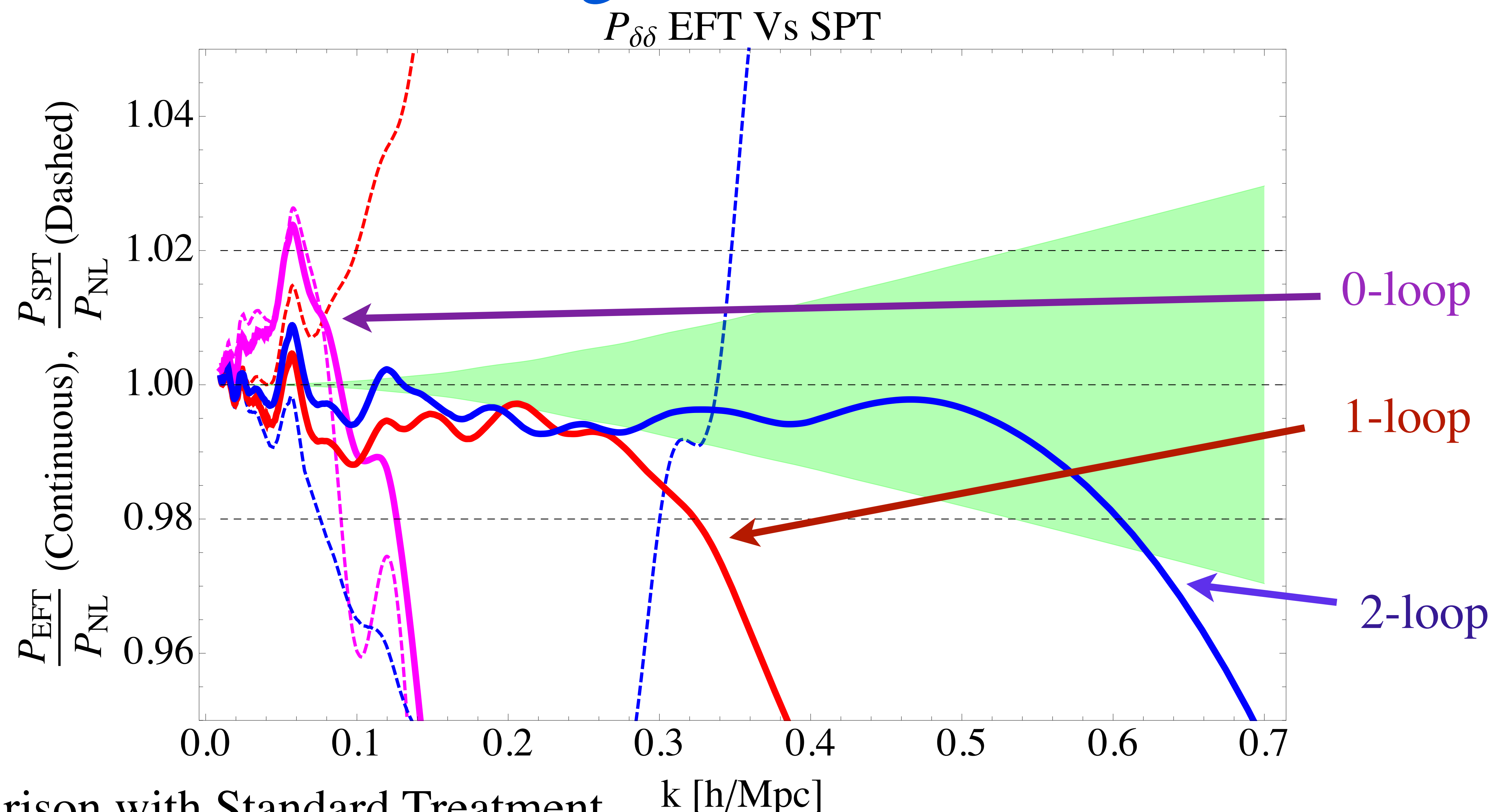
EFT of Large Scale Structures



- we fit until $k_{\rm max} \simeq 0.6\, h\, {\rm Mpc}^{-1}$, as where we should stop fitting
  - there are 200 more quasi linear modes than previously believed!

with Zaldarriaga **1404**

EFT of Large Scale Structures



- Comparison with Standard Treatment
- For the EFT, change from 1-loop to 2-loop predicted

$$P_{\text{EFT-2-loop}} = P_{11} + P_{\text{1-loop}} + P_{\text{2-loop}} - 2\,(2\pi)(c^2_{s(1)} + c^2_{s(2)})\frac{k^2}{k^2_{\rm NL}}P_{11} + (2\pi)c^2_{s(1)}P^{(c_{\rm s},p)}_{\text{1-loop}} + (2\pi)^2 c^4_{s(1)}\frac{k^4}{k^4_{\rm NL}}P_{11}$$

  – the other new terms are clearly important

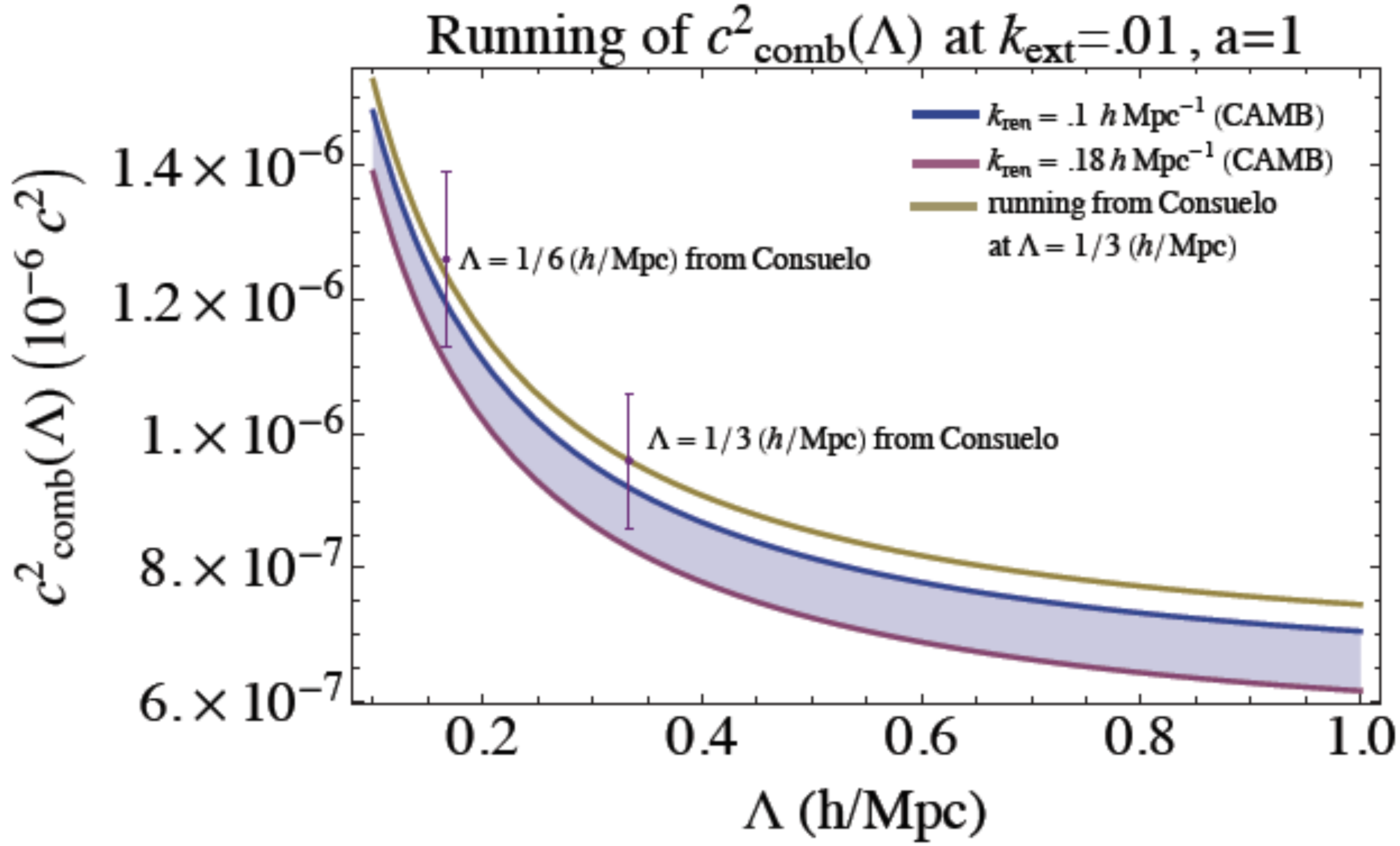
  – they `conspire' to the right answer

Measuring Parameters from small N-body Simulations

Measuring parameters from N-body sims.

- The EFT parameters can be measured from small N-body simulations
  - similar to what happens in QCD: lattice sims
- As you change smoothing scale, the result changes

$$\frac{d\,c_s}{d\Lambda} = \frac{d}{d\,\Lambda}\int^{\Lambda} d^3k\; P_{13}(k)$$

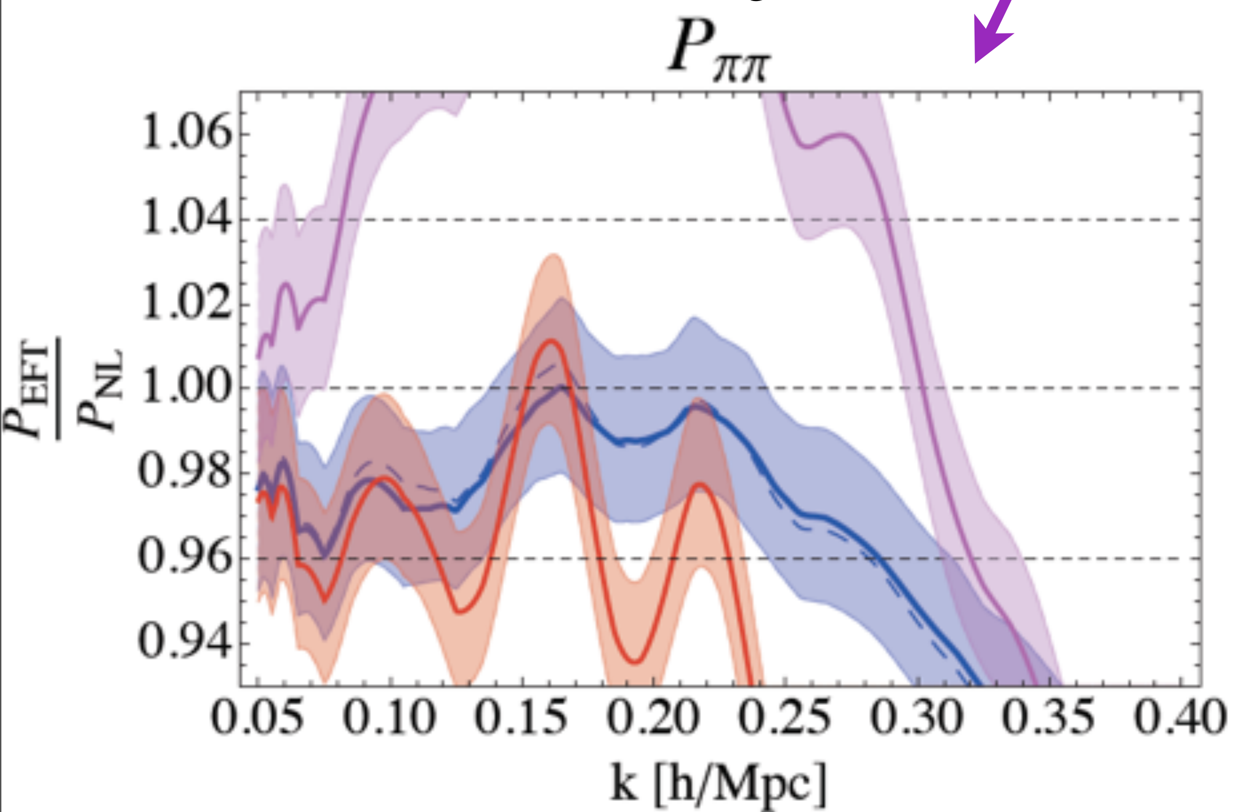


- Perfect agreement with fitting at low energies
  - like measuring F_π from lattice sims and $\pi\pi$ scattering

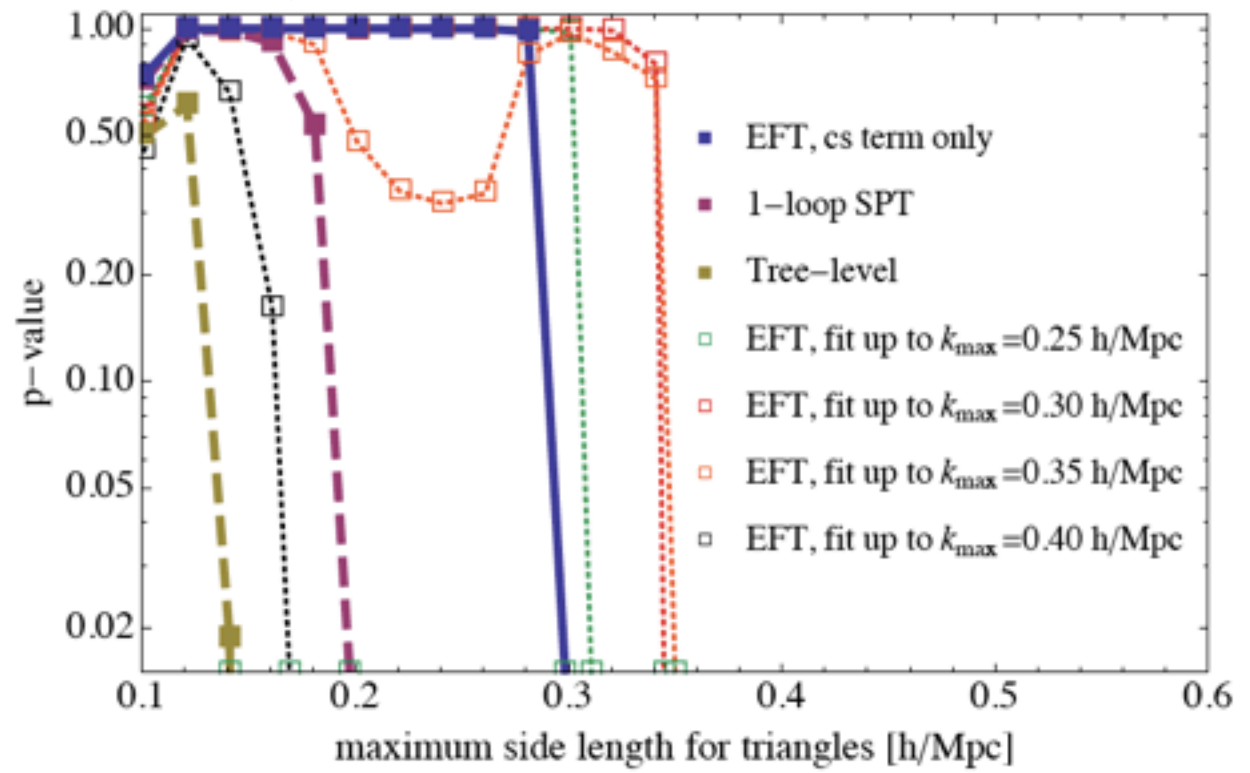
with Carrasco and Hertzberg **JHEP 2012**

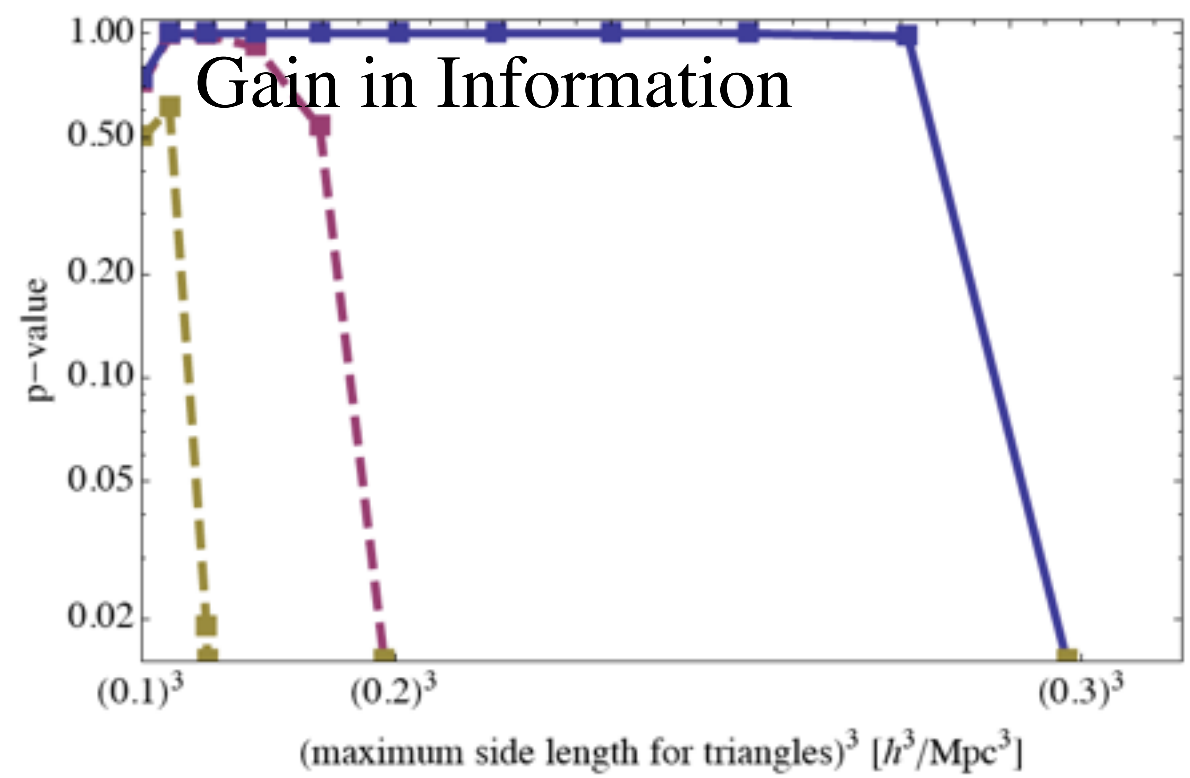
Momentum and Bispectrum

with Zaldarriaga **1404**

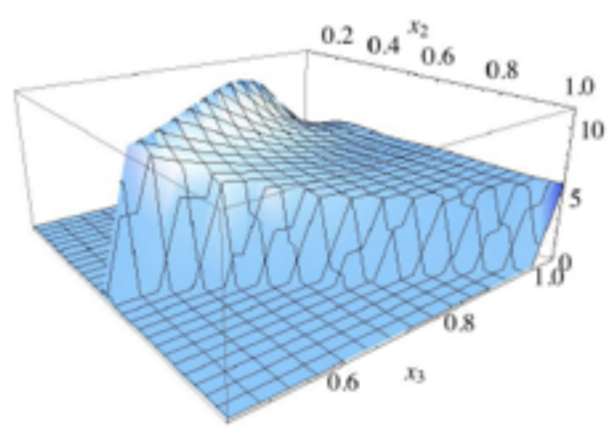


with Angulo, Foreman and Schmittful **1406**





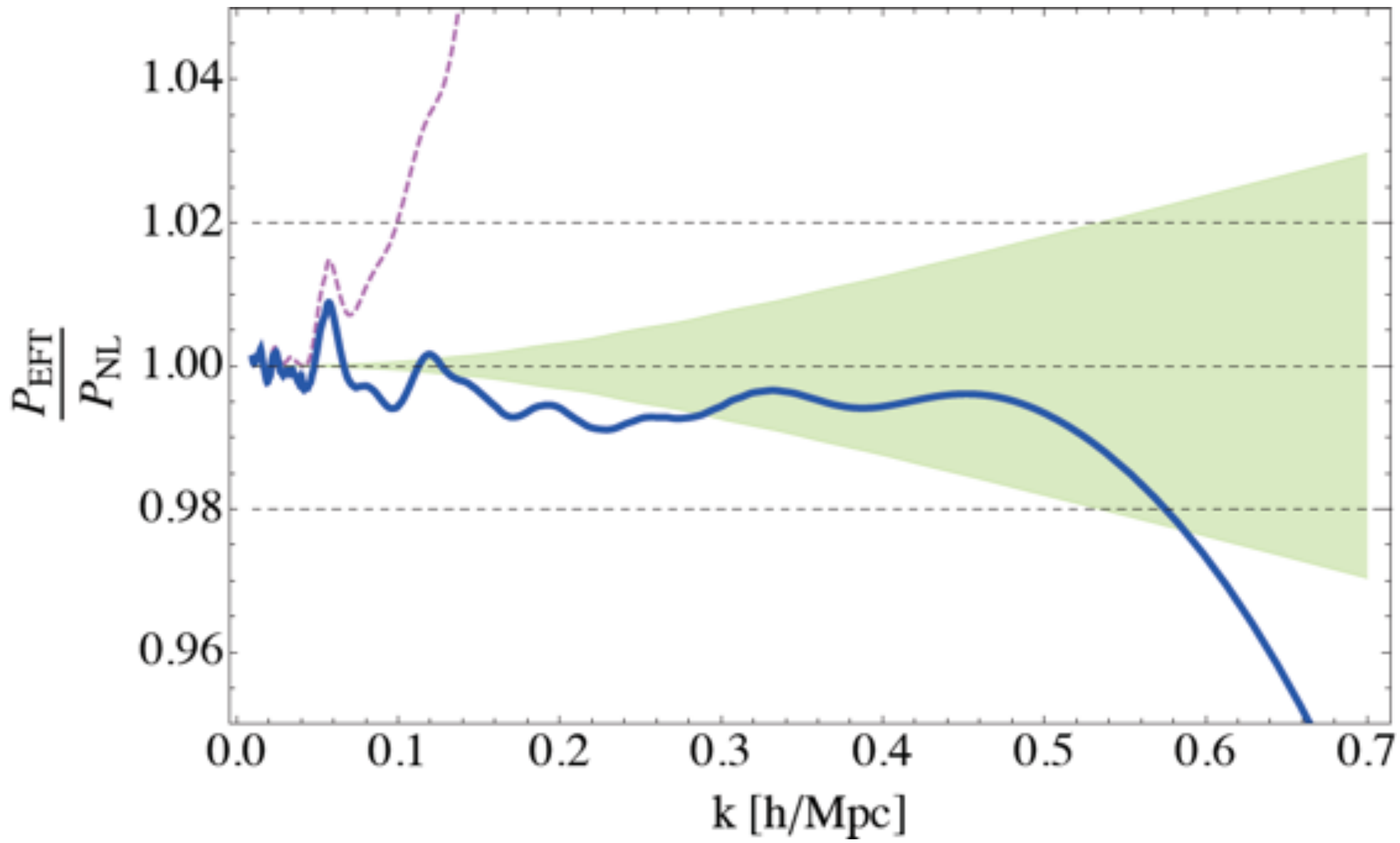
- At one-loop, similarly great results
  - with no additional parameter
  - as good as they should



- Similar formulas just worked out for Bias

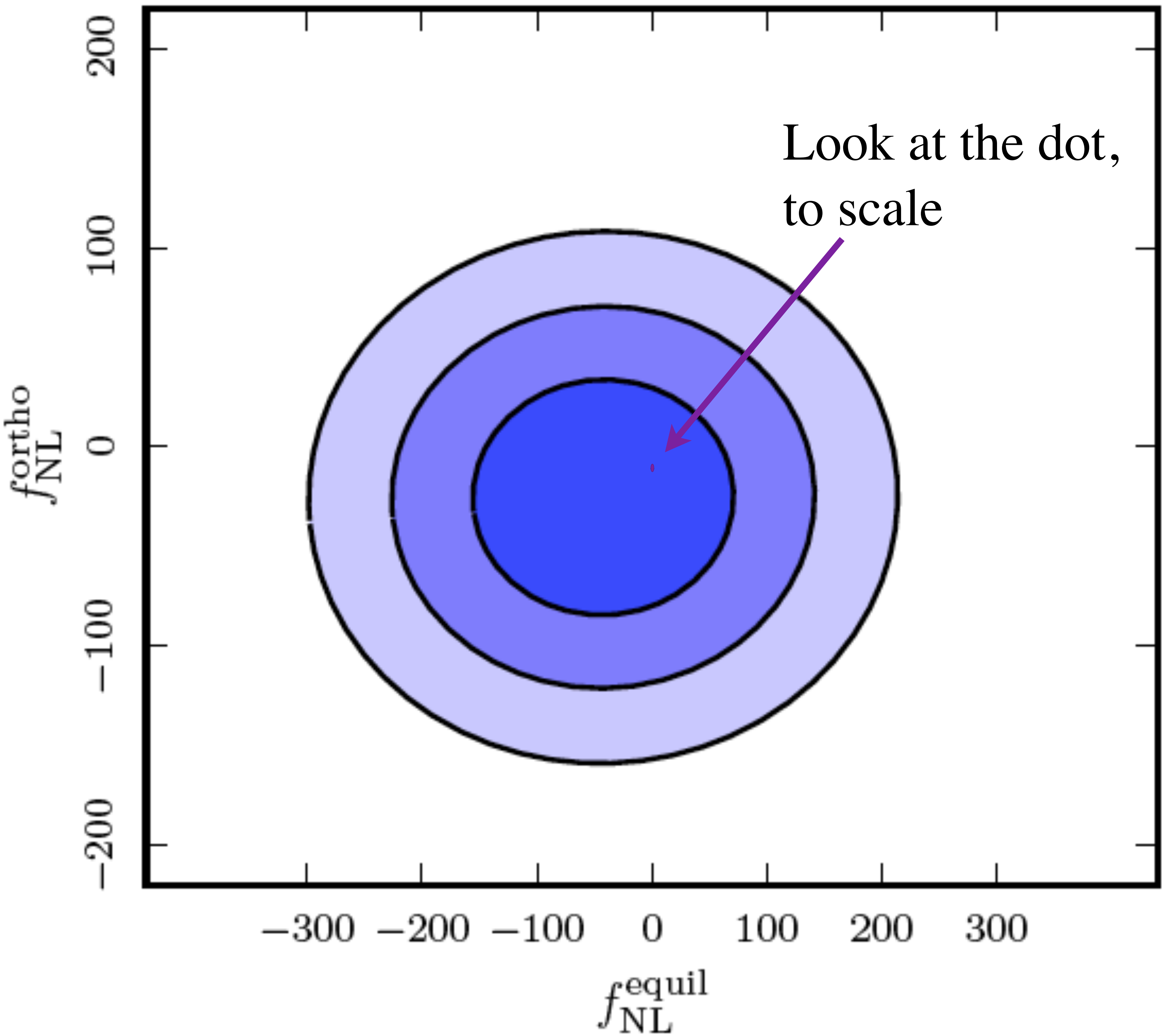
McDoland **0902** Senatore **1406**

EFT of Large Scale Structures



- A manifestly convergent perturbation theory $\left(\frac{k}{k_{NL}}\right)^L$
- we fit until $k_{max} \simeq 0.6\, h\,\mathrm{Mpc}^{-1}$, as where we should stop fitting
  - there are 200 more quasi linear modes than previously believed!
  - huge impact on possibilities, for ex: $f_{NL}^{\text{equil., orthog.}} \lesssim 1$
- Can all of us handle it?! This is an huge opportunity and a challenge for us

With this



Look at the dot,
to scale
200
100
0
−100
−200
f_{NL}^{ortho}
−300
−200
−100
0
100
200
300
f_{NL}^{equil}

Conclusions

- Many (most?) of the features of QFT appear in the EFT of LSS:
  - Loops, divergencies, counterterms and renormalization
  - non-renormalization theorems
  - Calculable and non-calculable terms
  - Measurements in lattice and lattice-running
  - IR-divergencies
- Results seem to be amazing, many calculations and verifications to do:
  - like if we just learned perturbative QCD, and LHC was soon turning on
    - higher n-point functions
    - Validation with simulation
  - With a growing number of groups (Caltech, Princeton, IAS, Cambridge, CEA, Zurich..., immediately after 2-loop result, a Princeton workshop was organized)
- If this works, the 10-yr future of Early Cosmology is good, even with no luck

The BAO peak in `5 minutes'

- The IR-resummation is crucial to get the BAO peak right.
  - we can do this very quickly.

with Zaldarriaga **1404**