Toward Systematics of Axion Monodromy

Based on works (2008-**present**) with
Westphal; McAllister; Wrase
Flauger; Dong, Horn; Dodelson, Torroba,
Senatore, Zaldarriaga; Mirabayi
as well as related works by Kaloper, Sorbo,
Lawrence, Pajer, Easther, Peiris, Xu,
Meerburg, Spergel, Wandelt,Roberts,
Dubovsky, D'Amico, Gobbetti, Kleban, Schillo,
Gur-Ari; Marchesano, Shiu, Uranga, Palti,
Weigand, Wenren, Schlaer, Lust, Hebecker,
Kraus, Witowski, Ibanez, Valenzuela, Dine,
Draper, Monteaux, Arends, Heimpel,
Mayrhofer, Schick, Yonekura, Higaki,
Kobayashi, Seto, Yamaguchi, Hassler, Massai,
Grimm, Ibe, Harigaya,...Kallosh-Linde(sugra)
cf the earlier N-flation scenario by
Dimopoulos, Kachru, McGreevy, Wacker;...
and 2-axion alignment Kim/Nilles/Peloso...
+chromo-natural, (gauge) inflation

As they map the universe with extraordinary coverage and precision, CMB/LSS experiments are reaching unprecedented observational sensitivity to `fundamental' physics including quantum gravity.

--BICEP2(+ input from BICEP1, Keck Array, SPT/ACT, Planck,WMAP,...): Tour de Force B-mode detection, at a level consistent with inflationary quantum gravitational waves (uncertain model-dependent amplitude), but could be consistent with foregrounds (uncertain, complicated) [Flauger Hill Spergel '14]
--Planck +Joint analysis (+ B3, Spider, CLASS,...)

--Chance to develop systematic understanding in the next months, years as r/dust, NG etc play out.

Outline

* Inflaton Field Range and quantum gravity
* String theory: large field range with underlying periodicity (monodromy)
* New examples and phenomenological range

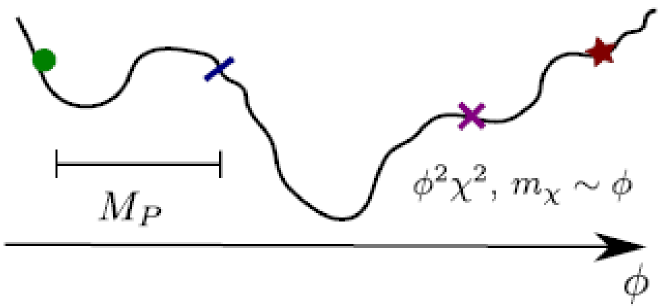
$$V \simeq \mu^{4-p}\phi^{p} + \Lambda^{4}(\phi)\sin\left(\frac{\phi}{f(\phi)} + \gamma\right)$$

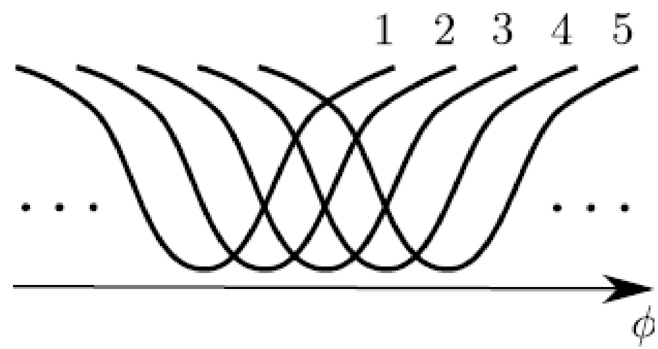
$p = 3, 2, 4/3, 2/3, \ldots$ (cf '08 $p = \cancel{2/3}, 1$; '10 $p \le 2$)

*Toward a systematic understanding of powers; n_s, r.
*Ocillatory templates for Planck2014

Parameterized ignorance of quantum grav.

String Theory axions (and duals)





New degrees of freedom each $\Delta\Phi \sim M_P$

From ubiquitous Axion-Flux couplings

No continuous global symm. in QG

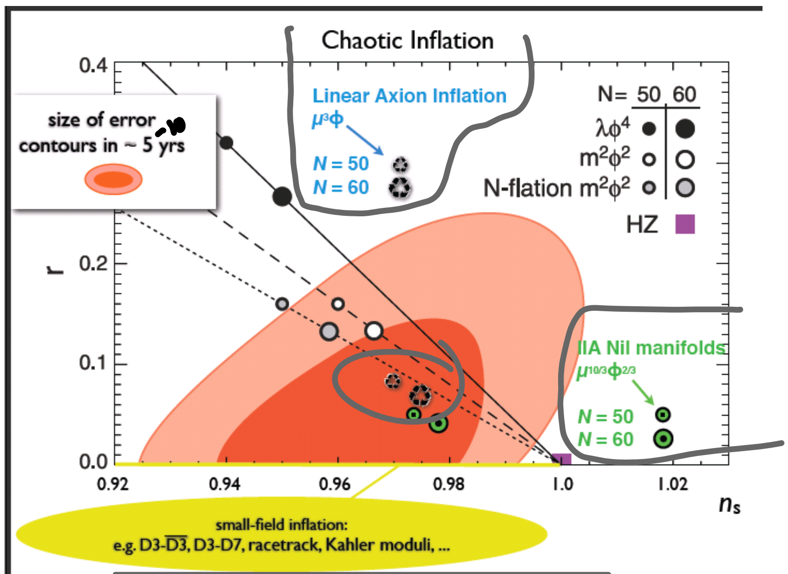
Discrete shift symm., f<<M_p

[cf Chaotic Infl.(Linde), Natural Infl. (Freese et al)]

This is a rather general mechanism exemplified by a variety of specific models.

One basic question is whether it was a coincidence that these landed within the phenomenologically viable regime in n_s, r. We will push on this later in the talk.

Result for '08 example



$r = 0.07$ $\quad V(\phi) = \mu^3\phi + \Lambda^4\cos\left(\frac{\phi}{2\pi f}\right)$

$n_s \simeq 0.98$

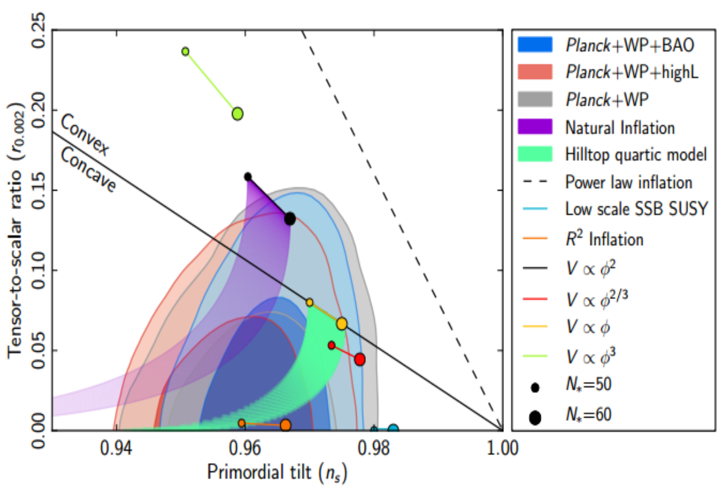
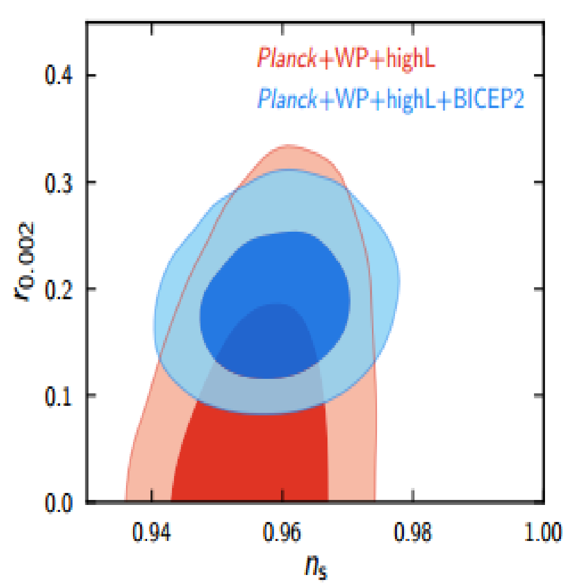


Fig. 1. Marginalized joint 68% and 95% CL regions for n_s and $r_{0.002}$ from *Planck* in combination with other data sets compared the theoretical predictions of selected inflationary models.



power law potentials with p=3,2,4/3, 1,2/3,...

r=.2, .13, .09, .07, .04,...

so far. We hope to get this understood more systematically in the B-mode era.

Inflation does not hinge on primordial B-modes, the TT power spectrum + E-mode polarization already provide a function's worth of evidence in favor of the paradigm. Despite interesting efforts, no consistent alternative theory is known (cf BH thermo, singularities).

What is at stake instead is a true observational lever to Quantum Gravity.

The particular ``model''

$$L_{Kinetic} - V(\phi)$$

is of interest insofar as it is connected to other physics. A tensor/scalar ratio r >.01 is strongly sensitive to quantum gravity via Lyth/Turner

Lyth/Turner Relation

$$N_e = \int \frac{da}{a} = \int \frac{da}{dt}\frac{dt}{a} = \int H\,dt$$

$$= \int \frac{H M_p}{\dot\phi}\frac{d\phi}{M_p} = \sqrt{8}\, r^{-\frac{1}{2}} \frac{\Delta\phi}{M_p}$$

using

$$r = \frac{\gamma\gamma}{\zeta\zeta} = \frac{\text{tensor}}{\text{Scalar}} \sim \frac{\frac{H^2}{M_p^2}}{\frac{H^4}{\dot\phi^2}}$$

and assuming no strong variation of $HM_p/\dot\phi$, and no exotic sources

$\Delta\Phi > M_P$ <=> $r > .01$

implies sensitivity of

$$V(\phi) = V_0 + \sum_n c_n \frac{(\phi-\phi_0)^n}{M_p^n}$$

$$\varepsilon \sim \frac{\dot{\phi}^2}{V} \sim \left(\frac{V' M_p}{V}\right)^2 \; ; \quad \eta \sim \frac{\ddot{H}}{H^2} \sim \frac{V'' M_p^2}{V}$$

to infinite sequence of *dangerously irrelevant* Planck-suppressed operators, compelling a UV complete treatment (shift symmetry gives radiative stability, Wilsonian naturalness, but still large assumption about classical theory).

String theory is a good candidate for QG

*Recover S=A/(4G)(special cases)
*AdS/CFT...
*UV finite amplitudes, singularity resolutions, dimensionality and topology changing transitions,...
*Intricate connections among different limits --->**Landscape of vacua, fitting with Weinberg et al's picture of late-time acceleration
**Not anything goes: f_axion <M_p; No hard Λ; Light d.o.f. at limits of moduli space [Ooguri/Vafa],...).

Monodromy generates symmetry-controlled large field range and observable B mode signal. (Other inflation mechanisms can yield low r.)

$$\int d^D x \sqrt{G} \sum_q \Big| \overbrace{F_q - C_{q-3} \wedge H + F_{q-\cdots} B \wedge \cdots \wedge B}^{\tilde{F}_q \text{ Gauge-invar.}} \Big|^2$$

fluxes

$$\int_{\Sigma_q} F_q = Q_q$$

axions $b = \int_{\Sigma_2} B$

(Direct Dependence)

This generalizes Stueckelburg couplings in electromagnetism

$$S = \int d^4x \left\{ F^2 - \rho^2 (\partial\theta - A)^2 \right\}$$

Gauge symmetry $A_\mu \to A_\mu + \partial_\mu \Lambda$
$\theta \to \theta - \Lambda$

In string theory, the string sources a $\underbrace{\text{2-index}}$ gauge potential B_{MN} analogously to how a charged particle sources A_μ in Electromagnetism

axions $=$ B_{MN}-modes
(and duals)

e.g. $D=10$ IIA

$$H = dB, \quad \tilde{F}_2 = dC_1 + F_0 B$$

$$\tilde{F}_4 = dC_3 + C_1 \wedge H_3 + \frac{1}{2} F_0 B \wedge B$$

This, its reductions & T-duals lead to

$$|\tilde{F}_q|^2 \sim |QB^n|^2$$

for various $n \equiv \frac{p_0}{2}$ ← fiducial power of b

$$\ldots \quad |\tilde{F}_q|^2 \sim |QB^n|^2$$

for various $n \equiv \frac{p_0}{2}$ fiducial power of b

Back reaction on moduli will reduce the power to $V \sim \phi^{p<p_0}$

To be systematic, we will later try to push p_0 $(\simeq p)$ high, to see if the theory allows that

Corrections

$$\sum_{k>1} g_s^{2k} |\tilde{F}_q|^{2k} \sim \sum_k \underset{\uparrow}{g_s^{2k}} \frac{Q_m^2}{\underset{\nearrow}{L^{2m}}} b^{2nk}$$

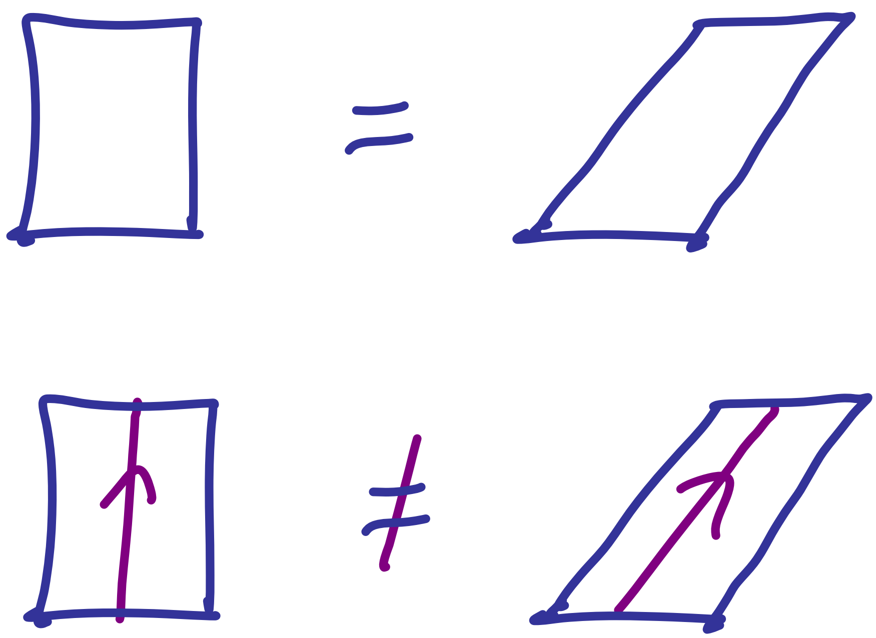
suppression

In general,
Work at large(-ish) radii & weak g_s for control, with or without low-energy SUSY.

Similarly, $\tau \to \tau + 1$

generically lifted

by fluxes



$$V = V_0(\chi) + V_1(\chi)\Big(\sum_n Q^{(2n)} \overset{\text{axion}}{b^n}\Big)^2 + \dots$$

Moduli

- Whole structure periodic

$$b \to b+1 \iff Q \to Q + \Delta Q$$

e.g. brane spectrum on Σ_2

- Each branch (fixed Q) has large range b

$$-b_{uv} < b < b_{uv}$$

$V(b_{uv} \gg 1) = V_{uv}$ (density at which lose control)

$$V = V_0(\chi) + V_1(\chi)\left(\sum_n Q^{(2n)} b^n\right)^2 + \dots$$

Moduli

$$\mathcal{L}_{kin} = \int d^4x \sqrt{-g}\, M_p^2 f(\chi)^2 \dot{b}^2$$

- Moduli adjust, flatten V [Dong et al '10]
- $\int db\, M_p f[\chi(b)] = \phi_b$ Canonically Normalized

e.g. $\dfrac{b}{L^2} \sim \dfrac{\phi_b}{M_p}$ (one scale)

$D=10$ Type II at $\phi_b \gg M_p$

$$V \sim M_p^4 \frac{g_s^4}{L^6} \frac{Q_n^2}{L^{2n}} \left(\frac{\phi^2}{M_p^2} + \frac{\phi^4}{M_p^4} + \mathcal{O}\left(\frac{g_s^2 Q_n^2 \phi^8}{L^{2n} M_p^8} \right) \right)$$

$$+ V_0(\chi = g_s, L, \ldots)$$

In specific models, find

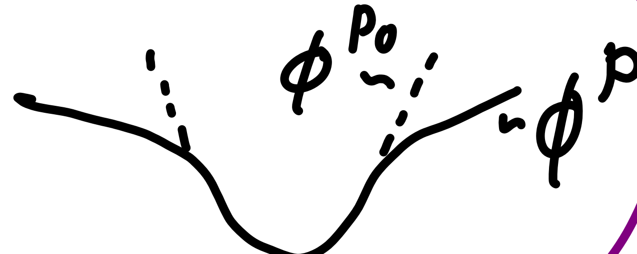
$$V \sim \hat{V}_1(\chi)\phi^{p_0} + V_0(\chi) \Big|_{\chi_{min}}$$

$$\simeq \mu^{4-p}\phi^p + \Lambda^4(\phi)\cos\left(\frac{\phi}{f(\phi)} + \gamma\right)$$

With $p < p_0$; $p = 3, 2, 4/3, 1, 2/3$

- $V_{\text{inflation}}$ helps stabilize moduli χ in 2-term structure: Flux quanta → $Q\ b$

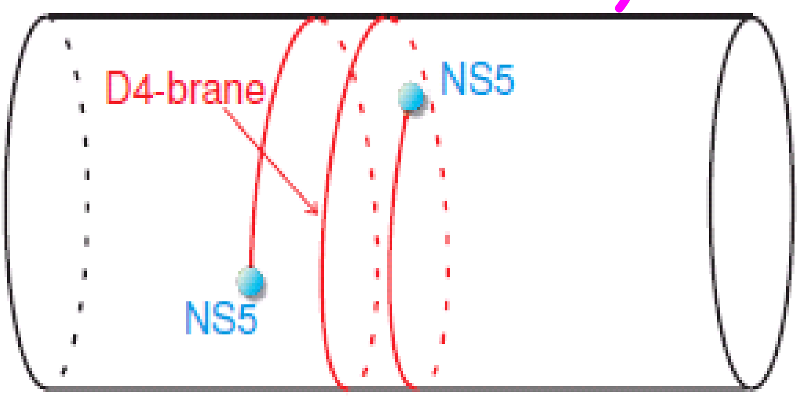
$$\left(\frac{L_2}{L_1}\right)^{n} Q_1^2 + \left(\frac{L_1}{L_2}\right)^{\hat{n}} (b Q_2)^2\, \hat{v}$$

(Backreaction flattens V in such cases: 

)

The specific example '08

(in GKP/KKLT)



(T-dual)

[McAllister ES Westphal; Flanagan et al]

$$V \sim \mu^3 \phi + \Lambda^4 \cos\left(\frac{\phi}{f} + \gamma\right)$$

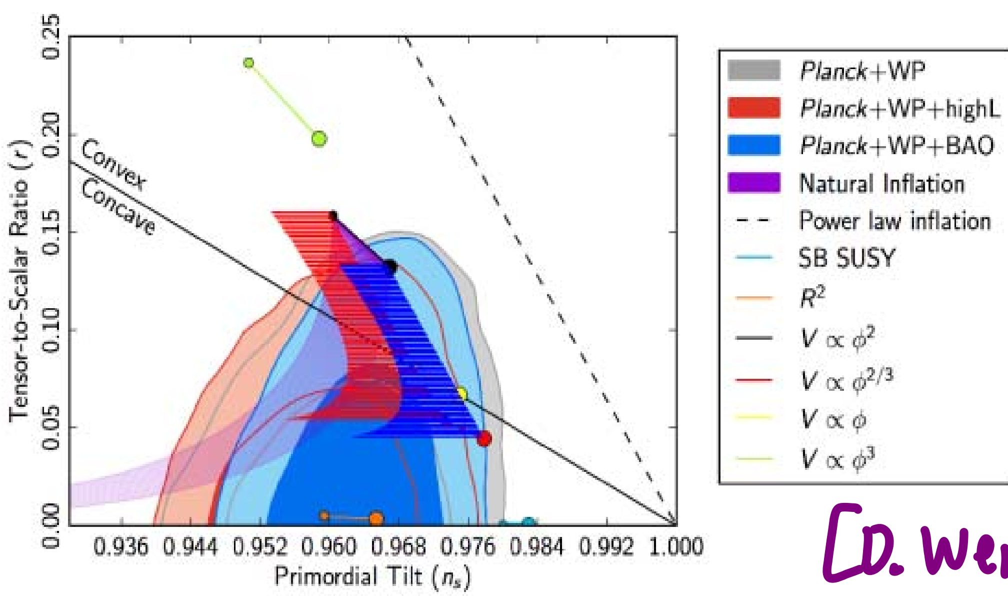
can be understood in this framework via AdS/CFT

as $p_0 = 2 \rightarrow p = 1$

[Dong Horn ES Westphal '10]

Multiple Axions (N-flation), each with monodromy, may be the generic case.

This centralizes n_s



[D. Wenren]

Now to new UV complete examples:

$$S_{II} = \int \frac{d^D x \sqrt{-G}}{\alpha'^{\frac{D-2}{2}}} \left\{ R + \frac{|dB|^2}{g_s^2} + \sum_q |\tilde{F}_q|^2 + \underbrace{\dots}_{\mathcal{O}(g_s^2 \tilde{F}^4)} \right\}$$

$D=10$ IIA $\quad H = dB \quad F_0 = Q_0$

Gauge-invariant generalized field strengths

$$\tilde{F}_2 = dC_1 + F_0 B$$

$$\tilde{F}_4 = dC_3 + C_1 \wedge H_3 + \frac{1}{2} F_0 B \wedge B$$

e.g. $\delta B = d\Lambda_1, \quad \delta C_1 = -F_0 \Lambda_1, \quad \delta C_3 = -F_0 \Lambda_1 \wedge B$

IIB $\quad H = dB, \quad F_1 = dC_0$

$$\tilde{F}_3 = dC_2 - C_0 H, \quad \tilde{F}_5 = dC_4 - \frac{1}{2} C_2 \wedge H + \frac{1}{2} B \wedge dC_2$$

IIB $\cong$ IIA on circle: $\quad C_0 = x^9 Q_0 + \hat{C}_0$
(T-duality) $\quad C_2 = x^9 Q_0 B + \hat{C}_2$

$$\Rightarrow |\tilde{F}_5|^2 \supset |F_1 \wedge B \wedge B|^2$$

[Bergshoeff et al]

New class of examples with larger range of r

Type IIB $\mathcal{L} \supset |F_1 \wedge B \wedge B|^2$

(T-dual of $F_0 \, B \wedge B$ in IIA)

Warmup:

To exhibit flattening effect, consider

e.g. $T^6 = T^2 \times T^2 \times T^2$

$$F_1 = \frac{Q_1}{L_1} \sum_{i=1}^{3} dy_1^{(i)}, \quad B = \sum_{i=1}^{3} \frac{b^{(i)}}{L^2} dy_1^{(i)} \wedge dy_2^{(i)}$$

fluxes

2-form potential field → axion $b = \int_{\Sigma_2} B$

$$F_3 = Q_{31} dy_1^{(1)} \wedge dy_1^{(2)} \wedge dy_1^{(3)} + Q_{32} dy_2^{(1)} \wedge dy_2^{(2)} \wedge dy_2^{(3)}$$

Basic Effect:

$$V \sim M_p^4 \frac{g_s^4}{L^{12}} \left(\frac{Q_1^2}{L^4} u b^4 + Q_{31}^2 u^3 + \frac{Q_{32}^2}{u^3} \right)$$

Inflationary potential & Q_{32} flux stabilize u during inflation

Fluxes stabilize u after inflation

$$u \sim 3^{\frac{1}{4}} L \sqrt{\frac{Q_{32}}{Q_1}} \cdot \frac{1}{b} \propto V_{eff} \propto b^3$$

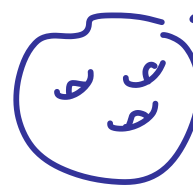
$$p_0 = 4, \quad p = 3$$

Checks:

- Kinetic terms negligible
- u does not go to extreme values with light degrees of freedom   cf Ooguri-Vafa
- Asymmetric axion directions stable ($|m^2| \ll H^2$, $\partial_{\phi_+} V \gg \partial_{\phi_-} V$)

The above mechanism (and generalizations) arises in string compactification on a product of Riemann Surfaces

(Saltman, ES '04) Classical, High-Scale.

• In full stabilization (on  ^{3})
g_s & L also adjust

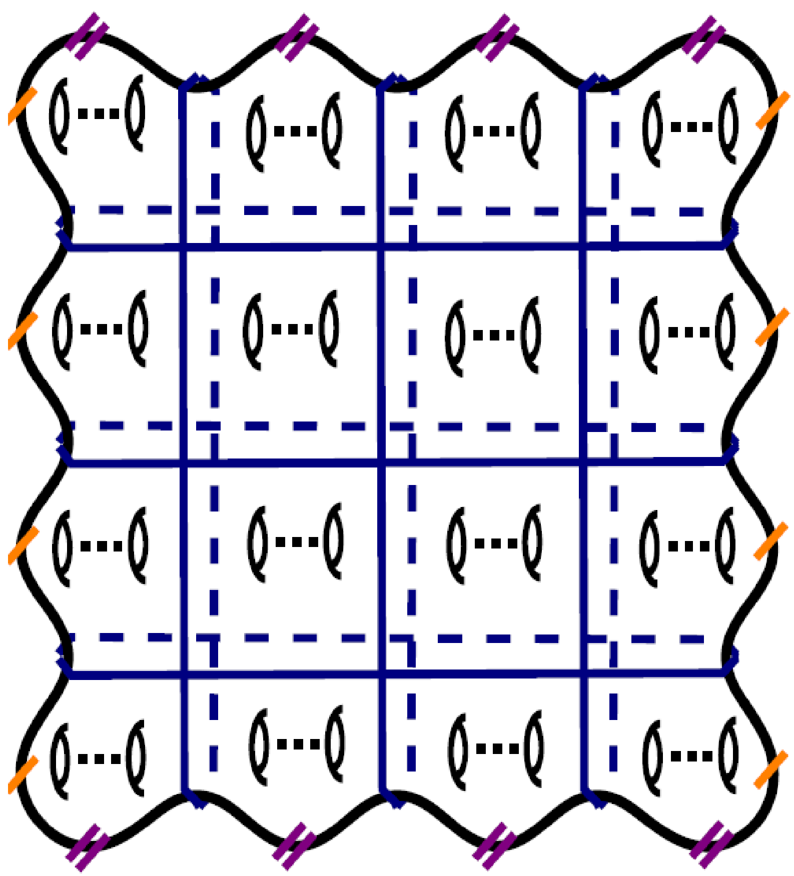
$$\rightarrow \; V \sim \phi^3, \; \phi^2, \; \phi^{4/3}, \; \phi^{2/3}$$

Microphysical constraints to satisfy (proofs of principle)

	Σ_1		Σ_2		Σ_3	
	1	2	3	4	5	6
7-brane	x		x		x	x
7'-brane	x	x		x	x	
7''-brane		x	x	x		x
e.g. F_1						x
e.g. B		x	x	x	x	

7-brane, 7'-brane, 7''-brane: trivial cycles

e.g. B: $\underbrace{\text{2, 3}}_{B^{(1)}}$, $\underbrace{\text{4, 5}}_{B^{(2)}}$ — nontrivial cycles; combinations that vanish on 7-branes



$\eta = g_s / V^{2/3}$ $\quad V =$ volume, $g_s =$ coupling

$$U \sim M_p^4 \Big\{ (h + n_7 - 1)\eta^2 - N_7 \eta^3 + \tilde{q}_5^2 \eta^4$$
$$+ n_3^2 \eta^2 + \tilde{q}_3^2 V^{2/3} \eta^4$$
$$+ q_1^2 V^{4/3} \eta^4 \Big\}$$

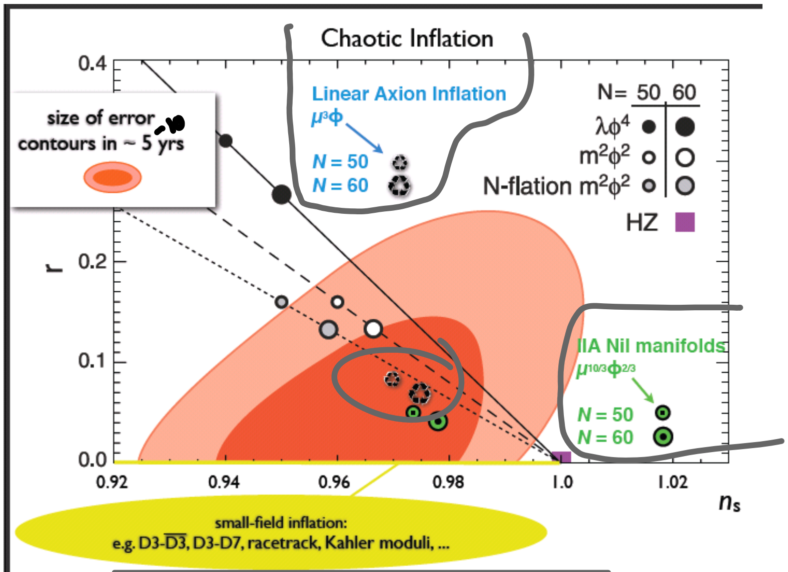
[Saltman - ES]

$$\tilde{q}_3^2 = q_3^2(u) + q_1^2(u) b^2$$

$$\tilde{q}_5^2 \sim \begin{cases} q_5^2(u) + 2 q_5(u) q_1(u) b^2 + q_1^2(u) b^4 & (i) \\ q_5^2(u) + q_1^2(u) b^4 & (ii) \end{cases}$$

Results depend on choices of flux & B ratios & distribution among cycles and Riemann Surfaces

Result for '08 example



$$\boxed{\begin{array}{ll} r = 0.07 & V(\phi) = \mu^3\phi + \Lambda^4\cos\left(\frac{\phi}{2\pi f}\right) \\ n_s \simeq 0.98 & \end{array}}$$

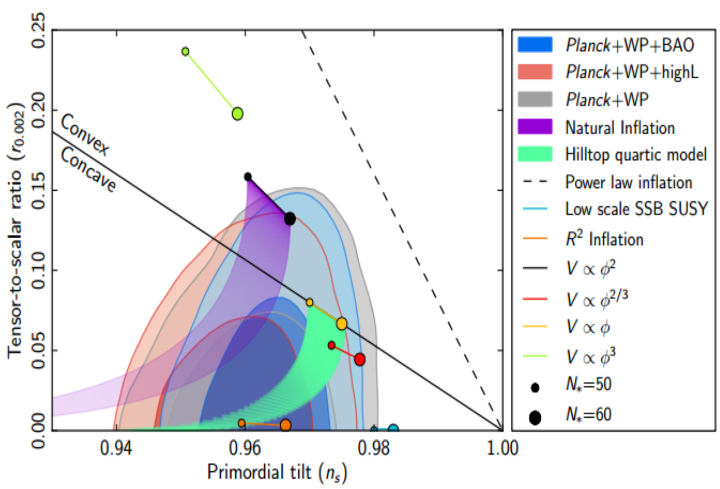
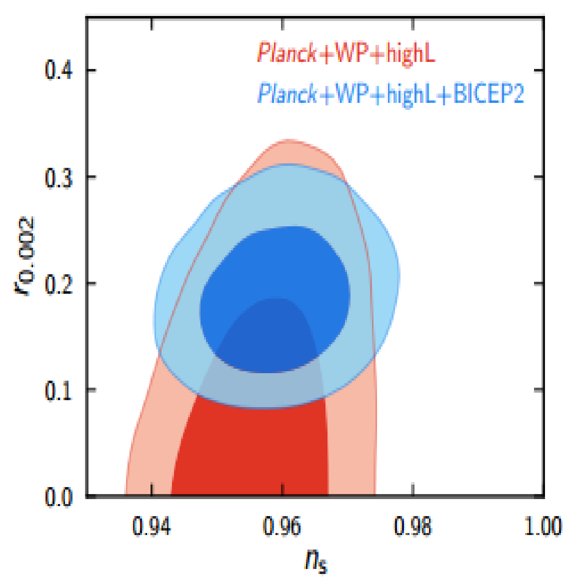


Fig. 1. Marginalized joint 68% and 95% CL regions for n_s and $r_{0.002}$ from *Planck* in combination with other data sets compared the theoretical predictions of selected inflationary models.



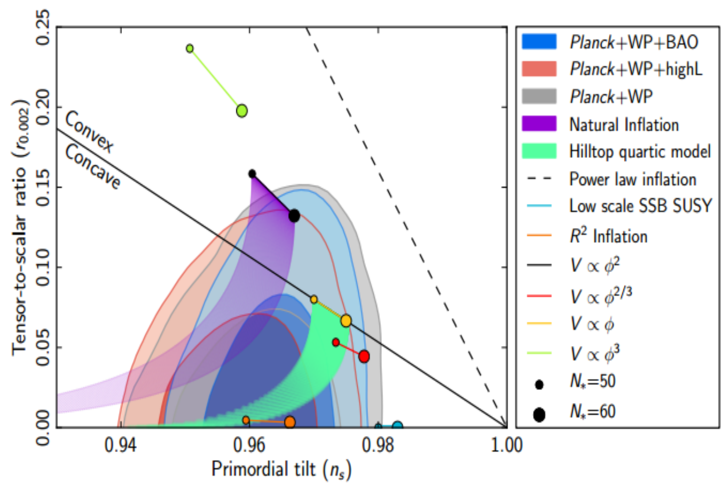
power law potentials with p=3,2,4/3, 1,2/3,...

r=.2, .13, .09, .07, .04,...

so far. We hope to get this understood more systematically in the B-mode era.

What is the UV-complete theory blob in $r, n_s, \ldots$?

[cf Dodelson, Creminelli et al '14...]



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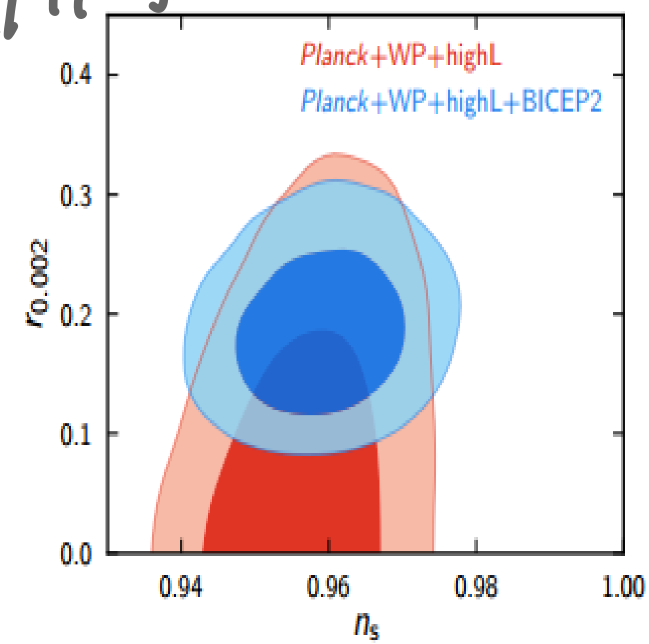

Are there theorems about range, rational values of $p \leftrightarrow n_s, r,$ etc.? cf EFT of perturbations but at $\Delta\Phi > M_p$

[Senatore et al]

Dual Axions

$B_{MN} \longleftrightarrow C_q$

$\longleftrightarrow$ Complex structure moduli

e.g. T^2

$$\rho = b + i\sqrt{G} \longleftrightarrow \tau = \frac{G_{12}}{G_{22}} + i\frac{\sqrt{G}}{G_{22}}$$

$\longleftrightarrow$ Brane positions

...

Axions are $> \frac{1}{2}$ the scalar fields in string theory:

- SUSY case $\Phi = r + i\theta$

  (and duals)

- ~~SUSY~~ limits $N_{\text{axion}} \sim 2^D$

  $N_{\substack{\text{other}\\ \text{moduli}}} \sim D^2$

It was a myth that string theory prefers small r, or that "most models" have that property — at least no credible argument for that.

Axion monodromy systematics

[Dodelson Dong ES Torroba '13 & in progress w/ McCandlish, Wenren]

$$V \sim \hat{V}(\chi)\,\phi^{p_0} + \cdots$$

$$\downarrow$$

$$V \sim \mu^{4-p}\,\phi^{p}$$

Strategy: look in theory space for extreme values of $p^0 \to p$ to see if phenomenological viability is robust.

In $D>10$, P_0 can naively be huge

$$|\tilde{F}_q|^2 = |F_q + B\wedge F_{q-2} + \cdots + F_0 B^{\frac{q}{2}}|^2$$

- But e.g. in product space we find that in appropriate field range, the $B^{\frac{q}{2}}$ is subdominant (cf N-flation).
- More generally, even for large P_0 there is room for strong 'flattening' by many adjusting fields.

Similarly, we can go back to twisted tori

ES westphal '08

Gur-Ari '13

$$(z, \xi^a) \simeq (z-1, \underset{\substack{\Uparrow \\ SL(n,\mathbb{Z})}}{M^a{}_b}\, \xi^b)$$

but in $D > 10$ again to see if the theory will generate extreme values of $p_0 \to p$, or not.

e.g. case $\underset{\underset{SL(n,\mathbb{Z})}{\cap}}{M} = e^{X}$ ← real

$$ds^2 = L_z^2 dz^2 + d\vec{\xi}^{\,T} (e^{zX})^T L^2 (e^{zX}) d\tilde{\xi}$$

wrap 4-brane on cycle $\sigma_a d\xi^a$

& compute DBI action →

$$S = -T_4 \int d^4x\, d\xi \left[V(z) \left(1 - \dot{z}^2 + \cdots\right)\right]$$

$$V(z)^2 = \sigma^T (e^{zX})^T e^{zX} \sigma$$

$$S = -T_4 \int d^4x\, d\xi \left[V(z)\left(1 - \dot{z}^2 + \cdots\right)\right]$$

$$V(z)^2 = \sigma^T \left(e^{zX}\right)^T e^{zX} \sigma$$

$\Rightarrow$ if σ an eigenvector of e^X with e-value $e^\gamma > 1$

will get $\partial\phi_{\text{canonical}} = e^{\gamma z}\partial z$

$$\Rightarrow \quad V(\phi) = \phi^2$$

More generally, will have

$$e^{zX} \sim \begin{pmatrix} 1 & z & \cdots & z^{D-5} \\ & \ddots & & \\ & & & 1 \end{pmatrix} \rightarrow p = \frac{2s}{s+2} < 2$$

$$s = 1, \ldots, O(D)$$

So far, even for extreme topology, D, etc. we don't (yet) find parametrically large p (or even p_0).

Goal (in progress): find exceptions or prove theorem

Oscillation Templates for Planck 2014

[Flauger McAllister ES Westphal]

$$V = V_0(\phi) + \Lambda^4(\phi)\cos\left(\frac{\phi}{f(\phi)} + \gamma\right)$$

[previous: Easther Flauger Peiris, Pajer Planck 2013, Meerburg Spergel Wandelt Aich et al]

low-ℓ anomalies & slow oscillation '14

$\Lambda^4 \cos(\cdots)$ generated by periodic effects such as worldsheet instantons (Large L_{Σ_2}) or particle/string production

Highly model-dependent, but interesting to search for

↳ Challenge: getting $f(\phi)$ wrong can wash out signal over $\ell_{min} \leq \ell \leq 2500$

$$\cos\left[\frac{\phi_k}{f(\phi_k)}\right]$$

$$\phi_k \simeq \sqrt{2p\left(N_* - \log\left(\frac{k}{k_*}\right)\right)}\, M_p$$

Template including effects described above (moduli drift)

$$V = V_0 + \mu^{4-p}\phi^{p}$$

$$+ \Lambda_0^4 e^{-C_0\left(\frac{\phi}{\phi_*}\right)^{\tilde{p}_1}} \cos\left[\gamma_0 + C_1\left(\frac{\phi}{\phi_*}\right)^{\tilde{p}_2}\right]$$

e.g. $p = \frac{4}{3}$, $\tilde{p}_1 = -\frac{1}{3}$, $\tilde{p}_2 = \frac{2}{3}$

Scan over $\tilde{p}_2$ as well as C_1 in the oscillatory part

More general effects such as multiple contribution to period-$f(\phi)$ effects, $\log(\phi)$ factors in $f(\phi)$ from non-perturbative stabilization mechanisms, etc may suggest additional templates (but can be degenerate).

low ℓ ?

There is a theoretical tension between the Wilsonian-natural shift-symmetry control of large-field inflation, and the low-ℓ anomalies if they become significant.

→ worth revisiting examples with flux hierarchies determining field range.