Holographic Cosmology Beyond Inflation?

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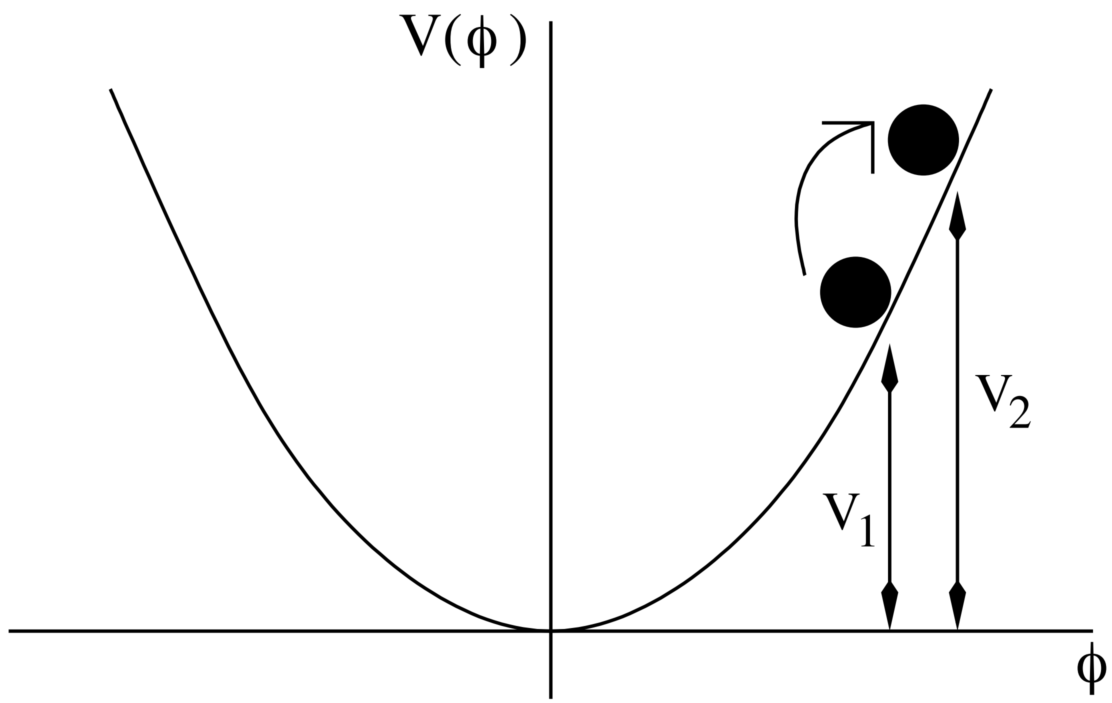
Questions

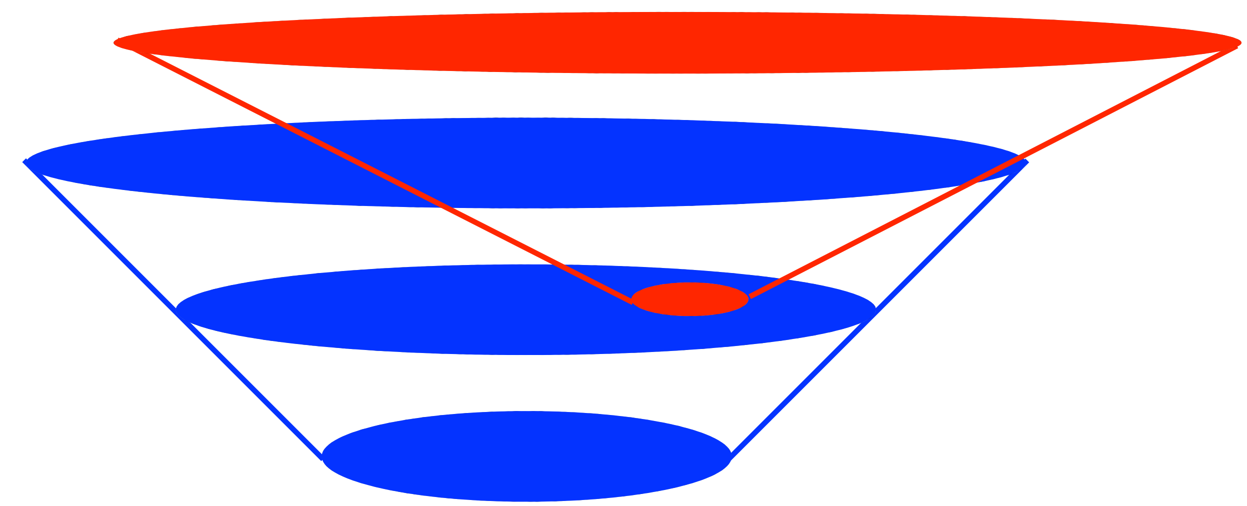
Haven't been thinking about inflation too much in recent years, but BICEP2 claim has rekindled my interest, like most people. Raises a number of old questions.

Some are initial conditions questions (I'm surprised to see many people with related questions here):

- To what extent should we worry about the initial conditions for inflation?
  - How should we picture space, time, dimensions and field content prior to inflation?
  - Within an effective field theory treatment, it may be that inflation requires homogeneity over a large patch, or other unlikely conditions, to begin.
  - This can make the probability low.
  - If we have a UV complete theory (such as string theory) can we address this question? Are there non-statistical answers? (Do we understand accelerating solutions even?)
  - Possibly inflation itself can overcome this with sufficient volume enhancement.
  - Expect (simple arguments; Dvali; Arkani-Hamed et al.) that there are limits on the classical number of efolds. Does this affect this initial condition calculation?
  - Do we *need* eternal inflation?

Eternal Inflation?





- Is eternal inflation a done-deal? Do we trust the naive intuition that it happens?
  - In the usual picture, it involves a quantum fluctuation in the inflaton, up the potential, over an extended region.
  - When is this within the semiclassical approximation? (See Creminelli, Dubovsky, Nicolis, Senatore & Zaldarriaga arXiv:0802.1067 [hep-th]).
  - If not, is there a way to understand it within a Quantum Gravity model, such as string theory? (Has implications beyond just the early universe).
  - Recent evidence to the contrary in 1+1d (Martinec & Moore)
  - How might we answer this generally?

Other Models?

- What do we really know about the space of theories that can simultaneously
  - Solve the flatness and horizon problems?
  - Generate the observed spectrum of density perturbations?
  - Make predictions for upcoming measurements (such as B-modes)

Motivated by these questions, been trying to learn more about frameworks in which might be useful to ask them.
One possibility: holographic cosmology (using AdS/CFT).

Possibilities for the Early Universe

- When we think about early universe, we extrapolate GR coupled to sources back into epochs of increasingly high curvature, and decreasing control.

- It is remarkable that we have fascinating possibilities to address our problems, and explain structure, such as inflation, in the regime in which we know how to calculate.

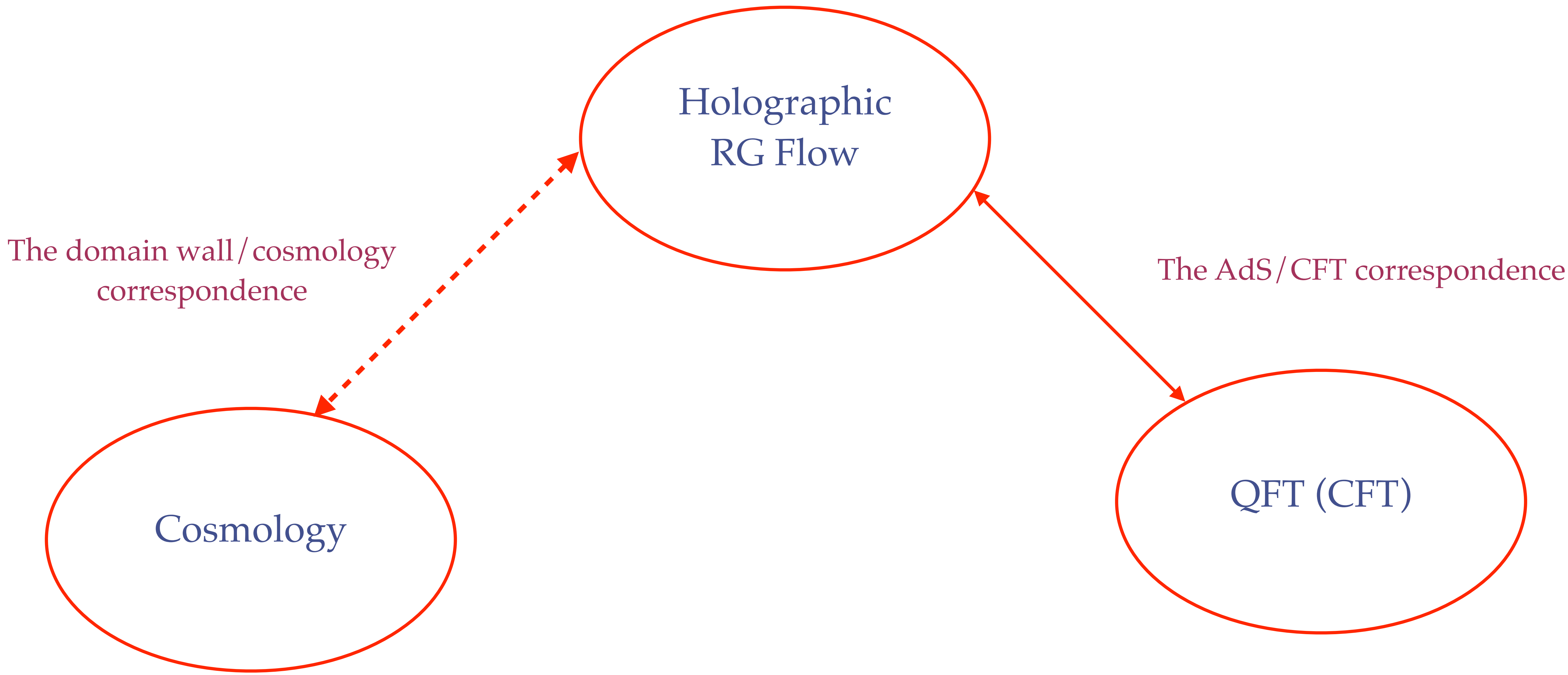
- Understanding the space of alternatives is difficult if we restrict ourselves to our perturbative, weakly-coupled description.

- Do we need the entire theory of quantum gravity to expand possibilities?

- One thing we've learned is that, in some cases, there exists a weak/strong coupling duality between gravity and field theory (AdS/CFT). Is there a way to use this to learn more?

- Obviously many people have considered this. One concrete proposal is to use the domain wall/cosmology connection (McFadden & Skenderis).

Domain Wall Cosmology



Here, “domain wall” just means “having a radially-dependent profile in the bulk”.

Domain Wall Cosmology

In cosmology, used to seeing the metric

$$ds^2 = \eta dz^2 + a(z)^2 \delta_{ij} dx^i dx^j \qquad \Phi = \Phi(z)$$

where $\eta = -1$ and z representing the time coordinate.

If we instead consider $\eta = +1$, then this describes a (Euclidean) “domain wall” (and if we made one of the x-coords timelike, it would be a Lorentzian one)

$$S = \eta \int d^4x \sqrt{|g|} \left[-\frac{1}{2\kappa^2} R + (\partial\Phi)^2 + V(\Phi) \right]$$

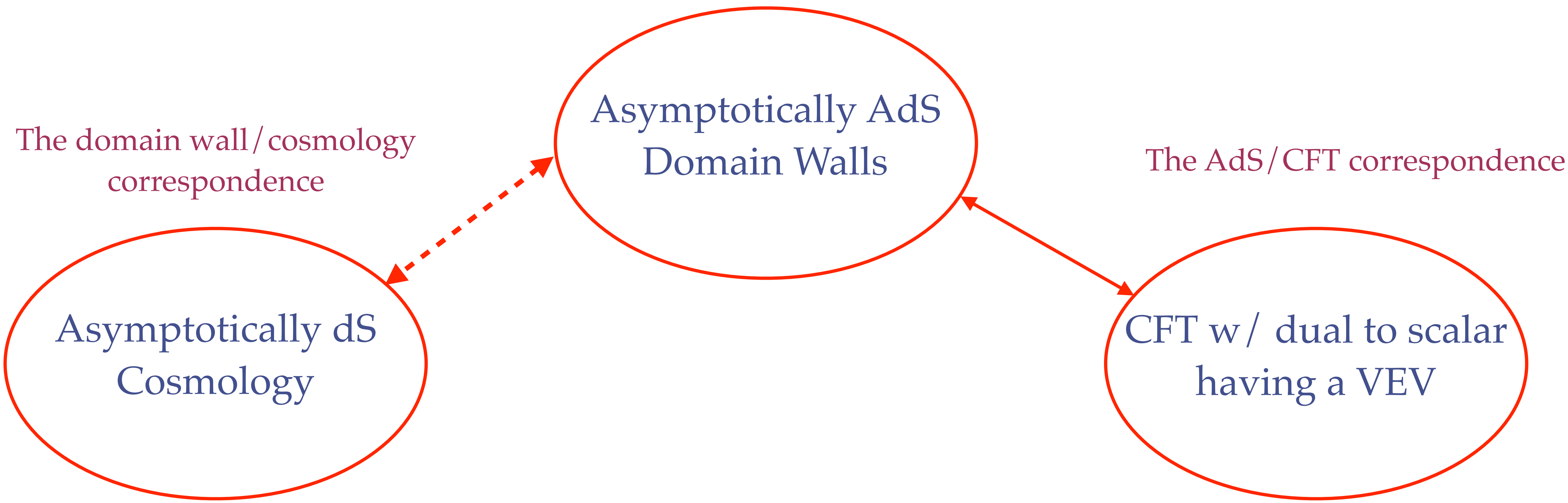
So, for every flat FRW solution with potential V there exists a related domain-wall solution with potential –V

This holds, for both background solutions, and linear perturbations. These solutions their properties can then be, in principle, related to behavior of boundary field theory.

For inflationary cosmologies, has been used to relate power spectra and holographic 2-point functions (and, ultimately, higher-point functions).

[McFadden & Skenderis]

For Example



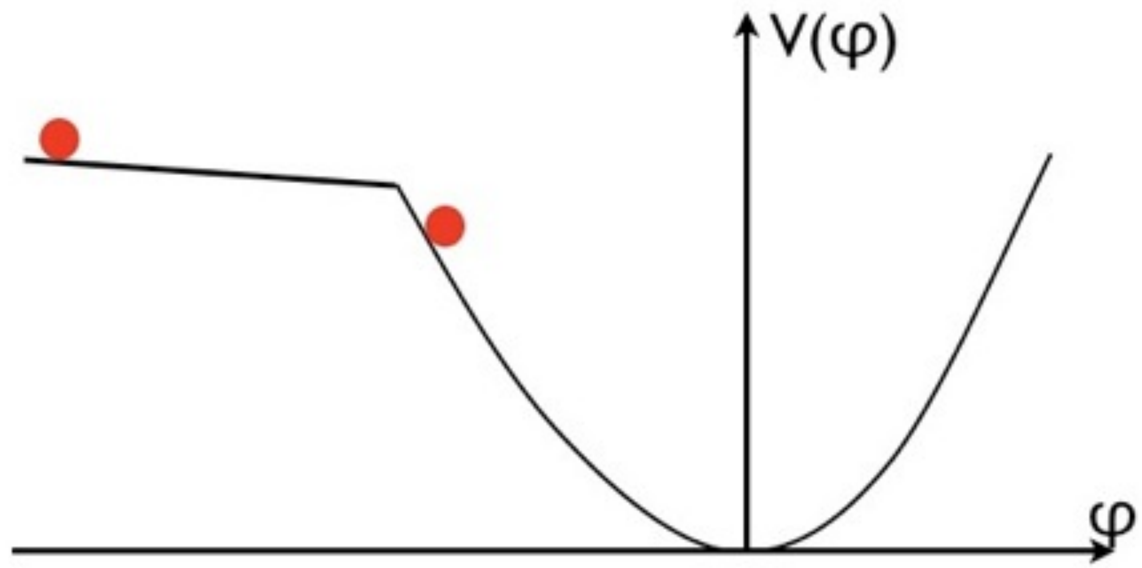
Different asymptotics & bulk slicings could probe (all?) other possible cosmologies

Inflation

We think of inflation as a solution to GR coupled to a fluid or scalar field so that

$$3M_P^2 H = \rho, \quad \rho \approx \text{cst} \longrightarrow ds^2 = -dt^2 + e^{Ht} d\vec{x}^2$$

with H approximately constant



If it was precisely de Sitter, then would have symmetry of dS group. But because quasi-dS, this is broken to the Poincaré algebra

$$\mathfrak{so}(1,4) \longrightarrow \mathfrak{iso}(3)$$

We use the fact that the spaces $dS_4,\ \mathbb{H}^4,\ AdS_4$ are closely related

Holographic Cosmology

$$ds^2_{dS_4} = \frac{H^2}{\eta^2}(-d\eta^2 + \delta_{ij}dx^i dx^j)$$

$$ds^2_{\mathbb{H}^4} = \frac{L^2}{z^2}(dz^2 + \delta_{ij}dx^i dx^j)$$

$$ds^2_{AdS_4} = \frac{L^2}{z^2}(dz^2 + \eta_{\mu\nu}dx^\mu dx^\nu)$$

These can be mapped into each other via

$$ds^2_{dS_4} \xrightarrow{L=iH,z=i\eta} ds^2_{\mathbb{H}^4} \xrightarrow{x^1=ix^0} ds^2_{AdS_4}$$

Corresponding symmetries

$$O(1,4) \longrightarrow O(1,4) \longrightarrow O(2,3)$$

This allows us to model quasi-dS by a non-unitary Euclidean CFT deformed by relevant operators. In this picture, slow roll-parameters get related to field theory parameters; e.g.

$$\beta^2 = 2M_P^2\epsilon$$

Thus, in the boundary QFT (and hence in the bulk) this realizes the required pattern

$$\mathfrak{so}(1,4) \longrightarrow \mathfrak{iso}(3)$$

Can we Extend to Other Models?

[K. Hinterbichler, J. Stokes and M.T., arXiv:1408.1955 [hep-th]]

Are there dualities like this for any possible evolution of early universe? Might provide alternative way to probe uniqueness of inflation as solution to questions of early cosmos.

First example, is there a bulk (gravitational) dual to the *Pseudo-Conformal Universe*?

[K. Hinterbichler & J. Khoury]

Works by SSB of conformal group in 4D to subalgebra isomorphic to dS isometries. Postulate early universe described by a CFT w/ scalar primary operators $\mathcal{O}_I$ w/conformal dimensions Δ_I *in Minkowski space*

Assume operators develop t-dep VEVs of form

$$\langle \mathcal{O}_I \rangle = \frac{c_I}{(-t)^{\Delta_I}} \qquad\qquad -\infty < t < 0$$

As required this breaks conformal group in 4D to subalgebra isomorphic to dS isometries.

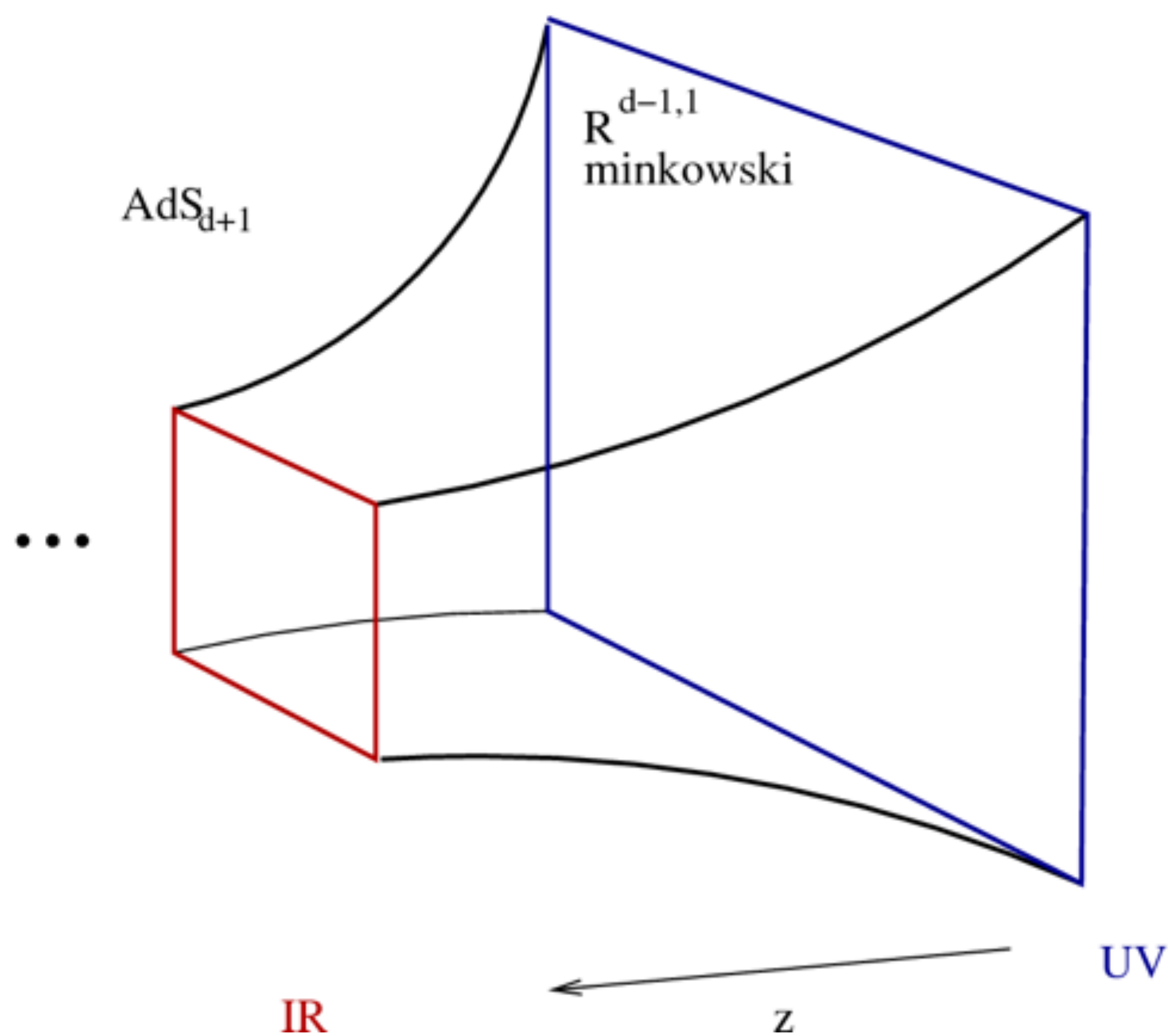
$$\mathfrak{so}(2,4) \longrightarrow \mathfrak{so}(1,4)$$

Breaks down at late times near t=0, need reheating transition to radiation-domination

SSB pattern means spectator fields w/ vanishing conformal dimension acquire scale-invariant spectra

For duals to inflation, physics of interest is in bulk and dual is boundary CFT. Here, physics of interest is non-gravitational CFT on boundary, and dual is bulk gravitational theory.

We're constructing bulk dual to a CFT in the pseudo-conformal phase.



[arXiv:0909.0518]

Holography of the PCU

We're looking for a gravitational dual that realizes the breaking

$$\mathfrak{so}(2,4) \longrightarrow \mathfrak{so}(1,4)$$

Important point is, of course, that generators of $\mathfrak{so}(2,4)$ and generators of AdS_5 are in one-to-one correspondence.

Start simply, with CFT in conformal vacuum. We know gravity dual is exact AdS space

$$ds^2 = dr^2 + e^{2r}\eta_{\mu\nu}dx^\mu dx^\nu , \qquad L = 1, \qquad z = e^{-r}$$

Killing vectors are

$$P_\mu = -\partial_\mu$$

$$L_{\mu\nu} = x_\mu \partial_\nu - x_\nu \partial_\mu$$

$$K_\mu = -x^2\partial_\mu + 2x_\mu x^\nu \partial_\nu - z^2 \partial_\mu + 2x_\mu z \partial_z$$

$$D = x^\mu \partial_\mu + z\partial_z$$

If we turn on a constant VEV in the field theory, then this will break $\mathfrak{so}(2,4) \longrightarrow \mathfrak{iso}(1,3)$
Bulk invariance under remaining symmetries then implies:

$$P_\mu \phi = L_{\mu\nu}\phi = 0 \implies \phi = \phi(r)$$

Domain Wall Cosmology for the PCU

So a constant VEV corresponds to a *domain wall* spacetime

$$ds^2 = dr^2 + e^{2A(r)}\eta_{\mu\nu}dx^\mu dx^\nu , \qquad \phi = \phi(r)$$

This will be asymptotically AdS if $A(r) \longrightarrow r$ as $r \longrightarrow \infty$

Consider a VEV that breaks $\mathfrak{so}(2,4) \longrightarrow \mathfrak{so}(1,4)$

There is a dS subalgebra of AdS, spanned by $D, \; P_i, \; L_{ij}, \; K_i$

$$P_i\phi = L_{ij}\phi = D\phi = 0 \implies \phi = \phi(x) \qquad\qquad x = t/z$$

Metric can then be written

$$ds^2 = \frac{dx^2}{x^2} - 2\frac{dx\,dt}{xt} + \frac{1}{t^2}\left[(1-x^2)dt^2 + x^2\delta_{ij}dx^i dx^j\right]$$

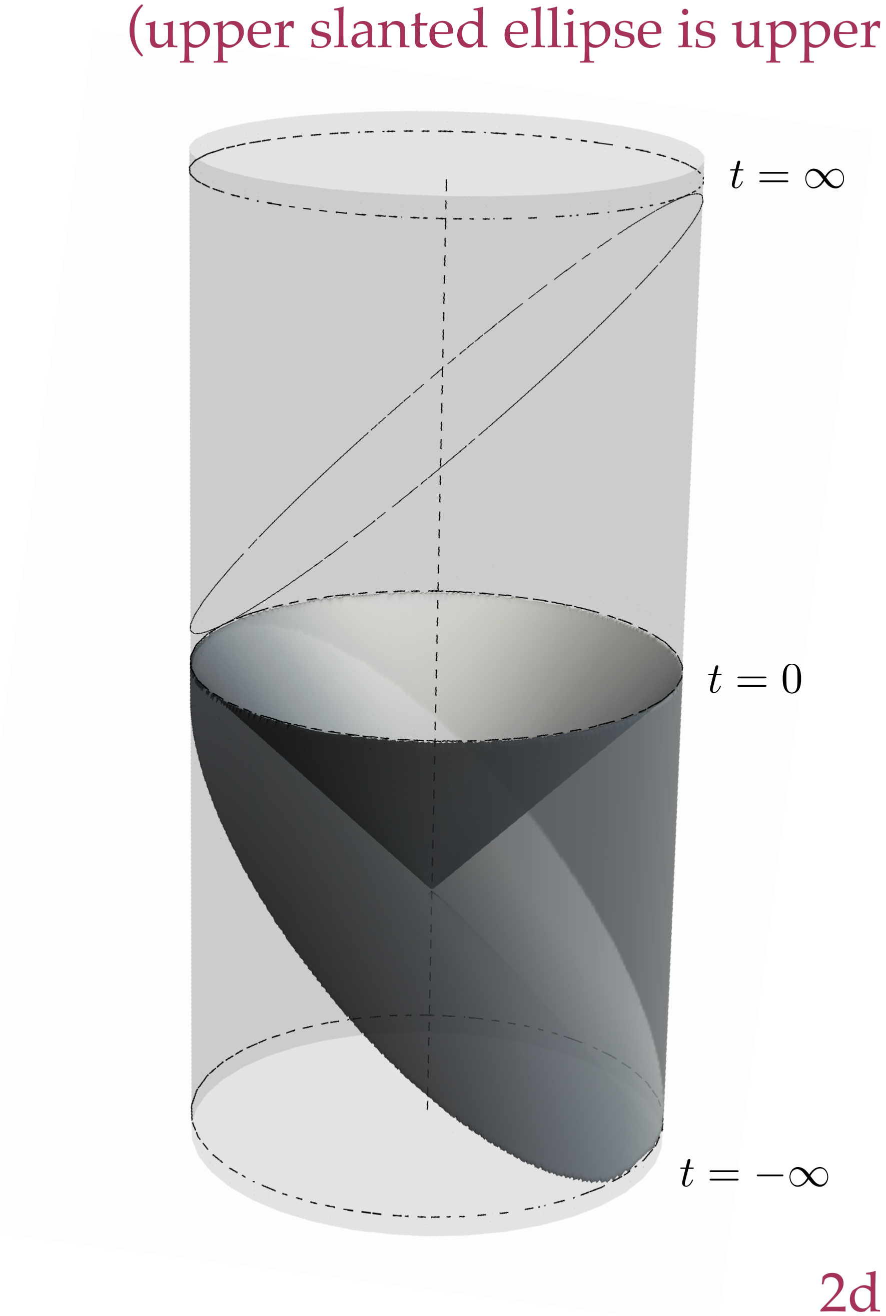
Which can be diagonalized as

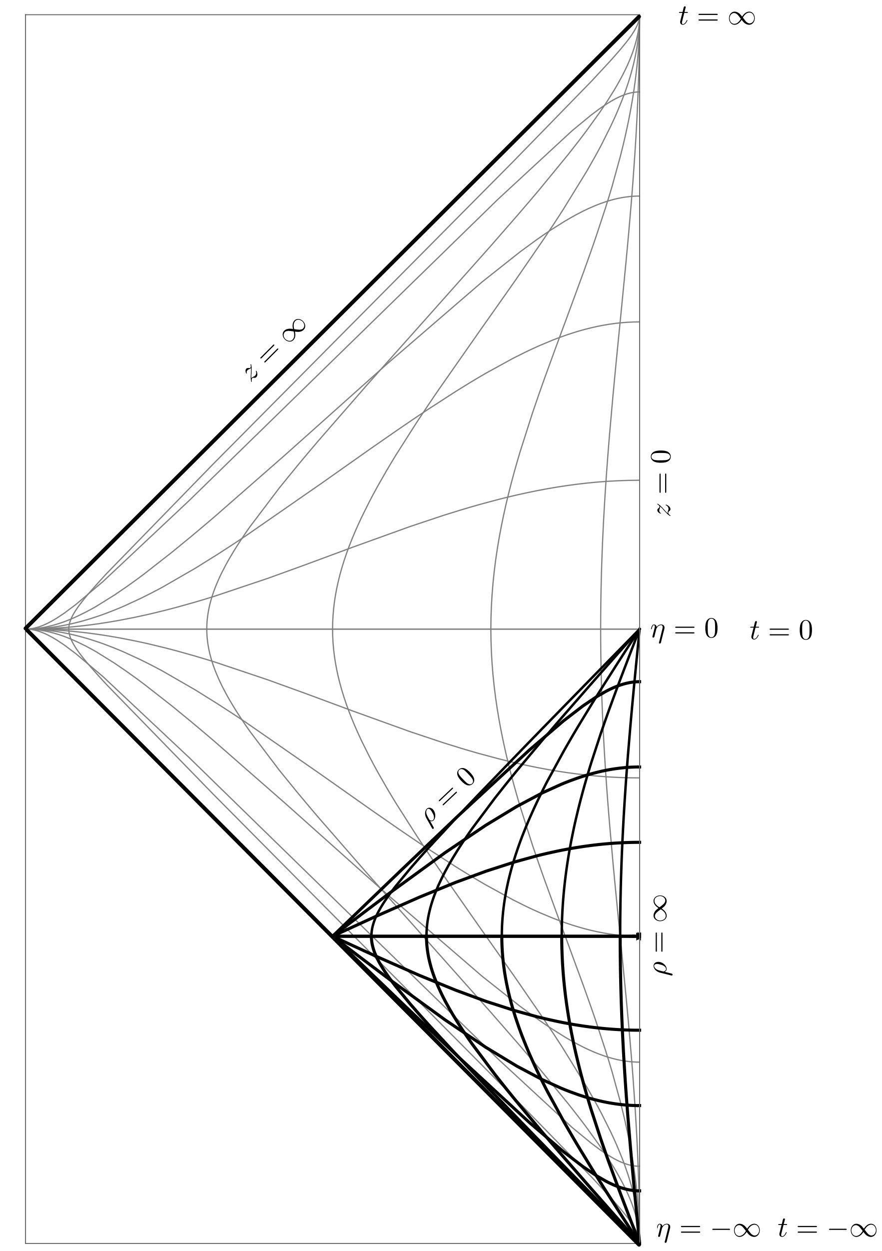
$$ds^2 = d\rho^2 + \sinh^2\rho\left[\frac{-d\eta^2 + \delta_{ij}dx^i dx^j}{\eta^2}\right]$$

Which is AdS_5 in the dS_4 slicing

Penrose Diagrams

Global AdS cylinder; dS slice coordinate region is bounded from above by lightcone running downward from t=0, and from below by slanted ellipse, also marking lower boundary of Poincaré patch (upper slanted ellipse is upper boundary of Poincaré patch).





2d slice down axis of AdS cylinder; thin lines are Poincaré lines of constant z and t, thick lines de Sitter slice lines of constant ρ and η

Check

Killing vectors for AdS_5 in the dS_4 slicing

$$
\begin{aligned}
p_i &= -\partial_i \\
j_{ij} &= x_i \partial_j - x_j \partial_i , \\
k_i &= -\vec{x}^2 \partial_i + 2 x_i x^j \partial_j + \eta^2 \partial_i + 2 x_i \eta \partial_\eta \\
d &= x^i \partial_i + \eta \partial_\eta \\
K_+ &= \frac{1}{\eta} \partial_\rho + \coth \rho \partial_\eta \\
K_- &= \frac{1}{\eta} (-\eta^2 + \vec{x}^2) \partial_\rho + (\eta^2 + \vec{x}^2) \coth \rho \partial_\eta + 2\eta \coth \rho x^i \partial_i \\
K_i &= \frac{1}{\eta} x_i \partial_\rho + x_i \coth \rho \partial_\eta + \eta \coth \rho \partial_i
\end{aligned}
$$

p_i , j_{ij} , k_i , d generate $\mathfrak{so}(1,4)$

Ansatz preserves dS isometries $\quad \phi = \phi(\rho) \implies p_i \phi = j_{ij} \phi = k_i \phi = d\phi = 0$

Including Back-Reaction

More generally, a metric consistent with the unbroken symmetries is

$$ds^2 = d\rho^2 + e^{2A(\rho)} ds^2_{dS_4}, \qquad \phi = \phi(\rho)$$

Where asymptotic AdS requires $A(\rho) \to \log \sinh \rho$ as $\rho \to \infty$

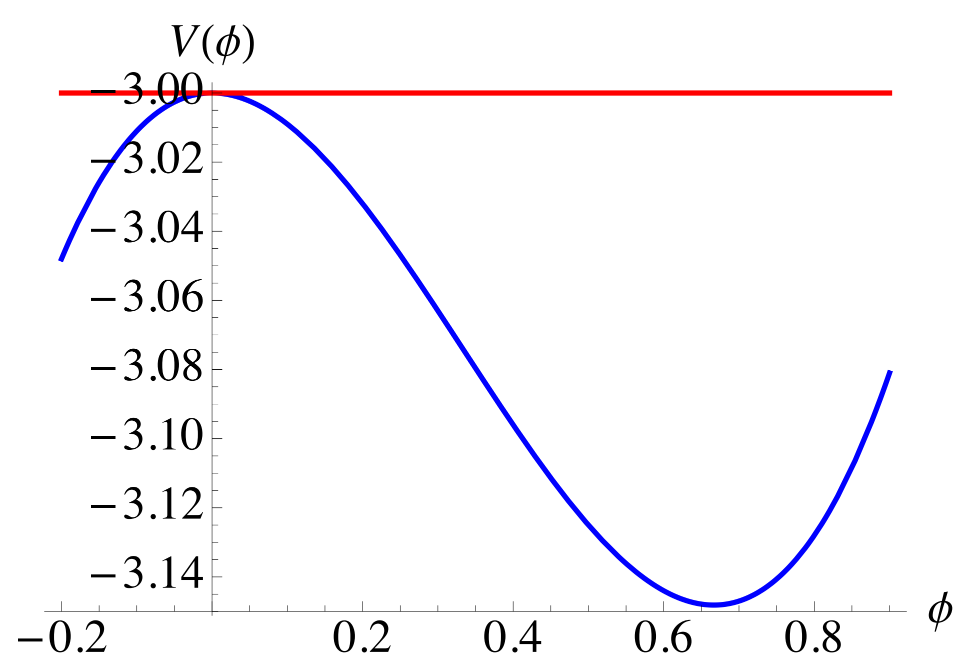
We want a domain wall with dS_4 leaves in asymptotic AdS_5

Action is

$$S = \int d^5x \sqrt{-g} \left[\frac{1}{4} R - \frac{1}{2} g^{MN} \partial_M \phi \partial_N \phi - V(\phi) \right] + \frac{1}{2} \int d^4x \sqrt{h} K$$

Give scalar potential a negative extremum at the origin

$$V = -3 + \frac{1}{2} m^2 \phi^2 + \mathcal{O}(\phi^3)$$



Background fields then obey

$$\phi''(\rho) + 4A'(\rho)\phi'(\rho) = \frac{\partial V}{\partial \phi}$$

$$3A'(\rho)^2 - 3e^{-2A(\rho)} = \frac{1}{2}\phi'(\rho)^2 - V$$

Simplest thing we could do is merely to choose V=-3. EOMs become

$$\phi'(\rho) = ce^{-4A(\rho)}, \qquad A'(\rho)^2 = 1 + e^{-2A(\rho)} + be^{-8A(\rho)}$$

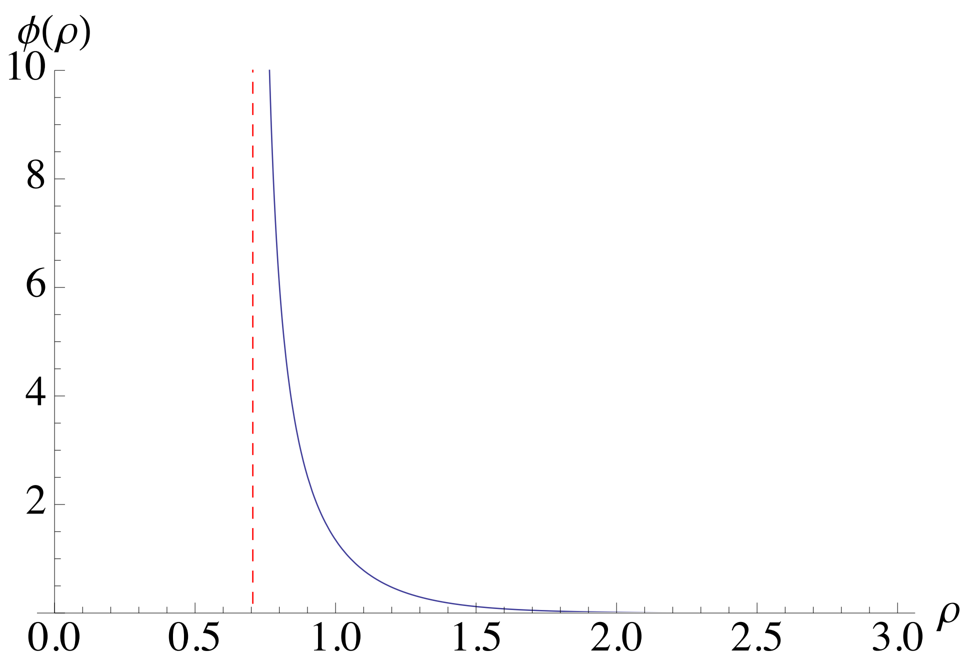
Metric can be written as

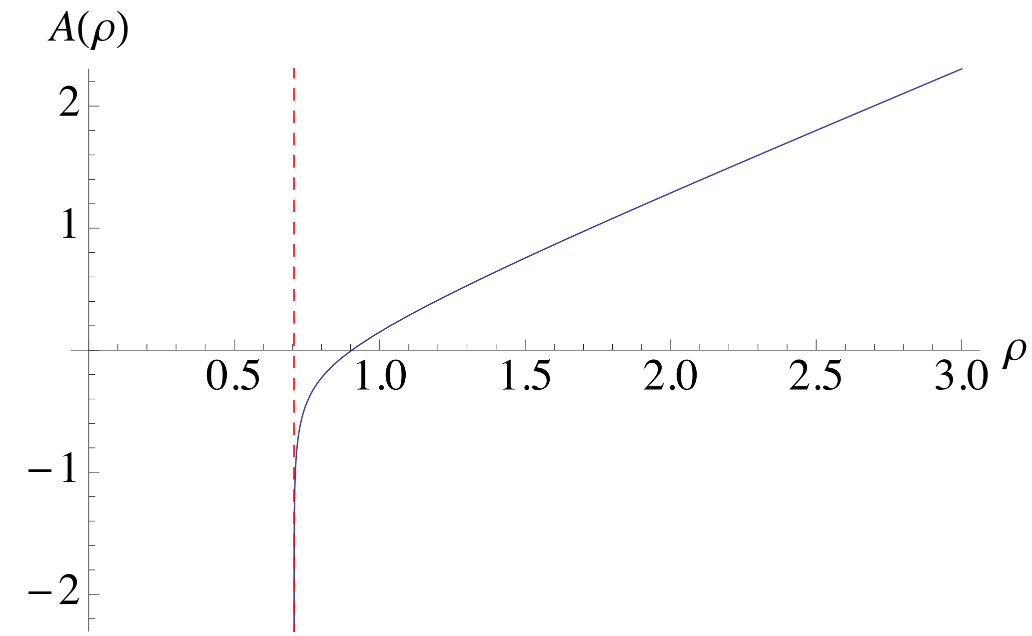
$$b = c^2/6$$

$$ds^2 = \frac{du^2}{4u^2(1+u+bu^4)} + \frac{1}{u}\left[\frac{-d\eta^2 + d\vec{x}^2}{\eta^2}\right] \qquad u = e^{-2A(\rho)}$$

Asymptotically:

$$\phi(\rho) \simeq \alpha + \beta e^{-4\rho}$$





1-Point Functions etc.

The holographic dictionary

- There exists a 1-1 correspondence: local gauge invariant operators of boundary QFT bulk modes.
  - Bulk metric corresponds to energy momentum tensor of boundary theory.
  - Bulk scalar fields correspond to scalar operators in boundary theory.
  - Can obtain correlation functions of gauge invariant operators from asymptotics of bulk solutions.

Want to determine VEVs of the stress energy tensor and the operator dual to the scalar field.

Correlation functions of gauge invariant operators can be extracted from asymptotics of bulk solutions.

Or, given correlation functions of dual operators, can reconstruct asymptotic solutions.

To perform holographic computations need some knowledge of structure of asymptotic solutions of field equations.

Fefferman-Graham Expansion

Any asymptotically AdS space admits expansion of form

$$ds^2 = \frac{dz^2 + g_{\mu\nu}(x,z)dx^\mu dx^\nu}{z^2}$$

$$g_{\mu\nu}(x,z) = g_{(0)\mu\nu}(x) + z^2 g_{(2)\mu\nu}(x) + z^4 g_{(4)\mu\nu}(x) + h_{(4)\mu\nu}(x) z^4 \log z^2 + \cdots .$$

$$\phi(x,z) = \phi_{(0)}(x) + z^4 \phi_{(4)}(x) + \cdots \qquad \text{(massless scalar)}$$

The GKPW prescription is

$$\left\langle e^{-\int d^4x\, \mathcal{O}(x)\phi_0(x)} \right\rangle = e^{-I_{\text{string}}[\phi_0]}\Big|_{\text{on-shell}}$$

Dual operator marginal

$$\Delta = 2 + \sqrt{4+m^2} \qquad \text{m=0} \qquad \Delta = 4$$

Holographic renormalization (most of the paper) gives 1-point functions

$$\langle \mathcal{O} \rangle = (4-2\Delta)\phi_{(4)}, \qquad \langle T_{\mu\nu} \rangle = g_{(4)\mu\nu}$$

How Does This Work?

Can put our metric
$$ds^2 = \frac{du^2}{4u^2(1+u+bu^4)} + \frac{1}{u}\left[\frac{-d\eta^2 + d\vec{x}^2}{\eta^2}\right]$$

in FG form. e.g. for b non-zero define

$$\sqrt{1+u+bu^4} = \frac{t}{\sqrt{t^2-z^2}} + b f_1(x) + \mathcal{O}(b^2) \qquad \eta = \sqrt{t^2-z^2} + bz f_2(x) + \mathcal{O}(b^2)$$

with
$$f_1(x) = \frac{xC_1}{(1-x^2)^{3/2}} + \frac{2+9x^2-6x^4-6x^2\left(-1+x^2\right)^2 \log\left[-1+\frac{1}{x^2}\right]}{8x\left(-1+x^2\right)^{7/2}}$$
$$f_2(x) = \frac{1}{4}\sqrt{-1+x^2}\left(\frac{17-42x^2+24x^4}{6\left(-1+x^2\right)^3} - \frac{4iC_1}{-1+x^2} + 4C_2 + 4\log\left[1-\frac{1}{x^2}\right] + \frac{3\log\left[-1+\frac{1}{x^2}\right]}{-1+x^2}\right)$$

Then transformed metric is of FG form to leading order in b

$$ds^2 = \frac{dz^2\left[1+\mathcal{O}(b^2)\right] + \left[-1+bc_3(x)+\mathcal{O}(b)^2\right]dt^2 + \left[1+bc_4(x)+\mathcal{O}(b)^2\right]d\vec{x}^2}{z^2}$$

FG Form Again

Form is fixed by unbroken O(1,4) symmetry

$$g_{tt}(z/t) = -1 + \frac{b}{8}\left(\frac{z}{t}\right)^8 + \mathcal{O}\left((z/t)^{10}\right) \qquad g_{11}(z/t) = 1 - \frac{b}{8}\left(\frac{z}{t}\right)^8 + \mathcal{O}\left((z/t)^{10}\right)$$

$$\langle T_{\mu\nu}\rangle = 0$$

Scalar equation is solved in FG coordinates by

$$\phi = \phi_{(0)} - \frac{c}{4}\left(\frac{z}{t}\right)^4 + \mathcal{O}\left((z/t)^6\right)$$

Setting $\Delta = 4$ yields

$$\langle \mathcal{O}\rangle = -4\phi_{(4)} = \frac{c}{t^4}$$

which is correct form required by O(1,4) symmetry: $\langle \mathcal{O}_I\rangle = \frac{c_I}{t^{\Delta_I}}$

[N.B. similar to interface CFT's which break $O(2,4) \to O(2,3)$ using $\langle \mathcal{O}_I\rangle = \frac{c_I}{y^{\Delta_I}}$]

Conclusions & Comments

- Have shown how to get a gravitational dual to one suggested alternative to inflation.
- Should also be possible to calculate other correlators (doing that now)

$$\langle \mathcal{O}(x)\mathcal{O}(y)\rangle \qquad \langle T_{\mu\nu}(x)\mathcal{O}(y)\rangle \qquad \langle T_{\mu\nu}(x)T_{\rho\sigma}(y)\rangle$$

- Ward identities (Justin's talk) have been computed, should be able to check them.
- Admits a supergravity embedding.
- Could consider holographic dual to turning on 4D gravity in boundary theory.
  - Dynamical gravity could be incorporated in boundary by moving boundary slightly into bulk, to a cut-off surface.
  - Local counter-terms now regarded as finite kinetic terms for graviton.
- Was just an example of how dualities extend beyond the usual application to inflation. A hope is that they might provide a route to understanding other possible early universe models.

Thank you for listening…

… and a big thank you to our organizers

See poster by James Stokes at COSMO-2014