String Inflation and the Planck Distance

Daniel Baumann

Department of Physics
Princeton University

with Liam McAllister
Based on

- D.B., Liam McAllister,
  A Microscopic Limit on Gravitational Waves from D-brane Inflation
Outline

Introduction

Tensors and the Planck Distance

Warped D-Brane Inflation

The Field Range Bound

Implications for Relativistic DBI Inflation

Conclusions
Non-Gaussianities in DBI Inflation
Outline

Introduction

Tensors and the Planck Distance

Warped D-Brane Inflation

The Field Range Bound

Implications for Relativistic DBI Inflation

Conclusions
Quantum Fluctuations

tensors \((\delta g_{ij})\)

\[ P_t \propto \frac{H^2}{M_P^2} \]

tensor-to-scalar ratio

\[ r \equiv \frac{P_t}{P_s} = 8 \left( \frac{d\phi}{dN_e} \frac{1}{M_P} \right)^2 \]

where

\[ dN_e \equiv H dt \]

scalars \((\delta \rho)\)

\[ P_s \propto H^2 \left( \frac{H}{\dot{\phi}} \right)^2 \]

\(\delta \phi \times (\delta \phi \rightarrow \delta \alpha)\)
The Lyth Bound

\[
r = 8 \left( \frac{d\phi}{dN_e} \frac{1}{M_P} \right)^2
\]

\[
\frac{\Delta \phi}{M_P} = \int dN_e \sqrt{\frac{1}{8} r(N_e)}
\]

\[
\equiv \sqrt{\frac{1}{8} r_*} N_{\text{eff}}
\]

\[r_* = r\] on observable scales.

\[
N_{\text{eff}} \equiv \int_{\phi_{\text{end}}}^{\phi_*} dN_e \sqrt{\frac{r(N_e)}{r_*}} \geq 30.
\]

\[
r_* = \frac{8}{(N_{\text{eff}})^2} \left( \frac{\Delta \phi}{M_P} \right)^2
\]
The Lyth Bound

\[ r_\star = \frac{8}{(N_{\text{eff}})^2} \left( \frac{\Delta \phi}{M_P} \right)^2 \]

Current observations imply \( N_{\text{eff}} \geq 30 \) and hence,

\[ \frac{r_\star}{0.01} < \frac{8}{9} \left( \frac{\Delta \phi}{M_P} \right)^2 \]
Useful constraint if we can compute $\frac{\Delta \phi}{M_P}$. 
Insensitive to the shape of the potential.
Observable tensors ($r_\star > 0.01$) require $\Delta \phi > M_P$:
Is this possible in string theory?
Tensors a **probe of fundamental physics** beyond the energy scale of inflation!
Relates abstract string theory calculations to cosmology.
Outline

Introduction

Tensors and the Planck Distance

Warped D-Brane Inflation

The Field Range Bound

Implications for Relativistic DBI Inflation

Conclusions
Inflation in String Theory

- Inflationary paradigm is highly successful, but no compelling microscopic theory.
- String theory provides:
  - considerable **UV control**
  - fundamental scalar fields
  - new perspectives on naturalness
- Dream:
  Use observations of the early universe to constrain string theory
  - given the energy scales, quite possibly a better test than TeV experiments.
D-Brane Inflation

Dvali and Tye

3+1

extra dimension

r

D3

D3
D-Brane Inflation

Dvali and Tye

\[ \text{extra dimension} \]

\[ \text{D3} \]

\[ \text{D3} \]

3+1
D-Brane Inflation

Dvali and Tye

3+1 extra dimension

D3 D3

r

extra dimension

r
D-Brane Inflation

Dvali and Tye

\[ \text{D3} \quad \text{D3} \]

3+1 extra dimension

\[ r \]

extra dimension
KKLMMT Scenario

 bulk CY
warped throat
 bulk CY
Outline

Introduction

Tensors and the Planck Distance

Warped D-Brane Inflation

The Field Range Bound

Implications for Relativistic DBI Inflation

Conclusions
Conifold Geometry

Warped cone over $X_5$

\[ ds^2 = h^{-1/2}(r) g_{\mu\nu} dx^\mu dx^\nu + h^{1/2}(r) (dr^2 + r^2 ds_{X_5}) \]

\[ h^{-1} = \left(\frac{r}{R}\right)^4, \quad T_3 R^4 = \frac{\pi}{2} \frac{N}{\text{Vol}(X_5)}. \]
**Dimensional Reduction**

\[
ds^2 = G_{AB} dX^A dX^B = h^{-1/2}(y) g_{\mu\nu} dx^\mu dx^\nu + h^{1/2}(y) g_{ij} dy^i dy^j
\]

10d Gravity

\[
S^{EH}_{10} = \frac{1}{2\kappa^2_{10}} \int d^{10}X \sqrt{G} \mathcal{R}(G)
\]

where

\[
\frac{1}{\kappa^2_{10}} = \frac{1}{\pi} (T_3)^2, \quad T_3 = \frac{1}{(2\pi)^3 g_s(\alpha')^2}.
\]

4d Gravity

\[
S^{EH}_{10} = \frac{1}{2\kappa^2_{10}} \times V^w_6 \times \int d^4x \sqrt{g} \mathcal{R}(g) + \cdots
\]

\[
= \frac{1}{2 M^2_P} \sqrt{g} \mathcal{R}(g) + \cdots
\]
Compactification Volume and 4d Planck Mass

Dimensional Reduction

$$M_F^2 = \frac{1}{\pi} (T_3)^2 V_6$$

Get conservative constraint by ignoring the bulk volume and computing the warped throat volume:

$$V_{throat} = \frac{1}{2} Vol(X_5) R^4 r_{max}^2 M_P^2 > T_3 r_{max}^2 N^4$$
Compactification Volume and 4d Planck Mass

Dimensional Reduction

$$M_P^2 = \frac{1}{\pi} (T_3)^2 V_6^w$$

$$V_6^w \equiv \int dy^6 \sqrt{g} h(y) = (V_6^w)_{\text{throat}} + (V_6^w)_{\text{bulk}}$$

Get conservative constraint by ignoring the bulk volume and computing the warped throat volume

$$(V_6^w)_{\text{throat}} = \frac{1}{2} \text{Vol}(X_5) R^4 r_{\text{max}}^2$$
Compactification Volume and 4d Planck Mass

Dimensional Reduction

\[ M_P^2 = \frac{1}{\pi} (T_3)^2 V_6^w \]

\[ V_6^w \equiv \int dy^6 \sqrt{g} h(y) = (V_6^w)_{\text{throat}} + (V_6^w)_{\text{bulk}} \]

\[ > (V_6^w)_{\text{throat}} \]

Get \textit{conservative constraint} by ignoring the bulk volume and computing the warped throat volume

\[ (V_6^w)_{\text{throat}} = \frac{1}{2} \text{Vol}(X_5) R^4 R_{\text{max}}^2 \]

\[ M_P^2 > T_3 r_{\text{max}}^2 \frac{N}{4} \]
4d-Planck Mass

\[ M_P^2 > T_3 r_{\text{max}}^2 \frac{N}{4} \]

\[ \frac{\Delta \phi}{M_P} < \frac{2}{\sqrt{N}} \]

Canonical Inflaton Field

\[ \phi^2 = T_3 r^2 < T_3 r_{\text{max}}^2 \]
Primordial Tensors?

Theoretical consistency requires:

\[ \frac{\Delta \phi}{M_P} < \frac{2}{\sqrt{N}} \]

\[ N \gg 1 \]

i.e.

\[ \Delta \phi \ll M_P \]

NO observable tensors from brane inflation!
Outline

Introduction

Tensors and the Planck Distance

Warped D-Brane Inflation

The Field Range Bound

Implications for Relativistic DBI Inflation

Conclusions
Case Study: Quadratic DBI

\[ V(\phi) = \frac{1}{2} m^2 \phi^2, \quad f_{AdS}^{-1}(\phi) \equiv T_3 h_{AdS}^{-1}(\phi) = \frac{2}{\pi} \frac{\text{Vol}(X_5)}{N} \phi^4. \]

implies:

\[ r_\star f_{NL} \sim M_P^2 \left( \frac{H'}{H} \right)^2 \sim \left( \frac{M_P}{\phi_\star} \right)^2 > \frac{N}{4} \]

**LOWER limit on** \( f_{NL} \):

\[ f_{NL} > \frac{N}{4r_\star} > 1 \]
Case Study: Quadratic DBI

\[ f_{AdS}^{-1}(\phi) = \frac{2}{\pi} \frac{\text{Vol}(X_5)}{N} \phi^4, \quad r_* f_{NL} \sim \left( \frac{M_P}{\phi} \right)^2 > \frac{N}{4} \]

- Field Range Bound + Non-Gaussianity Limits

\[ N < \frac{27}{70} r_* f_{NL} < 38 \]
Case Study: Quadratic DBI

\[ f_{AdS}(\phi) = \frac{2}{\pi} \frac{\text{Vol}(X_5)}{N} \phi^4, \quad r_* f_{NL} \sim \left( \frac{M_P}{\phi} \right)^2 > \frac{N}{4} \]

- Field Range Bound + Non-Gaussianity Limits

\[ N < \frac{27}{70} r_* f_{NL} < 38 \]

- Scalar Amplitude \((P_s)\) + Non-Gaussianity Limits

\[ N = \left( \frac{32}{3\pi} \right)^3 \frac{3\text{Vol}(X_5)}{(r_* f_{NL})^2 P_s^*} \quad > \quad 10^8 \]
Generalized Field Range Bound:
Lidsey and Huston  arXiv:0705.0240 [hep-th]

Dimensional Reduction

\[ M_P^2 > \frac{\text{Vol}(X_5)}{\pi} \int_{\phi_{\text{IR}}}^{\phi_{\text{UV}}} d\phi \phi^5 f(\phi) \]

\[ \int_{\phi_{\text{UV}}}^{\phi_{\text{IR}}} d\phi \phi^5 f(\phi) > \Delta \phi^* \phi^5 f^* > (\Delta \phi^*)^6 f^* \]

\[ \left( \frac{\Delta \phi^*}{M_P} \right)^6 < \frac{\pi}{\text{Vol}(X_5)} \left( f^* M_P^4 \right)^{-1} \]

\[ (f^* M_P^4)^{-1} = \frac{\pi^2}{16} P^*_s r^*_s \left( 1 + \frac{1}{3f_{NL}} \right) \]

\[ \left( \frac{\Delta \phi^*}{M_P} \right)^6 < \frac{\pi^3}{16\text{Vol}(X_5)} P^*_s r^*_s \left( 1 + \frac{1}{3f_{NL}} \right) \]
Tensor Bound

- **Lyth**
  \[
  \left( \frac{\Delta \phi_*}{M_P} \right)^2 \approx \frac{r_*}{8} (\Delta N_*)^2
  \]

- **Field Range Bound for Relativistic DBI Models**
  \[
  \left( \frac{\Delta \phi_*}{M_P} \right)^6 < \frac{\pi^3}{16 \text{Vol}(X_5)} P^*_s r_*^2
  \]

- **Upper limit for Relativistic DBI Models**
  \[
  r_* < \frac{32 P^*_s}{(\Delta N_*)^6 \text{Vol}(X_5)} \frac{\pi^3}{10^{-8}}
  \]
The upper limit on tensors

\[ r_\star < 10^{-8} \]

is in tension with a lower limit on tensors for relativistic DBI models with \( n_s < 1 \) (see Lidsey and Huston)

\[ r_\star > \frac{4(1 - n_s)}{\sqrt{3} f_{NL}}. \]
Outline

Introduction

Tensors and the Planck Distance

Warped D-Brane Inflation

The Field Range Bound

Implications for Relativistic DBI Inflation

Conclusions
Are detectable primordial tensors possible in a consistent microscopic theory?

Can we use (non)observation of tensors to constrain string theory?
Models which obey the field range bound are of two types:

1. Slow roll models with $f_{NL} \ll 1$
   (But see "A Delicate Universe").

2. Relativistic DBI models with $f_{NL} > 1$, but $n_s > 1$. 
Thank you for your attention

and thanks to my collaborators:

Liam McAllister.

Anatoly Dymarsky, Igor Klebanov, Juan Maldacena, Arvind Murugan and Paul Steinhardt.

Hiranya Peiris, Asantha Cooray and Brett Friedman.