String Inflation and the Planck Distance

Daniel Baumann

Department of Physics
Princeton University

with Liam McAllister

Based on

- D.B., Liam McAllister,
  A Microscopic Limit on Gravitational Waves from D-brane Inflation

Outline

Outline

Non-Gaussianities in DBI Inflation

Outline

Quantum Fluctuations

tensors (δg_{ij})

$$P_t \propto \frac{H^2}{M_P^2}$$

scalars $(\delta\rho)$

$$P_s \propto H^2 \left(\frac{H}{\dot{\phi}}\right)^2$$

$\delta\phi \times (\delta\phi \to \delta a)$

tensor-to-scalar ratio

$$r \equiv \frac{P_t}{P_s} = 8 \left(\frac{d\phi}{dN_e}\frac{1}{M_P}\right)^2$$

where

$$dN_e \equiv H dt$$

The Lyth Bound

$$r = 8\left(\frac{d\phi}{dN_e}\frac{1}{M_P}\right)^2$$

$$\frac{\Delta\phi}{M_P} = \int dN_e \sqrt{\frac{1}{8} r(N_e)}$$
$$\equiv \sqrt{\frac{1}{8} r_\star}\; N_{\rm eff}$$

$r_\star = r$ on observable scales.

$N_{\rm eff} \equiv \int_{\phi_{\rm end}}^{\phi_\star} dN_e \sqrt{\frac{r(N_e)}{r_\star}} \geq 30$.

$$r_\star = \frac{8}{(N_{\rm eff})^2}\left(\frac{\Delta\phi}{M_P}\right)^2$$

The Lyth Bound

$$r_\star = \frac{8}{(N_{\rm eff})^2}\left(\frac{\Delta\phi}{M_P}\right)^2$$

Current observations imply $N_{\rm eff} \geq 30$ and hence,

$$\frac{r_\star}{0.01} < \frac{8}{9}\left(\frac{\Delta\phi}{M_P}\right)^2$$

Comments

$$\frac{r_\star}{0.01} < \frac{8}{9}\left(\frac{\Delta\phi}{M_P}\right)^2$$

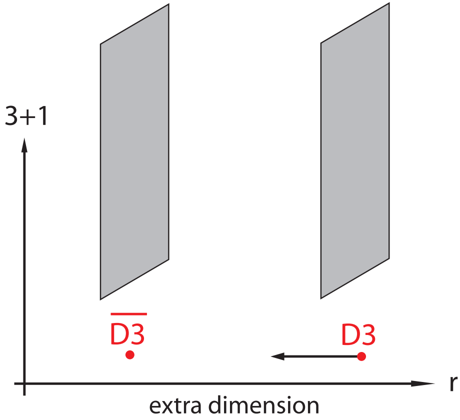
- Useful constraint if we can compute $\frac{\Delta\phi}{M_P}$.
- Insensitive to the shape of the potential.
- Observable tensors ($r_\star > 0.01$) require $\Delta\phi > M_P$:
  Is this possible in string theory?
- Tensors a probe of fundamental physics
  beyond the energy scale of inflation!
- Relates abstract string theory calculations
  to cosmology.

Outline

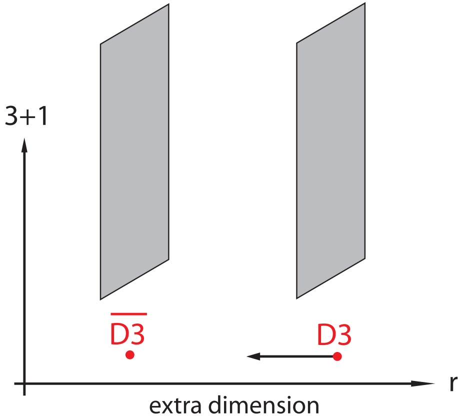
Inflation in String Theory

- Inflationary paradigm is highly successful, but no compelling microscopic theory.
- String theory provides:
  - considerable UV control
  - fundamental scalar fields
  - new perspectives on naturalness
- Dream:
  Use observations of the early universe to constrain string theory
  - given the energy scales, quite possibly a better test than TeV experiments.

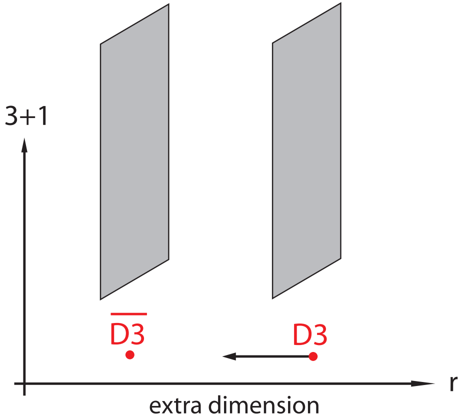
Dvali and Tye



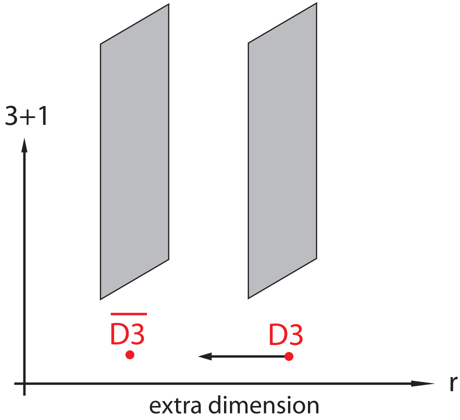
Dvali and Tye



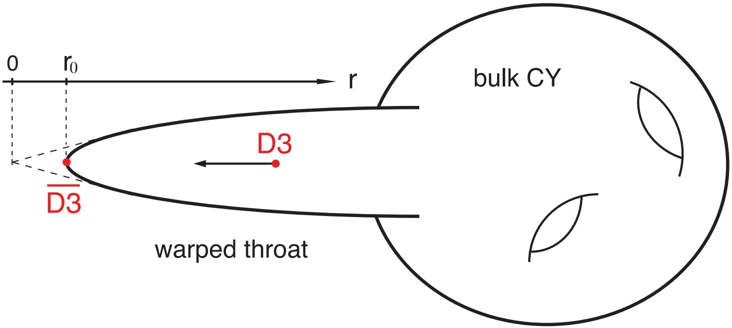
Dvali and Tye



Dvali and Tye



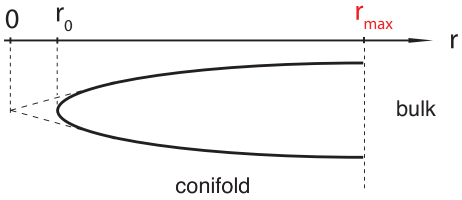
KKLMMT Scenario



Outline

Conifold Geometry

Warped cone over X_5



$$ds^2 = h^{-1/2}(r) g_{\mu\nu} dx^\mu dx^\nu + h^{1/2}(r)(dr^2 + r^2 ds_{X_5})$$

$$h^{-1} = \left(\frac{r}{R}\right)^4 , \qquad T_3 R^4 = \frac{\pi}{2} \frac{N}{\mathrm{Vol}(X_5)} .$$

Dimensional Reduction

$$ds^2 = G_{AB} dX^A dX^B = h^{-1/2}(y) g_{\mu\nu} dx^\mu dx^\nu + h^{1/2}(y) g_{ij} dy^i dy^j$$

10d Gravity

$$S_{10}^{EH} = \frac{1}{2\kappa_{10}^2} \int d^{10}X \sqrt{G}\, \mathcal{R}(G)$$

where

$$\frac{1}{\kappa_{10}^2} = \frac{1}{\pi}(T_3)^2 \,, \qquad T_3 = \frac{1}{(2\pi)^3 g_s (\alpha')^2} \,.$$

4d Gravity

$$S_{10}^{EH} = \underbrace{\frac{1}{2\kappa_{10}^2} \times V_6^w}_{\frac{1}{2}M_P^2} \times \int d^4x \sqrt{g}\, \mathcal{R}(g) + \cdots$$

Compactification Volume and 4d Planck Mass

Dimensional Reduction

$$M_P^2 = \frac{1}{\pi}(T_3)^2 V_6^w$$

Compactification Volume and 4d Planck Mass

Dimensional Reduction

$$M_P^2 = \frac{1}{\pi}(T_3)^2 V_6^w$$

$$\begin{aligned} V_6^w \equiv \int dy^6 \sqrt{g}\, h(y) =& (V_6^w)_{\rm throat} + (V_6^w)_{\rm bulk} \\ >& (V_6^w)_{\rm throat} \end{aligned}$$

Get conservative constraint by ignoring the bulk volume and computing the warped throat volume

$$(V_6^w)_{\rm throat} = \frac{1}{2}{\rm Vol}(X_5) R^4 r_{\rm max}^2$$

Compactification Volume and 4d Planck Mass

Dimensional Reduction

$$M_P^2 = \frac{1}{\pi}(T_3)^2 V_6^w$$

$$V_6^w \equiv \int dy^6 \sqrt{g}\, h(y) = (V_6^w)_{\rm throat} + (V_6^w)_{\rm bulk} > (V_6^w)_{\rm throat}$$

Get conservative constraint by ignoring the bulk volume and computing the warped throat volume

$$(V_6^w)_{\rm throat} = \frac{1}{2}{\rm Vol}(X_5) R^4 r_{\rm max}^2$$

$$M_P^2 > T_3 r_{\rm max}^2 \frac{N}{4}$$

Compactification Constraint on the Field Variation
D.B. and L. McAllister [hep-th:0610285]

4d-Planck Mass

$$M_P^2 > T_3 r_{\rm max}^2 \frac{N}{4}$$

$$+$$

Canonical Inflaton Field

$$\phi^2 = T_3 r^2 < T_3 r_{\rm max}^2$$

$$\frac{\Delta\phi}{M_P} < \frac{2}{\sqrt{N}}$$

Primordial Tensors?

$$\frac{\Delta\phi}{M_P} < \frac{2}{\sqrt{N}}$$

Theoretical consistency requires:

$$N \gg 1$$

i.e.

$$\Delta\phi \ll M_P$$

NO observable tensors from brane inflation!

Outline

Case Study: Quadratic DBI

$$V(\phi) = \frac{1}{2} m^2 \phi^2 \, , \qquad f_{AdS}^{-1}(\phi) \equiv T_3 h_{AdS}^{-1}(\phi) = \frac{2}{\pi} \frac{\mathrm{Vol}(X_5)}{N} \, \phi^4 \, .$$

implies:

$$r_\star f_{NL} \sim M_P^2 \left(\frac{H'}{H} \right)^2 \sim \left(\frac{M_P}{\phi_\star} \right)^2 > \frac{N}{4}$$

LOWER limit on f_{NL}:

$$f_{NL} > \frac{N}{4 r_\star} > 1$$

Case Study: Quadratic DBI

$$f_{AdS}^{-1}(\phi) = \frac{2}{\pi}\frac{\mathrm{Vol}(X_5)}{N}\,\phi^4\,, \qquad r_\star f_{NL} \sim \left(\frac{M_P}{\phi}\right)^2 > \frac{N}{4}$$

- Field Range Bound + Non-Gaussianity Limits

$$N < \frac{27}{70} r_\star f_{NL} < 38$$

Case Study: Quadratic DBI

$$f_{AdS}^{-1}(\phi) = \frac{2}{\pi}\frac{\mathrm{Vol}(X_5)}{N}\,\phi^4\,, \qquad r_\star f_{NL} \sim \left(\frac{M_P}{\phi}\right)^2 > \frac{N}{4}$$

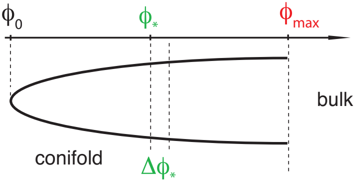
- Field Range Bound + Non-Gaussianity Limits

$$N < \frac{27}{70} r_\star f_{NL} < 38$$

- Scalar Amplitude (P_s) + Non-Gaussianity Limits

$$N = \left(\frac{32}{3\pi}\right)^3 \frac{3\mathrm{Vol}(X_5)}{(r_\star^2 f_{NL})^2 P_s^\star} > 10^8$$

Generalized Field Range Bound: Lidsey and Huston arXiv:0705.0240 [hep-th]



Dimensional Reduction

$$M_P^2 > \frac{\mathrm{Vol}(X_5)}{\pi} \int_{\phi_{\mathrm{IR}}}^{\phi_{\mathrm{UV}}} \mathrm{d}\phi \, \phi^5 f(\phi)$$

$$\int_{\phi_{\mathrm{IR}}}^{\phi_{\mathrm{UV}}} \mathrm{d}\phi \, \phi^5 f(\phi) > \Delta\phi_\star \, \phi_\star^5 f_\star > (\Delta\phi_\star)^6 f_\star$$

$$\left(\frac{\Delta\phi_\star}{M_P}\right)^6 < \frac{\pi}{\mathrm{Vol}(X_5)} (f_\star M_P^4)^{-1}$$

$$(f_\star M_P^4)^{-1} = \frac{\pi^2}{16} P_s^\star r_\star^2 \left(1 + \frac{1}{3 f_{NL}}\right)$$

$$\left(\frac{\Delta\phi_\star}{M_P}\right)^6 < \frac{\pi^3}{16 \mathrm{Vol}(X_5)} P_s^\star r_\star^2 \left(1 + \frac{1}{3 f_{NL}}\right)$$

Tensor Bound

- Lyth

$$\left(\frac{\Delta\phi_\star}{M_P}\right)^2 \approx \frac{r_\star}{8}(\Delta N_\star)^2$$

- Field Range Bound for Relativistic DBI Models

$$\left(\frac{\Delta\phi_\star}{M_P}\right)^6 < \frac{\pi^3}{16\mathrm{Vol}(X_5)} P_s^\star r_\star^2$$

- Upper limit for Relativistic DBI Models

$$r_\star < \frac{32 P_s^\star}{(\Delta N_\star)^6}\frac{\pi^3}{\mathrm{Vol}(X_5)} < 10^{-8}$$

Tension with the Data

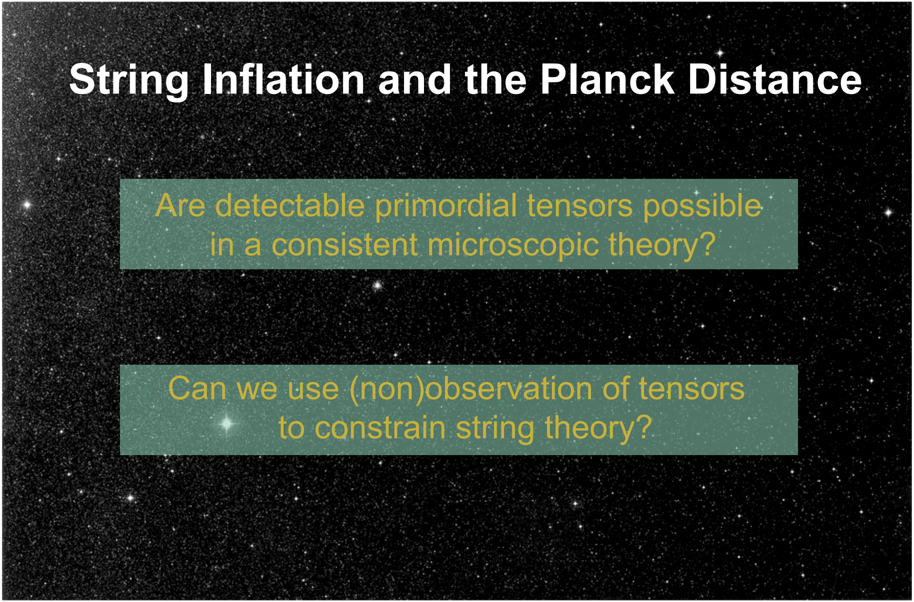
The upper limit on tensors

$$r_\star < 10^{-8}$$

is in tension with a *lower* limit on tensors for relativistic DBI models with $n_s < 1$ (see Lidsey and Huston)

$$r_\star > \frac{4(1-n_s)}{\sqrt{3 f_{NL}}} \,.$$

Outline


String Inflation and the Planck Distance
Are detectable primordial tensors possible in a consistent microscopic theory?
Can we use (non)observation of tensors to constrain string theory?

Models which obey the field range bound are of two types:

1. Slow roll models with $f_{NL} \ll 1$
   (But see "A Delicate Universe").

2. Relativistic DBI models with $f_{NL} > 1$, but $n_s > 1$.

Thank you for your attention

and thanks to my collaborators:

Liam McAllister.

Anatoly Dymarsky, Igor Klebanov, Juan Maldacena, Arvind Murugan and Paul Steinhardt.

Hiranya Peiris, Asantha Cooray and Brett Friedman.