Observational Signatures and Non-Gaussianities of General Single Field Inflation

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hep-th/0605045, with M-x.Huang, S.Kachru, G.Shiu;
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Inflation Models and Observations

- WMAP measurement on CMBR

Spectral index: $n_S = 0.951^{+0.015}_{-0.019}$

Running of spectral index: $dn_S/d\ln k = -0.102^{+0.050}_{-0.043}$

Tensor to scalar ratio: $r_{0.002} < 0.55$

Non-Gaussianity: $|f_{NL}| < 300$

- Inflation mechanisms and models

Slow-roll inflation --- using flat potential;

DBI inflation --- using speed-limit in warped space;

K-inflation --- inflation driven by kinetic energy.

Most General Non-Gaussianities in Single Field Theory

- Single field inflation:
  - Inflaton is responsible for density perturbations;
  - Lagrangian is arbitrary function of $g_{\mu\nu}\partial^\mu\phi\partial^\nu\phi$ and ϕ ;
  - Arbitrary sound speed c_s and λ (to be defined).
- Motivations
  - Null hypothesis on specific models;
    Fit or constrain parameters model-independently;
  - Several string models has distinctive predictions on non-Gaussianities;
  - Straightforward evaluation of non-Gaussianities for future models in this general class.

Outline

- Review of several classes of models
- General formalism
- General form of non-Gaussianities
- Using non-G to probe string theory

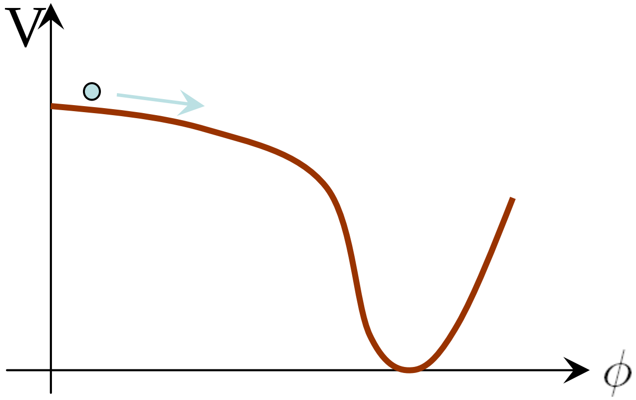
Review of Several Classes of Models

1. Slow-roll inflation (Linde 82; Albrecht & Steinhardt 82)

- Lagrangian

$$P(X,\phi) = X - V(\phi), \qquad X = -\frac{1}{2} g_{\mu\nu}\partial^\mu\phi\partial^\nu\phi$$

Inflaton rolling on a flat potential $V(\phi)$



Slow-roll parameters:

$$\epsilon_V = \frac{M_{pl}^2}{2}\left(\frac{V'}{V}\right)^2 \ll 1$$

$$\eta_V = M_{pl}^2 \frac{V''}{V} \ll 1$$

- Scalar and tensor power spectrum

$$P_k^\zeta = \frac{1}{12\pi^2 M_{pl}^6}\frac{V^3}{V'^2}\ , \qquad P_k^h = \frac{2V}{3\pi^2 M_{pl}^4}$$

- Spectral index

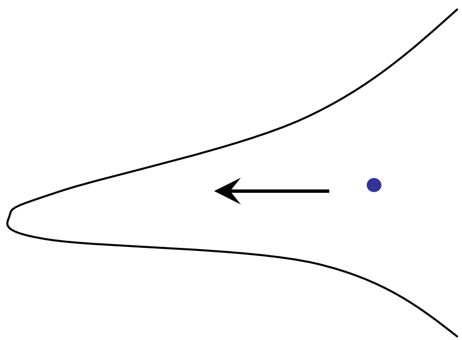
$$n_s - 1 = \frac{d\ln P_k^\zeta}{d\ln k} = M_{pl}^2\left(-3\frac{V'^2}{V^2} + 2\frac{V''}{V}\right)$$

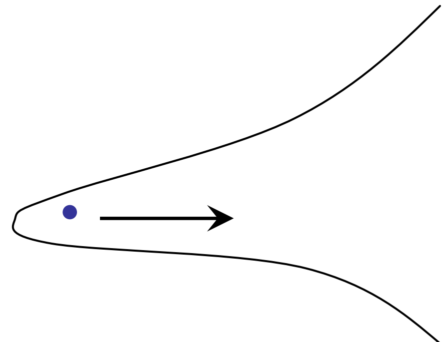
- ➢ dS inflation; Power-law inflation;
  Large field inflation; Small field inflation;

- ➢ String models:
  Branes; Tachyons; Axions; Radions.

2. DBI inflation (Silverstein, Tong & Alishahiha, 03,04; X.C. 04,05)

$$ds^2 \propto \frac{\phi^2}{\sqrt{\lambda}} ds_4^2 + \frac{\sqrt{\lambda}}{\phi^2} d\phi^2 \quad \Longrightarrow \quad |\dot{\phi}| \leq \frac{\phi^2}{\sqrt{\lambda}}$$





UV model (Silverstein, Tong, 03)

$$V(\phi) \approx \frac{1}{2} m^2 \phi^2$$

$$m \gg M_{\rm Pl}/\sqrt{\lambda}$$

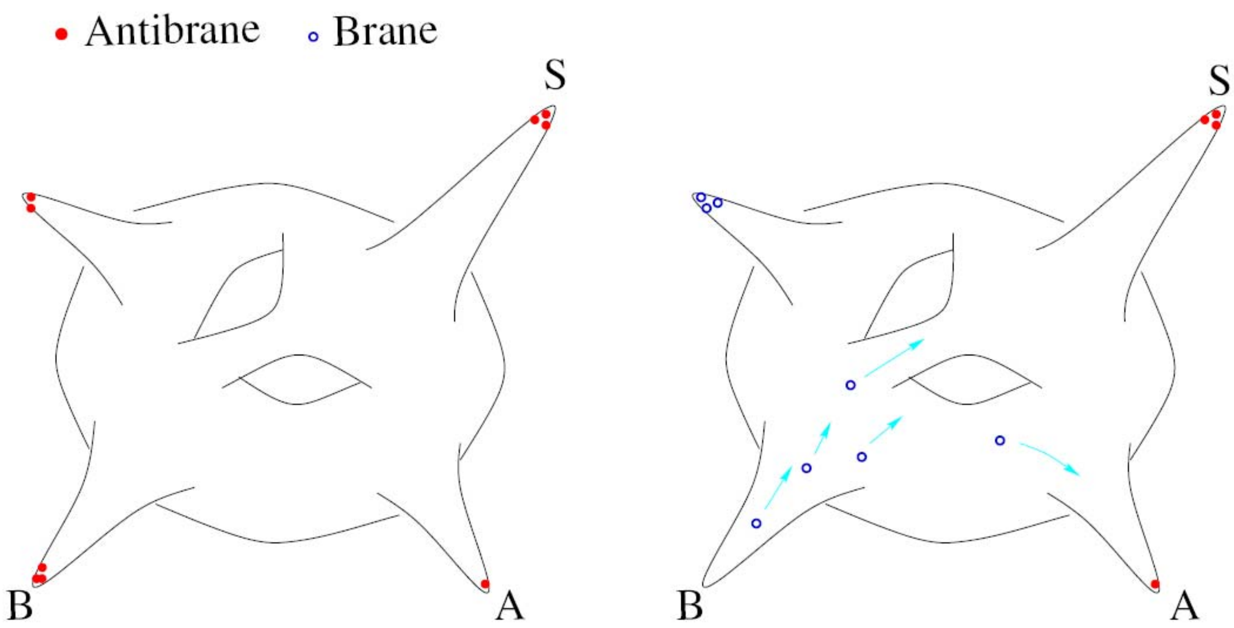
IR model (X.C. 04)

$$V(\phi) \approx V_0 - \frac{1}{2} m^2 \phi^2$$

$$m \sim H$$

- Lagrangian $\quad P = -f(\phi)^{-1}\sqrt{1 - 2Xf(\phi)} + f(\phi)^{-1} - V(\phi)$

- Multi-throat brane inflation (X.C. 04)



- ➢ Antibrane-flux annihilation (Kachru, Pearson, Verlinde, 01)
- ➢ Generate branes as candidate inflatons
- ➢ Exit B-throat, roll through bulk, settle down in another throat
- ➢ Enough warping: DBI inflation;
  Flat potential: slow-roll inflation.

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A General Formalism (Garriga & Mukhanov, 99)

- Lagrangian

$$S = \frac{1}{2}\int d^4x\sqrt{-g}\left[M_{pl}^2 R - 2P(X,\phi)\right]$$

$$\text{where} \quad X = -\frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi$$

- Metric

$$ds^2 = -dt^2 + a^2(t)dx_3^2$$

- Define parameters

$$c_s^2 = \frac{dP}{dE} = \frac{P_{,X}}{P_{,X} + 2XP_{,XX}} \quad \text{(sound speed)}$$

$$\lambda = X^2 P_{,XX} + \frac{2}{3}X^3 P_{,XXX}$$

$$\Sigma = XP_{,X} + 2X^2 P_{,XX} = \frac{H^2\epsilon}{c_s^2}$$

- Slow variation parameters

$$\epsilon = -\frac{\dot{H}}{H^2}\ , \quad \eta = \frac{\dot{\epsilon}}{\epsilon H}\ , \quad s = \frac{\dot{c}_s}{c_s H}\ , \quad l = \frac{\dot{\lambda}}{\lambda H}\ .$$

- More general than the usual slow-roll parameters
  - Flat potential v.s. steep potential (DBI) or no potential (k-inflation)
  - Non-relativistic slow-roll v.s. ultra-relativistic fast-roll

- Power spectrum

$$P_k^{\zeta} = \frac{1}{36\pi^2 M_{pl}^4}\frac{E^2}{c_s(P+E)} = \frac{1}{8\pi^2 M_{pl}^2}\frac{H^2}{c_s\epsilon} \qquad P_k^h = \frac{2}{3\pi^2}\frac{E}{M_{pl}^4}$$

- Spectral index

$$n_s - 1 = \frac{d\ln P_k^{\zeta}}{d\ln k} = -2\epsilon - \eta - s \qquad n_T = \frac{d\ln P_k^h}{d\ln k} = -2\epsilon$$

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ADM Formalism

(Maldacena, 02; Seery & Lidsey 05; X.C., Huang, Kachru & Shiu, 06)

- Metric

$$ds^2 = -N^2 dt^2 + h_{ij}(dx^i + N^i dt)(dx^j + N^j dt)$$

$$\text{Comoving gauge: } \delta\phi = 0 \quad , \quad h_{ij} = a^2 e^{2\zeta} \delta_{ij} \quad .$$

- N and N^i are Lagrangian multipliers

In order to get the cubic expansion of $\mathcal{L}$, it is enough to solve N and N^i to the first order, because

$$S \supset \int d^4x \text{ [zeroth order constraint eqn]} \cdot N^{(n)}$$
$$+ \int d^4x \text{[first order constraint eqn]} \cdot N^{(n-1)} + \cdots$$

The Cubic Part

- The exact cubic action for scalar perturbation ζ

$$S_3 = \int dt d^3x \{ -a^3 (\Sigma(1 - \frac{1}{c_s^2}) + 2\lambda) \frac{\dot{\zeta}^3}{H^3} + \frac{a^3\epsilon}{c_s^4}(\epsilon - 3 + 3c_s^2)\zeta\dot{\zeta}^2$$
$$+ \frac{a\epsilon}{c_s^2}(\epsilon - 2s + 1 - c_s^2)\zeta(\partial\zeta)^2 - 2a\frac{\epsilon}{c_s^2}\dot{\zeta}(\partial\zeta)(\partial\chi)$$
$$+ \frac{a^3\epsilon}{2c_s^2}\frac{d}{dt}(\frac{\eta}{c_s^2})\zeta^2\dot{\zeta} + \frac{\epsilon}{2a}(\partial\zeta)(\partial\chi)\partial^2\chi + \frac{\epsilon}{4a}(\partial^2\zeta)(\partial\chi)^2 + 2f(\zeta)\frac{\delta L}{\delta\zeta}|_1 \} ,$$

where

$$\partial^2\chi = a^2\frac{\epsilon}{c_s^2}\dot{\zeta} , \qquad \frac{\delta L}{\delta\zeta}|_1 = a\left(\frac{d\partial^2\chi}{dt} + H\partial^2\chi - \epsilon\partial^2\zeta\right) ,$$

$$f(\zeta) = \frac{\eta}{4c_s^2}\zeta^2 + \text{ (terms with at least one derivative on } \zeta) .$$

The 3-Point Function

$$\langle \zeta(t,\mathbf{k}_1)\zeta(t,\mathbf{k}_2)\zeta(t,\mathbf{k}_3)\rangle = -i\int_{t_0}^{t} dt' \langle [\zeta(t,\mathbf{k}_1)\zeta(t,\mathbf{k}_2)\zeta(t,\mathbf{k}_3), H_{int}(t')]\rangle \ ,$$

where H_{int} is the interaction Hamiltonian given by $S_3 = \int dt dx^3 L_3$,

$$H_{int} = -\int dx^3 L_3 \ .$$

- Define

$$\langle \zeta(\mathbf{k}_1)\zeta(\mathbf{k}_2)\zeta(\mathbf{k}_3)\rangle = (2\pi)^7 \delta^3(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3)(P_k^\zeta)^2 \frac{1}{\Pi_i k_i^3}\ \mathcal{A}$$

The Cubic Part

- The exact cubic action for scalar perturbation ζ

$$S_3 = \int dt d^3x \{-a^3(\Sigma(1-\frac{1}{c_s^2}) + 2\lambda)\frac{\dot{\zeta}^3}{H^3} + \frac{a^3\epsilon}{c_s^4}(\epsilon - 3 + 3c_s^2)\zeta\dot{\zeta}^2$$
$$+ \frac{a\epsilon}{c_s^2}(\epsilon - 2s + 1 - c_s^2)\zeta(\partial\zeta)^2 - 2a\frac{\epsilon}{c_s^2}\dot{\zeta}(\partial\zeta)(\partial\chi)$$
$$+ \frac{a^3\epsilon}{2c_s^2}\frac{d}{dt}(\frac{\eta}{c_s^2})\zeta^2\dot{\zeta} + \frac{\epsilon}{2a}(\partial\zeta)(\partial\chi)\partial^2\chi + \frac{\epsilon}{4a}(\partial^2\zeta)(\partial\chi)^2 + 2f(\zeta)\frac{\delta L}{\delta\zeta}|_1\} ,$$

where

$$\partial^2\chi = a^2\frac{\epsilon}{c_s^2}\dot{\zeta} , \qquad \frac{\delta L}{\delta\zeta}|_1 = a\left(\frac{d\partial^2\chi}{dt} + H\partial^2\chi - \epsilon\partial^2\zeta\right) ,$$

$$f(\zeta) = \frac{\eta}{4c_s^2}\zeta^2 + \text{(terms with at least one derivative on } \zeta) .$$

Various Terms

- $\left(\frac{1}{c_s^2}-1\right)\epsilon\,,\quad \frac{2\lambda}{\Sigma}\epsilon\;;$ $\Longrightarrow$ $\mathcal{A}\propto\left(\frac{1}{c_s^2}-1\right)\,,\quad \mathcal{A}\propto\frac{2\lambda}{\Sigma}\;;$
- $\frac{1}{c_s^2}\epsilon^2\,,\quad \frac{1}{c_s^2}\epsilon s\,,$ $\Longrightarrow$ $\mathcal{A}\propto\frac{\epsilon}{c_s^2}\,,\quad \mathcal{A}\propto\frac{s}{c_s^2}\;;$
- $2f(\zeta)\frac{\delta L}{\delta\zeta}\Big|_1$ $\Longrightarrow$ $\mathcal{A}\propto\frac{\eta}{c_s^2}\;;$

The last term can be absorbed by a redefinition:

$$\zeta\to\zeta_n+f(\zeta_n)\;,$$

which introduces an extra term in $\langle\zeta^3\rangle$:

$$\begin{aligned}\langle\zeta(\mathbf{x}_1)\zeta(\mathbf{x}_2)\zeta(\mathbf{x}_3)\rangle &= \langle\zeta_n(\mathbf{x}_1)\zeta_n(\mathbf{x}_2)\zeta_n(\mathbf{x}_3)\rangle\\ &+\frac{\eta}{2c_s^2}(\langle\zeta_n(\mathbf{x}_1)\zeta_n(\mathbf{x}_2)\rangle\langle\zeta_n(\mathbf{x}_1)\zeta_n(\mathbf{x}_3)\rangle+\mathrm{sym})+\mathcal{O}(\eta^2)\;.\end{aligned}$$

The Cubic Part

- The exact cubic action for scalar perturbation ζ

$$S_3 = \int dt d^3x \{ -a^3 (\Sigma(1 - \frac{1}{c_s^2}) + 2\lambda) \frac{\dot{\zeta}^3}{H^3} + \frac{a^3\epsilon}{c_s^4}(\epsilon - 3 + 3c_s^2)\zeta\dot{\zeta}^2$$
$$+ \frac{a\epsilon}{c_s^2}(\epsilon - 2s + 1 - c_s^2)\zeta(\partial\zeta)^2 - 2a\frac{\epsilon}{c_s^2}\dot{\zeta}(\partial\zeta)(\partial\chi)$$
$$+ \frac{a^3\epsilon}{2c_s^2}\frac{d}{dt}(\frac{\eta}{c_s^2})\zeta^2\dot{\zeta} + \frac{\epsilon}{2a}(\partial\zeta)(\partial\chi)\partial^2\chi + \frac{\epsilon}{4a}(\partial^2\zeta)(\partial\chi)^2 + 2f(\zeta)\frac{\delta L}{\delta\zeta}|_1 \} ,$$

where

$$\partial^2\chi = a^2\frac{\epsilon}{c_s^2}\dot{\zeta} , \qquad \frac{\delta L}{\delta\zeta}|_1 = a\left(\frac{d\partial^2\chi}{dt} + H\partial^2\chi - \epsilon\partial^2\zeta\right) ,$$

$$f(\zeta) = \frac{\eta}{4c_s^2}\zeta^2 + \text{(terms with at least one derivative on } \zeta) .$$

Various Terms

- $\left(\frac{1}{c_s^2}-1\right)\epsilon \ , \quad \frac{2\lambda}{\Sigma}\epsilon \ ;$ ⇒ $\mathcal{A} \propto \left(\frac{1}{c_s^2}-1\right) \ , \quad \mathcal{A} \propto \frac{2\lambda}{\Sigma} \ ;$
- $\frac{1}{c_s^2}\epsilon^2 \ , \quad \frac{1}{c_s^2}\epsilon s \ ,$ ⇒ $\mathcal{A} \propto \frac{\epsilon}{c_s^2} \ , \quad \mathcal{A} \propto \frac{s}{c_s^2} \ ;$
- $2f(\zeta)\frac{\delta L}{\delta \zeta}|_1$ ⇒ $\mathcal{A} \propto \frac{\eta}{c_s^2} \ ;$
- $\frac{\epsilon}{c_s^2}\frac{d}{dt}\left(\frac{\eta}{c_s^2}\right) \ , \quad \epsilon^3 \ .$

⇒ Negligible, unless there are sharp features.

(X.C., Easther & Lim, 06)

Final Results (X.C., Huang, Kachru, Shiu, 06)

- The 3-pt function for a general single field inflation to $\mathcal{O}(\epsilon)$:

$$\langle \zeta(\mathbf{k}_1)\zeta(\mathbf{k}_2)\zeta(\mathbf{k}_3)\rangle = (2\pi)^7\delta^3(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3)(\tilde{P}_K^\zeta)^2\frac{1}{\Pi_i k_i^3} \times (\mathcal{A}_\lambda+\mathcal{A}_c+\mathcal{A}_o+\mathcal{A}_\epsilon+\mathcal{A}_\eta+\mathcal{A}_s) ,$$

where we have decomposed the shape into six parts ($K \equiv k_1+k_2+k_3$)

$$\mathcal{A}_\lambda = \left(\frac{1}{c_s^2}-1-\frac{\lambda}{\Sigma}[2-(3-2\mathbf{c}_1)l]\right)_K \frac{3k_1^2k_2^2k_3^2}{2K^3} ,$$

$$\mathcal{A}_c = \left(\frac{1}{c_s^2}-1\right)_K \left(-\frac{1}{K}\sum_{i>j}k_i^2k_j^2+\frac{1}{2K^2}\sum_{i\neq j}k_i^2k_j^3+\frac{1}{8}\sum_i k_i^3\right) ,$$

$$\mathcal{A}_o = \left(\frac{1}{c_s^2}-1-\frac{2\lambda}{\Sigma}\right)_K (\epsilon F_{\lambda\epsilon}+\eta F_{\lambda\eta}+sF_{\lambda s}) + \left(\frac{1}{c_s^2}-1\right)_K (\epsilon F_{c\epsilon}+\eta F_{c\eta}+sF_{cs}) ,$$

$$\mathcal{A}_\epsilon = \epsilon\left(-\frac{1}{8}\sum_i k_i^3+\frac{1}{8}\sum_{i\neq j}k_ik_j^2+\frac{1}{K}\sum_{i>j}k_i^2k_j^2\right) ,$$

$$\mathcal{A}_\eta = \eta\left(\frac{1}{8}\sum_i k_i^3\right) ,$$

$$\mathcal{A}_s = sF_s .$$

- Completely specified by 5 parameters: c_s , $\frac{\lambda}{\Sigma}$, ϵ , η , s.

Size, Shape, and Running of Non-Gaussianities

- Size of non-Gaussianities

WMAP ansatz: $\langle\zeta(\mathbf{k}_1)\zeta(\mathbf{k}_2)\zeta(\mathbf{k}_3)\rangle = (2\pi)^7\delta^3(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3)(-\frac{3}{10}f_{NL}(P_k^\zeta)^2)\frac{\Sigma_i k_i^3}{\Pi_i k_i^3}$

Taking equilateral limit in our results:

$$f_{NL}^{\lambda} = -\frac{5}{81}\left(\frac{1}{c_s^2}-1-\frac{2\lambda}{\Sigma}\right)$$
$$f_{NL}^{c} = \frac{35}{108}\left(\frac{1}{c_s^2}-1\right) ,$$

$$f_{NL}^{o} = \mathcal{O}\left(\frac{\epsilon}{c_s^2} , \frac{\epsilon\lambda}{\Sigma}\right) , \quad f_{NL}^{\epsilon,\eta,s} = \mathcal{O}(\epsilon) .$$

Large non-Gaussianity → Small c_s or large λ/Σ

- Current bound: $|f_{NL}| < 300$ for the first two, $|f_{NL}| < 100$ for the rest.

  (WMAP team, 06; Creminelli, Nicolis, Senatore, Tegmark & Zaldarriaga,05)

- WMAP will eventually reach $|f_{NL}| \sim 40$; Planck $|f_{NL}| \sim 20$.

Slow-Roll Inflation

- $c_s \approx 1 \ \& \ \lambda/\Sigma \approx 0$

- In this limit, our formulae recover the slow-roll results of Maldacena, 02; Seery & Lidsey, 05.

- In slow-roll inflation, the non-Gaussianity is unobservable, $f_{NL} = \mathcal{O}(\epsilon)$.

DBI Inflation

- $\frac{2\lambda}{\Sigma} = \frac{1}{c_s^2} - 1$, so the leading order of $\mathcal{A}_\lambda$ vanishes.

  In this limit, our formulae recovers the DBI result:

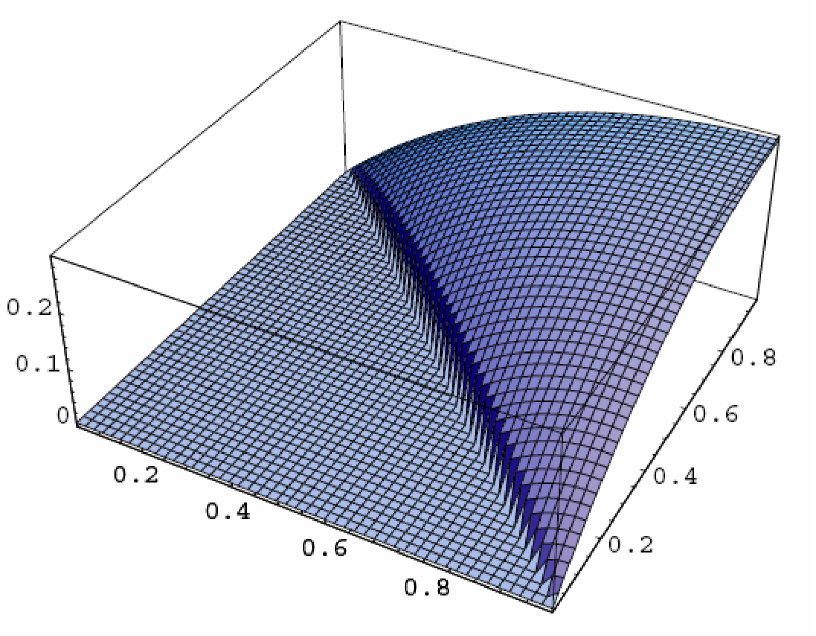
  (Alishahiha, Silverstein & Tong, 04)

$$\mathcal{A}_c = \left(\frac{1}{c_s^2} - 1\right)\left(-\frac{1}{K}\sum_{i>j} k_i^2 k_j^2 + \frac{1}{2K^2}\sum_{i\neq j} k_i^2 k_j^3 + \frac{1}{8}\sum_i k_i^3\right)$$

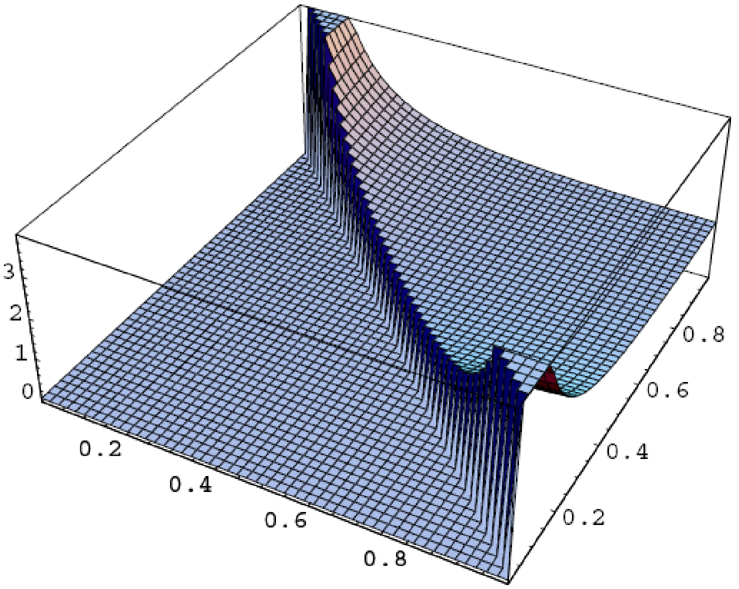
- Potentially observable $f_{NL} \approx 0.32 c_s^{-2}$.

Shape of Non-Gaussianities

$$\langle \zeta(\mathbf{k}_1)\zeta(\mathbf{k}_2)\zeta(\mathbf{k}_3)\rangle$$



DBI inflation



Slow-roll inflation

Babich, Creminelli, Zaldarriaga, 04; X.C., Huang, Kachru, Shiu, 06.

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Constrain String Models

- In the UV DBI model (Silverstein, Tong & Alishahiha, 03,04)

$$f_{NL} \approx 1.3 \frac{p^2 M_{\rm Pl}^4}{\phi^4} , \qquad p \gg 1$$

Viable only if $\phi \gtrsim M_{\rm Pl}$

- But ϕ is restricted by the size of the throat R, $\phi \ll M_{\rm Pl}$

→ Excessive non-Gaussianities

(X.C., 05; Baumann, Mcallister, 06; Bean, Shandera, Tye & Xu, 07)

- In the IR DBI model (X.C. 04,05)

$$f_{NL} \approx 0.036 \beta^2 N_{e\rm DBI}^2 \qquad \beta \sim 1$$

- ➢ Compatible with current observations
- ➢ Testable in future, $f_{NL} \sim 100$.

Probing Geometry in String Compactification

- Running of non-Gaussianity

$\Longrightarrow$ Shape of geometry in extra dimension (X.C. 05)

$$\text{Define:} \quad n_{NG} - 1 \equiv \frac{d \ln |f_{NL}|}{d \ln k} \approx -2s \quad .$$

$$\text{AdS geometry}: \quad n_{NG} - 1 < 0 \ ;$$
$$\text{constant warp factor}: \quad n_{NG} - 1 > 0 \ .$$

- Combining with the correlated feature in 2-pt function

(Shiu, Underwood, 06)

Stringy Physics in Hubble-induced Hagedorn Phase Transition

(X.C., 04, 05; Bean, X.C., Xu, in progress)

- Warped space ⇒ { Provides speed limit; Redshifts string scale } (Randall, Sundrum, 99)

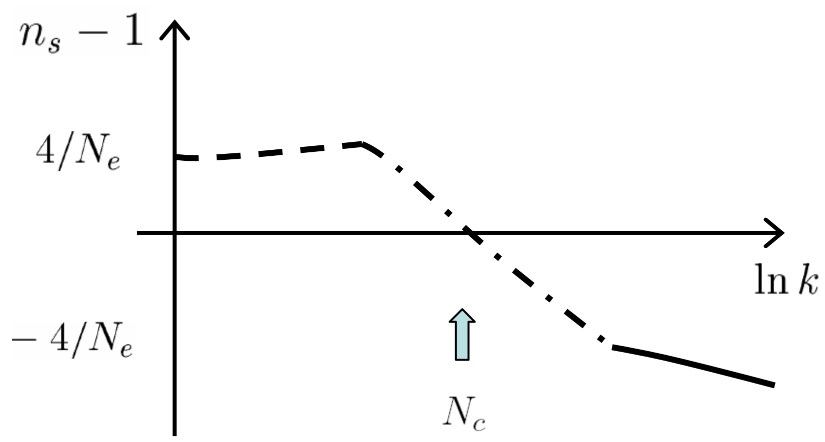
- In IR DBI inflation, at earlier times, i.e. larger scales,
  Hubble energy > redshifted string scale. (Phase transition)

  ⇒ Not only scalar fluctuations, but also stringy fluctuations.

- Happens at

$$N_c \sim \frac{\lambda_B^{1/8}}{\beta^{1/2} n_B^{1/8}} \sim \mathcal{O}(10)$$

$$n_s - 1 \approx -\frac{4}{N_e^{\rm DBI}} \longrightarrow \frac{4}{N_e^{\rm DBI}}$$



IR DBI Brane Inflation Predictions

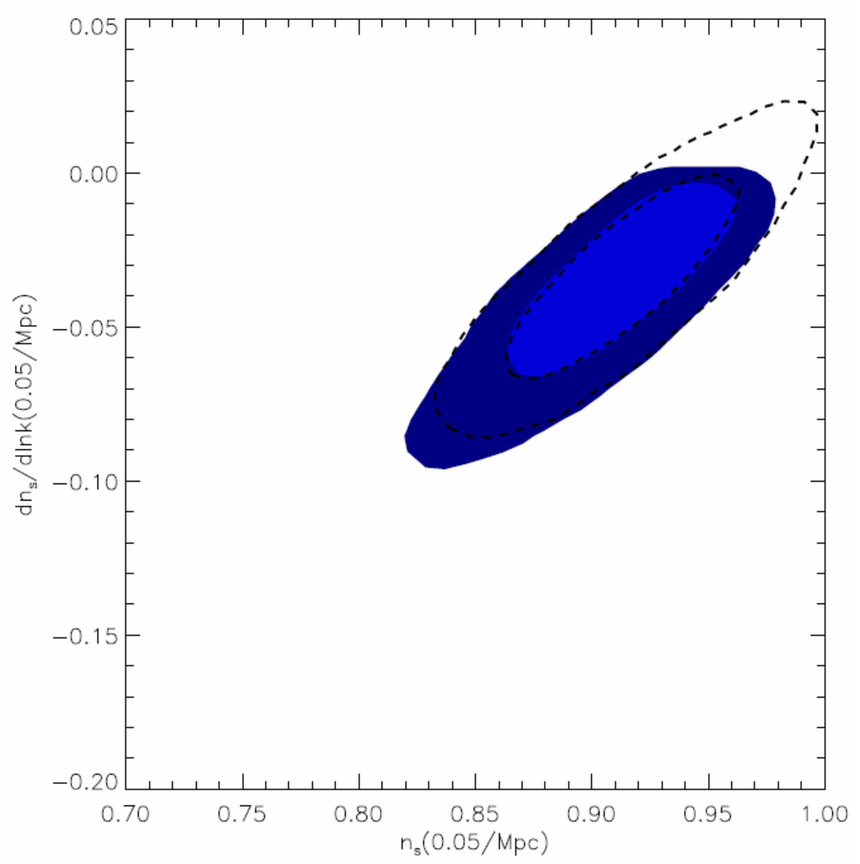
- Large, but regional, running of spectral index

Signaling the phase transition happens within CMB scales

- Large non-Gaussianity (stringy at larger scales)

Stringy non-G or even estimation is currently unavailable.

Experiments ahead of string theory!



Conclusions

- A full non-Gaussianity in general single field inflation specified by 5 parameters;
- Explicit form of momentum dependence, including a few potentially observable;
- Recovered all previously known results, explore unknown regions.
- Probing string theory models, including field theoretic with strong string motivations and completely stringy physics.