The Bispectrum in Renormalized Perturbation Theory

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Renormalized Cosmological Perturbation Theory

M. Crocce and R. Scoccimarro, Phys. Rev. D **73** 063519 & 063520

RPT: Beyond two-point observables

F. Bernardeau, M. Crocce and R. Scoccimarro, *in preparation*

Overview

* Problems with standard PT
* Renormalized PT approach :

> Power Spectrum and two-point Propagator (Baryon Acoustic Oscillations)

> Bispectrum and three-point Propagator (work in *progress*)

analytic and N-body results

Conclusions

Linear order $\delta_{\rm L}(k,z) = D_+(z)\delta_0(k) \xrightarrow[\langle\delta(\mathbf{k})\delta(-\mathbf{k})\rangle]{} P_{\rm lin}(k,z) = [D_+(z)]^2 P_0(k)$

Standard perturbation theory expands the density contrast in terms of the linear solution,

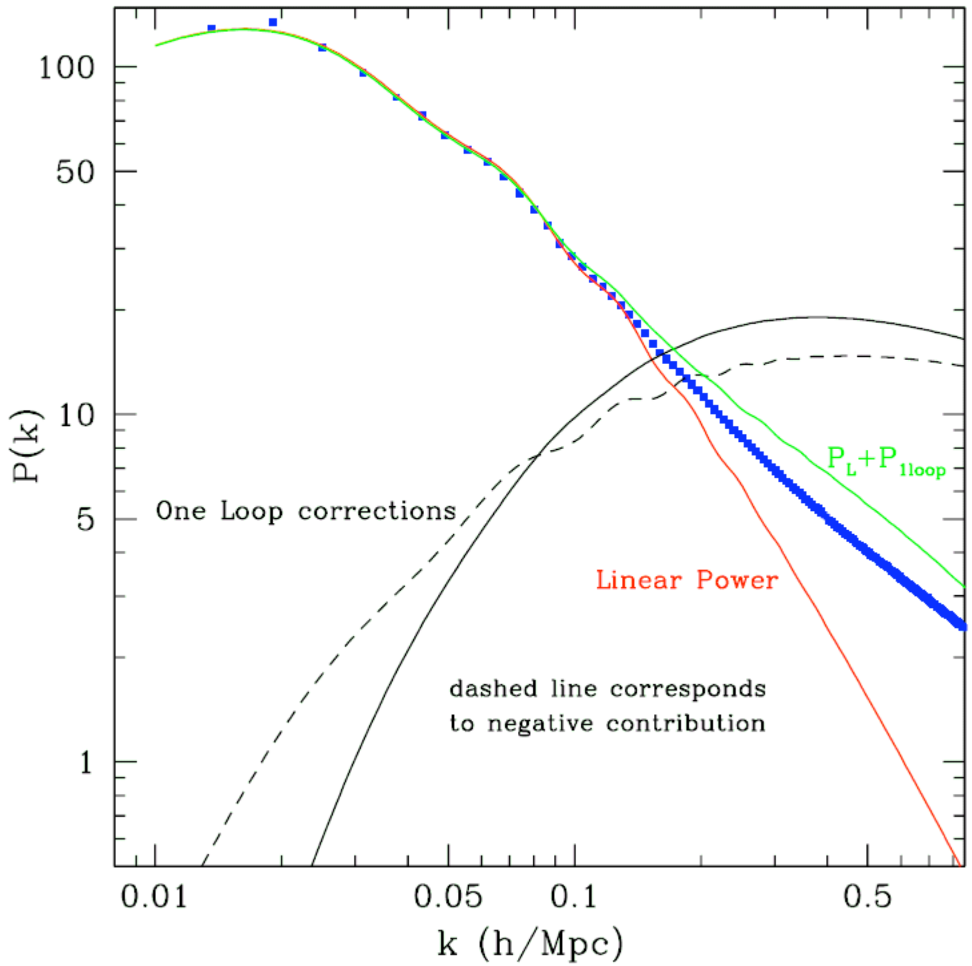
$$P(k,z) = D_+^2(z)\,P_0(k) + P_{\rm 1loop}(k,z) + P_{\rm 2loop}(k,z) + \ldots$$

$$P_{\rm 1loop} \sim \mathcal{O}(P_{\rm lin}\,\Delta_{\rm lin})$$
$$P_{\rm 2loop} \sim \mathcal{O}(P_{\rm lin}\,\Delta_{\rm lin}^2)$$

$$\Delta_{\rm lin} = 4\pi k^3 P_{\rm lin}$$

This expansion is valid at large scales where fluctuations are small, but it brakes down when approaching the nonlinear regime where $\Delta_{\rm lin} \gtrsim 1$.

One needs to sum up all orders to get meaningful answers !!



RPT and two-point statistics : Power Spectrum

$$P(k,z) = D_+^2(z)P_0(k) + P_{13}(k,z) + P_{22}(k,z) + \dots$$

$$P_{22}(k,z) \equiv 2\int [F_2^{(s)}(\mathbf{k}-\mathbf{q},\mathbf{q})]^2 P_{\rm lin}(|\mathbf{k}-\mathbf{q}|,z)P_{\rm lin}(q,z)\mathrm{d}^3\mathbf{q}$$ one loop irreducible

$$P_{13}(k,z) \equiv D_+^2(z)\,P_0(k)\,6\int F_3^{(s)}(\mathbf{k},\mathbf{q},-\mathbf{q})P_{\rm lin}(q,z)\mathrm{d}^3\mathbf{q}$$ one loop reducible

$$P(k,z) = D_+^2(z)\left[1+6\int F_3^{(s)}P_{\rm lin}\mathrm{d}^3\mathbf{q}+\dots\right]P_0(k) + P_{\rm irreducibles}(k,z)$$

all orders can be systematically incorporated

$$\boxed{P(k,z) = G_\delta^2(k,z)\times P_0(k) + P_{\rm Mode\,Coupling}(k,z)}$$

G is the nonlinear propagator
(or two-point propagator)

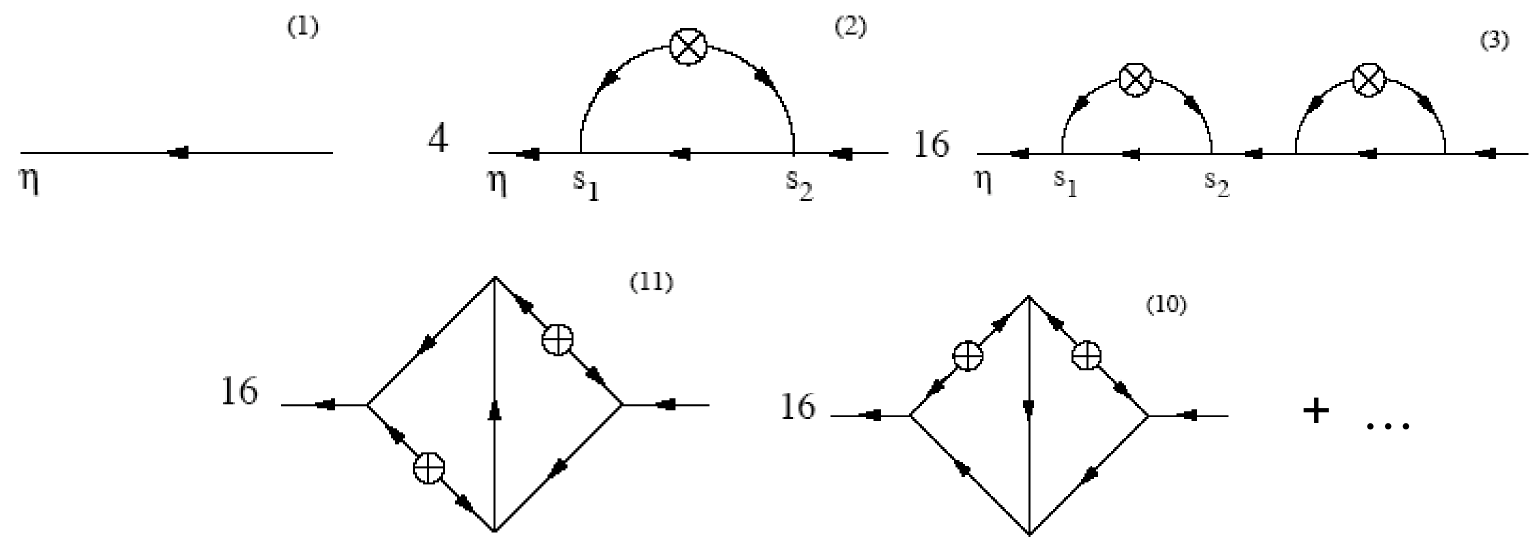
final density / vel divergence

$$G_{ab}(k,\eta)\,\delta_{\rm D}(\mathbf{k}-\mathbf{k}') \equiv \left\langle \frac{\delta\Psi_a(\mathbf{k},\eta)}{\delta\phi_b(\mathbf{k}')}\right\rangle$$

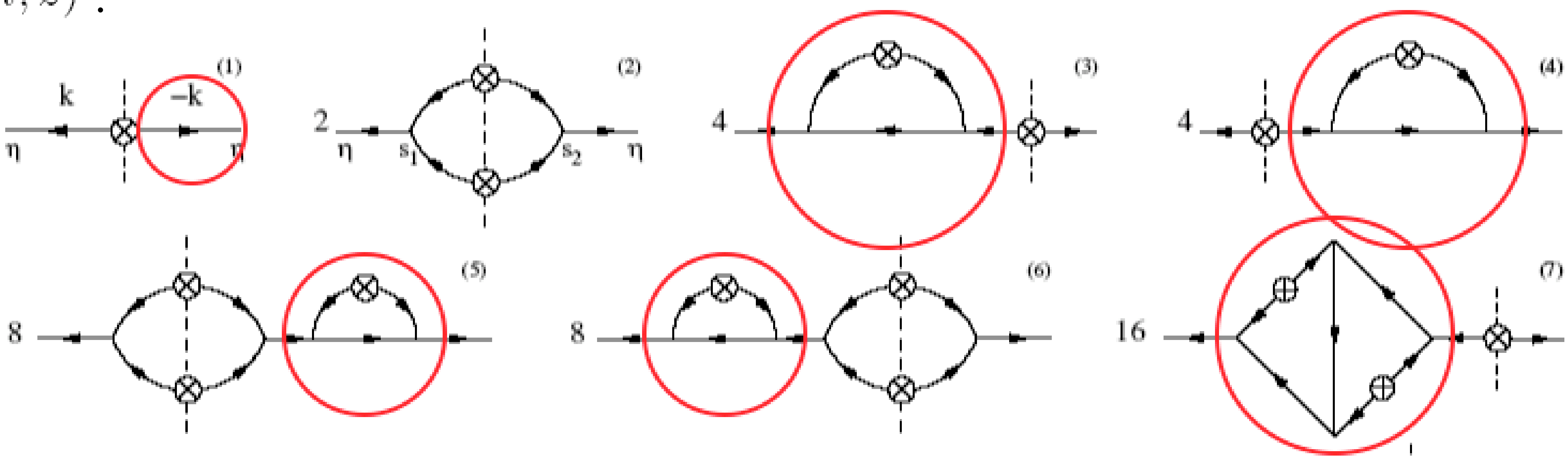
Initial conditions

In reality we do this diagrammatically keeping all sub-leading time dependencies (i.e. nonlinear corrections involve integrals of momentum and time)

$G_\delta(k, z)$:



$P(k, z)$:



$$P(k,z) = G_\delta^2(k,z) \times P_0(k) + P_{\rm Mode\,Coupling}(k,z)$$

$G_\delta(k,z)$ is well defined and has physical meaning

* The **propagator** is a measure of the *memory to the initial conditions*

- At large scales it reduces to the usual growth factor in linear theory (i.e. memory is "preserved"): $G_\delta(k \to 0) \to D_+$
- At smaller scales it receives nonlinear corrections due to mode coupling that drive it to zero (the final field "does not remember" the initial distribution): $G_\delta(k \to \infty) \to 0$

* The rest of the diagrams (still an infinite number) can be thought of as the effect of **Mode Coupling**. The propagator can be re-summed in them as well

They measure generation of structure at small scales

They dominate in a narrow range of scales drastically improving convergence. Now small scale large amplitude fluctuations are exponentially damped

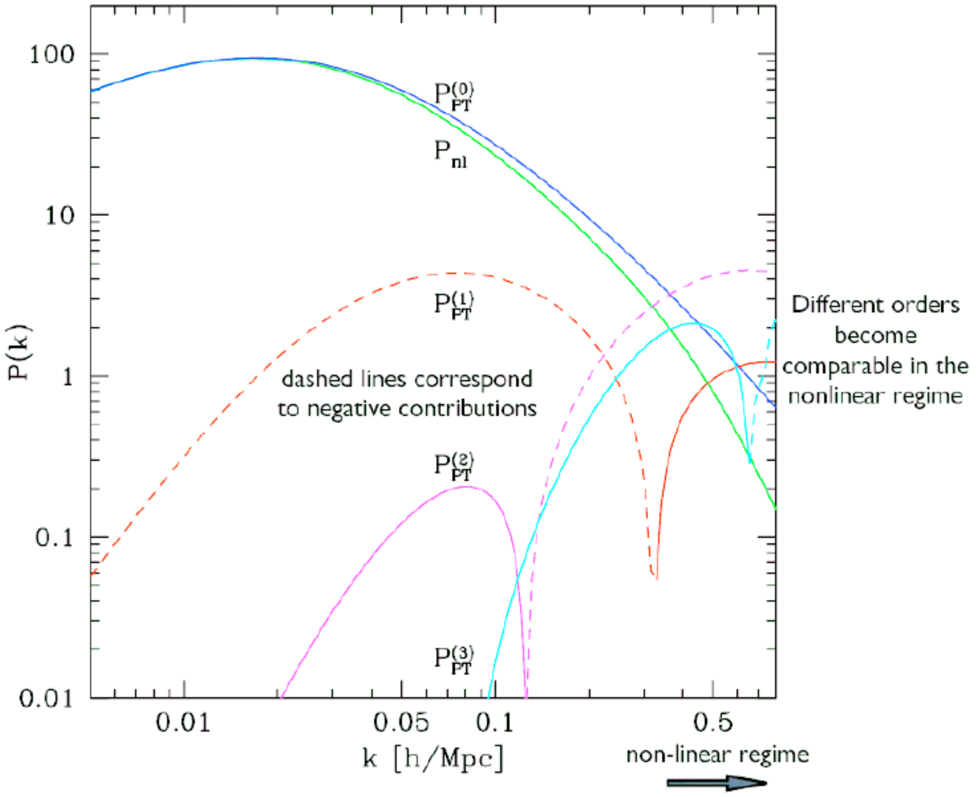
$$P_{\rm MC}(k,z) \sim \mathcal{O}([G^2(k',z')P_0(k')])$$

Example : Zel'dovich Approximation (particles moving in straight lines according to their primordial gravitational forces)

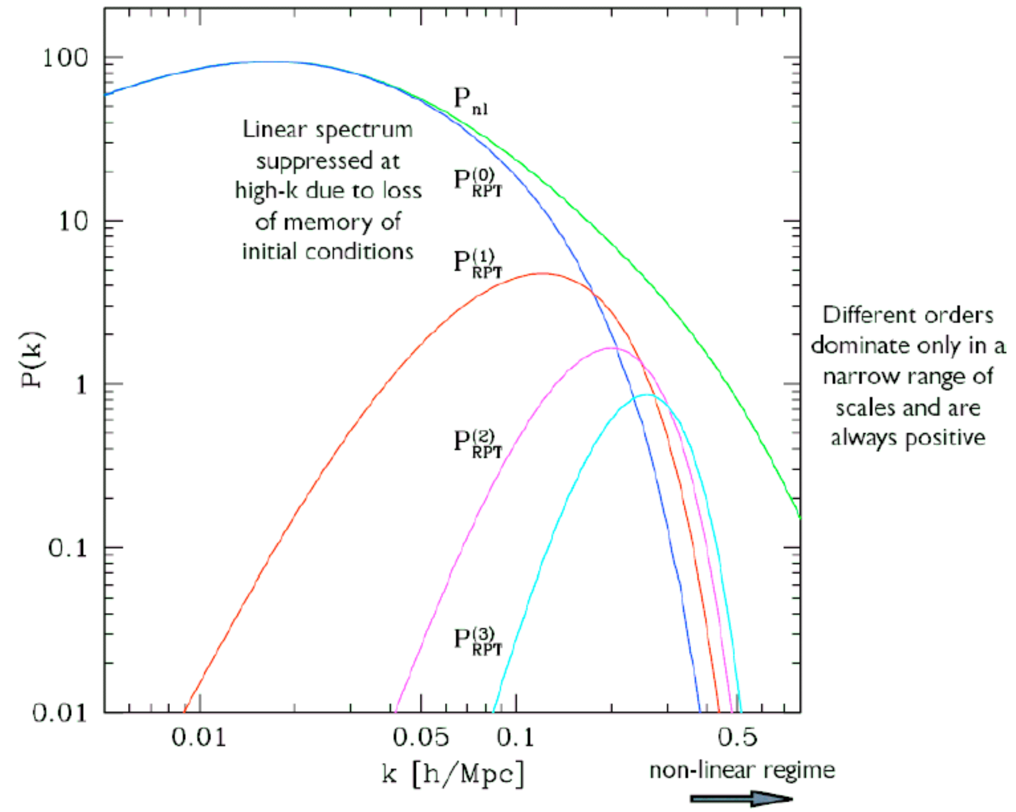
* In this approx. is possible to compute the nonlinear propagator and mode coupling power exactly

$$\boxed{G_\delta^{ZA} = a\, \exp(-k^2\sigma_v^2/2)} \qquad \left(\sigma_v^2 \equiv (1/3)\int d^3q P_L(q,a)/q^2\right)$$

ZA Nonlinear Power Spectrum in standard PT expansion



ZA Nonlinear Power Spectrum in RPT expansion



$$P(k,z) = [D_+(z)]^2\; P_0(k) + P^{(1)}(k,z) + \cdots$$

depends on $D_+^2(z)P_0(k)$

$$P(k,z) = G_\delta^2(k,z) \times P_0(k) + P_{1\,\mathrm{Loop}}(k,z) + \cdots$$

depends on $G^2(k,z)P_0(k)$

RPT and three-point statistics : Bispectrum

$$B(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3) \equiv D_+^4(z)\,P_0(k_1)\,P_0(k_2)\,2\,F_2^{(s)}(\mathbf{k}_1,\mathbf{k}_2) + \underbrace{B_{222} + B_{321}^{I} + B_{321}^{II} + B_{411}}_{\text{one-loop corrections}} + \text{permutations}$$

$$B_{321}^{II} \equiv 6D_+^4(z)P_0(k_1)P_0(k_2)F_2^{(s)}(\mathbf{k}_1,\mathbf{k}_2)\int P_{\rm lin}(q,z)F_3^{(s)}(\mathbf{k}_1,\mathbf{q},-\mathbf{q})\,\mathrm{d}^3\mathbf{q} \longrightarrow \text{renormalises } D_+$$

$$B_{411} \equiv 12D_+^4(z)P_0(k_1,z)P_0(k_2,z)\int P_{\rm lin}(q,z)F_4^{(s)}(\mathbf{q},-\mathbf{q},-\mathbf{k}_2,-\mathbf{k}_3)\,\mathrm{d}^3\mathbf{q} \longrightarrow \text{renormalises } F_2^{(s)}$$

B_{222} , B_{321}^{I} $\longrightarrow$ one loop irreducible

$$B(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3) \equiv G_\delta^2(k_1,z)P_0(k_1)\,G_\delta^2(k_2,z)P_0(k_2)\left(2\,F_2^{(s)}(\mathbf{k}_1,\mathbf{k}_2) + 12\int P_{\rm lin}\,F_4^{(s)}\mathrm{d}^3\mathbf{q} + \ldots\right) + B_{\rm irreducibles}$$

$$\Downarrow$$

$$\boxed{B(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3) \equiv G_\delta^2(k_1,z)P_0(k_1)\,G_\delta^2(k_2,z)P_0(k_2)\,\Gamma_\delta^{(2)}(\mathbf{k}_1,\mathbf{k}_2) + B_{\rm irreducibles} + \text{perm}}$$

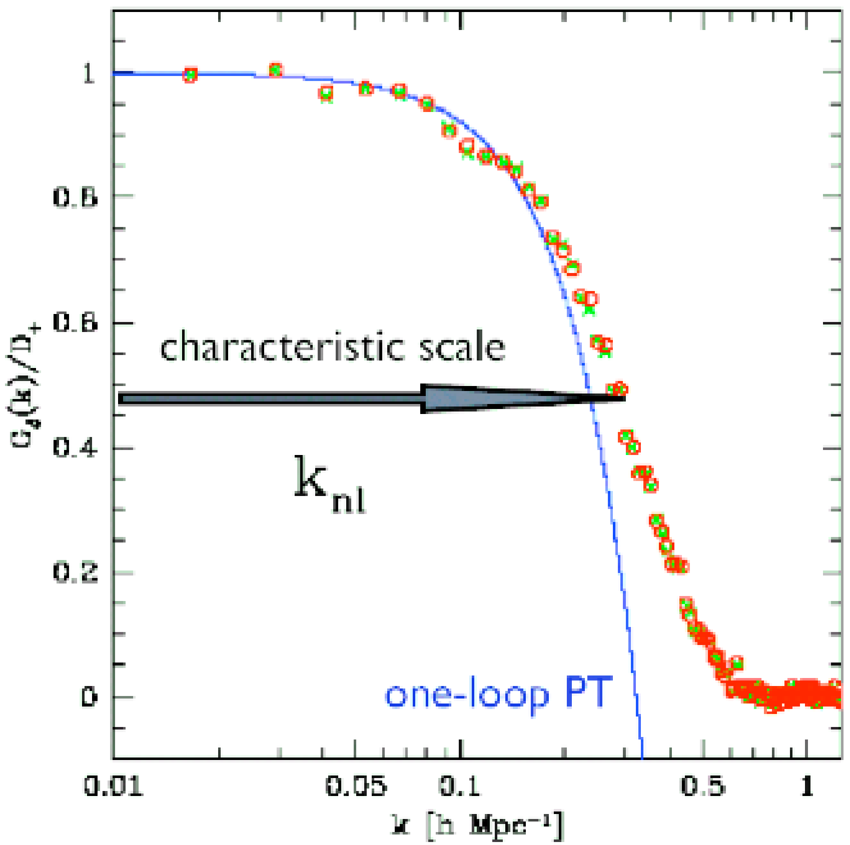
three-point propagator $\quad \Gamma_{abc}^{(2)}(\mathbf{k}_1,\mathbf{k}_2,z)\;\delta_{\rm D}(\mathbf{k}-\mathbf{k}_1-\mathbf{k}_2) \equiv \left\langle \dfrac{\delta^2\Psi_a(\mathbf{k},z)}{\delta\phi_b(\mathbf{k}_1)\delta\phi_c(\mathbf{k}_2)}\right\rangle$

What else can be said about propagators ? (as they became key ingredients)

Two-point propagator :

* For Gaussian init cond it can be shown that $G_\delta = \langle \delta(\mathbf{k}, z)\delta_0(-\mathbf{k}, z)\rangle / P_0(k)$

Thus it can be measured from N-body !



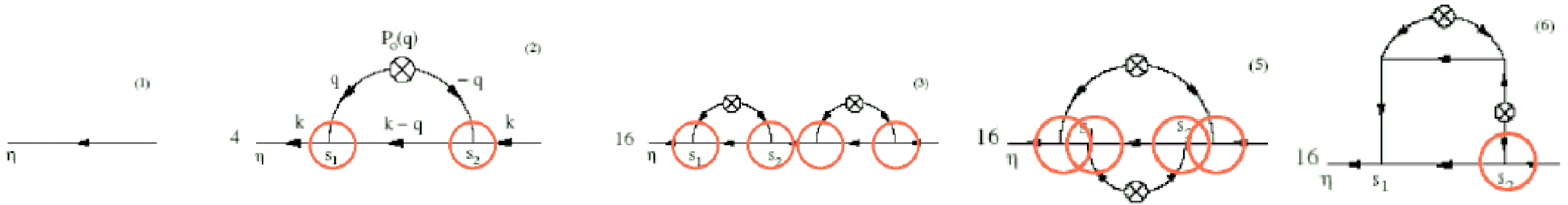
Nearly Gaussian decay

The departure from linear evolution at large scales is well described by the one-loop diagram (first nonlinear correction)

low-k limit :

$$G_\delta(k, z) \approx D_+(z)\left(1 - \frac{61}{210}k^2\sigma_v^2 D_+^2\right)$$

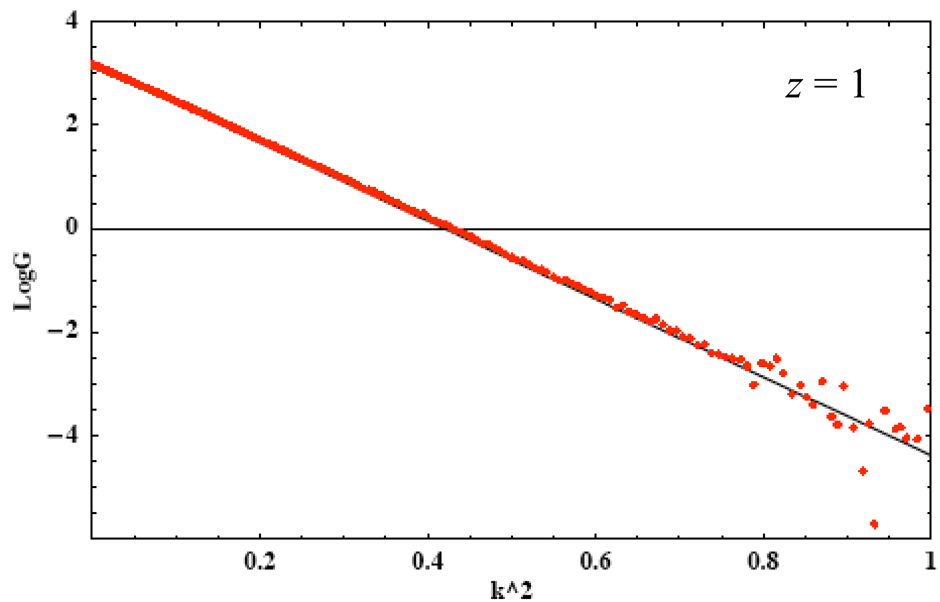
The nonlinear regime ⇨ large-k limit



We were able to re-sum the dominant contribution to all orders !

$$\boxed{G_\delta(k,z) \simeq D_+(z)\exp\left(-\frac{1}{2}k^2\sigma_v^2(D_+(z)-1)^2\right)}$$

$$\sigma_v^2 \equiv \frac{1}{3}\int d^3q\frac{P_0(q)}{q^2}$$

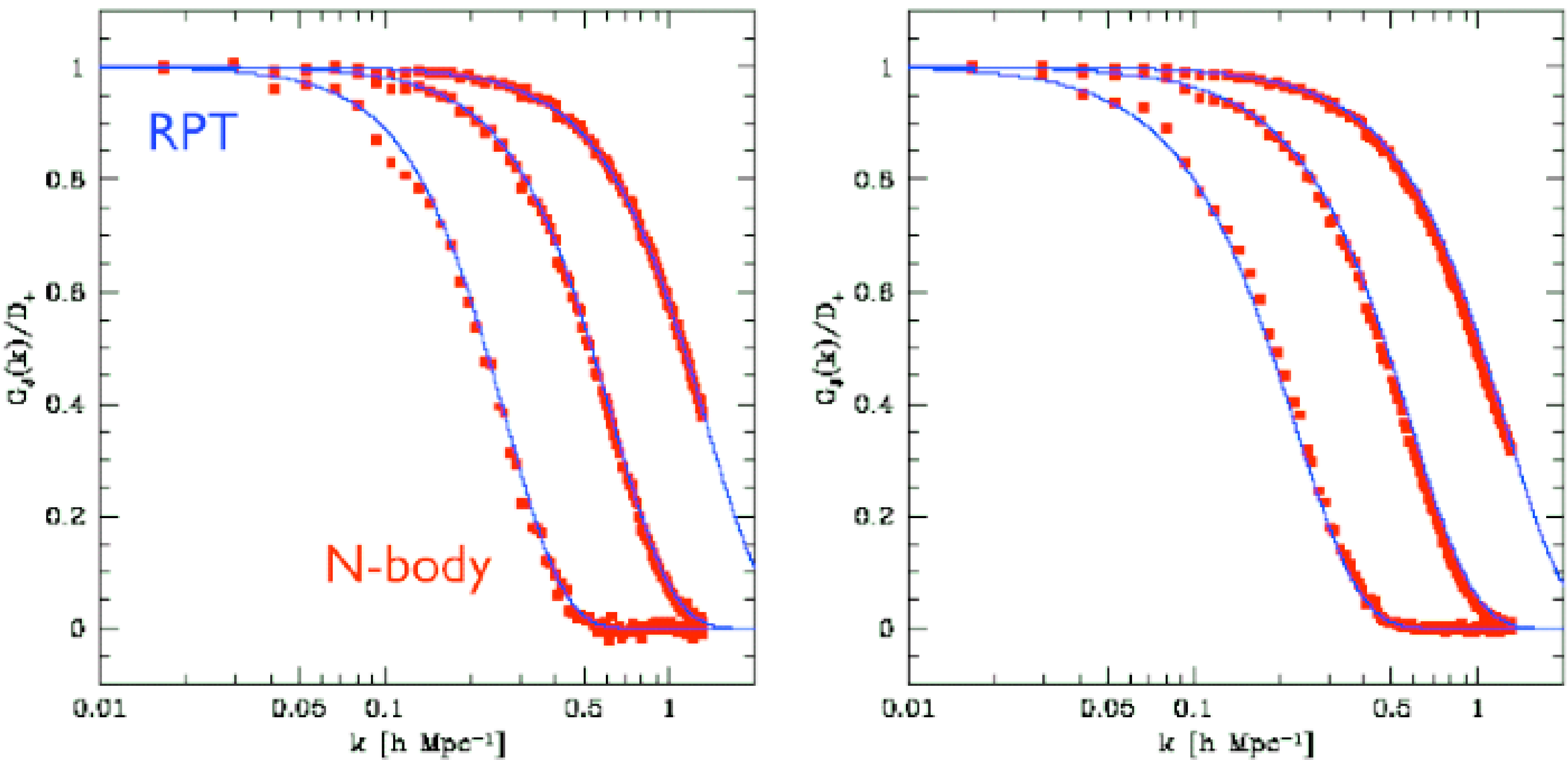


To recover the two-point propagator for all scales we match this asymptotic result with to the low-k (one-loop correction) expression by

* regarding the one-loop propagator as the power series expansion of a Gaussian

- must decay monotically as k increases for fixed time
- must decay monotically as time increases for fixed k

Comparison between RPT and N-Body Simulations
(z = 0, 2, 5)



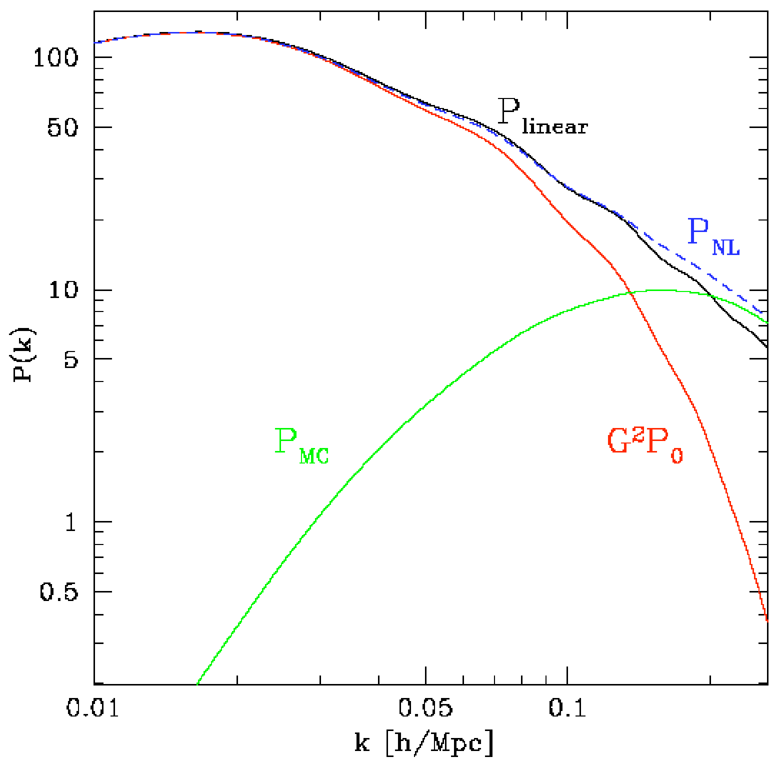
- The RPT predictions match simulations, even into the nonlinear regime, for density and velocity fields, without introducing any free parameters.

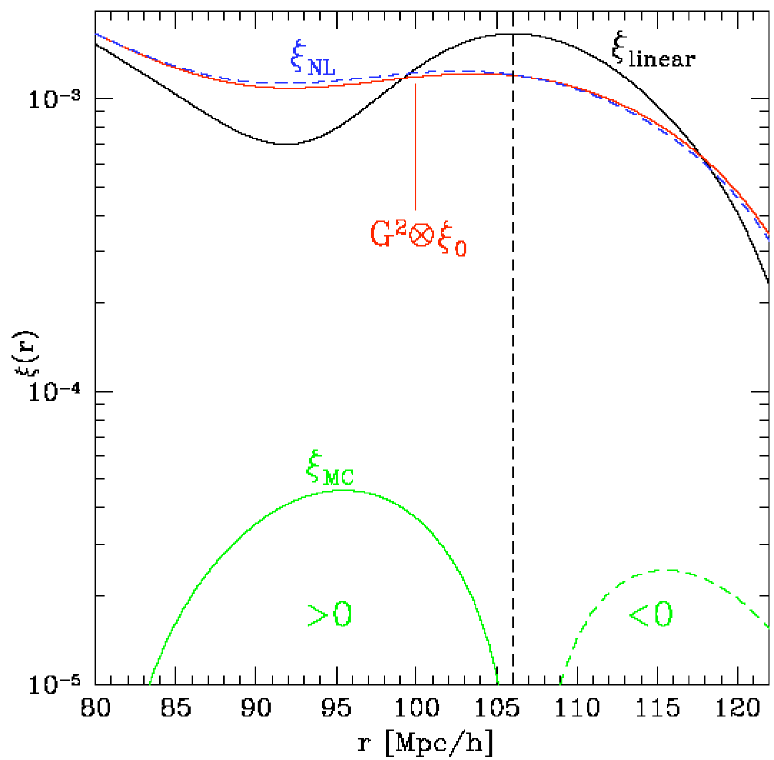
Not only of academic interest ⟹ application to Baryon Acoustic Oscillations (astro-ph/0704.2783)

RPT is a perfect match for BAO because it can describe accurately the nonlinear scales where the acoustic signature extends to

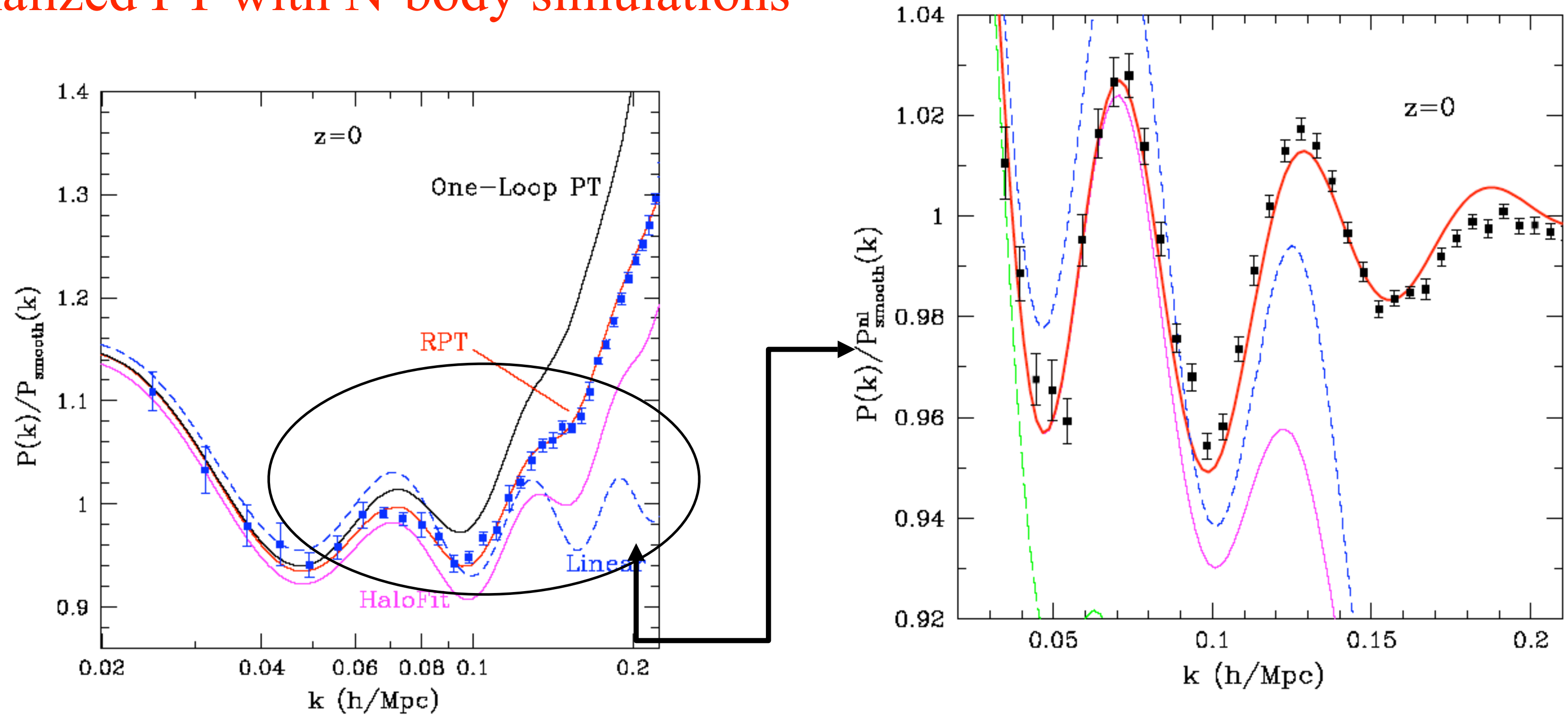
Provided with the prescription for the two-point propagator we computed $P_{\text{Mode Coupling}}$ "up to two loops" (only one irreducible contribution at each order)

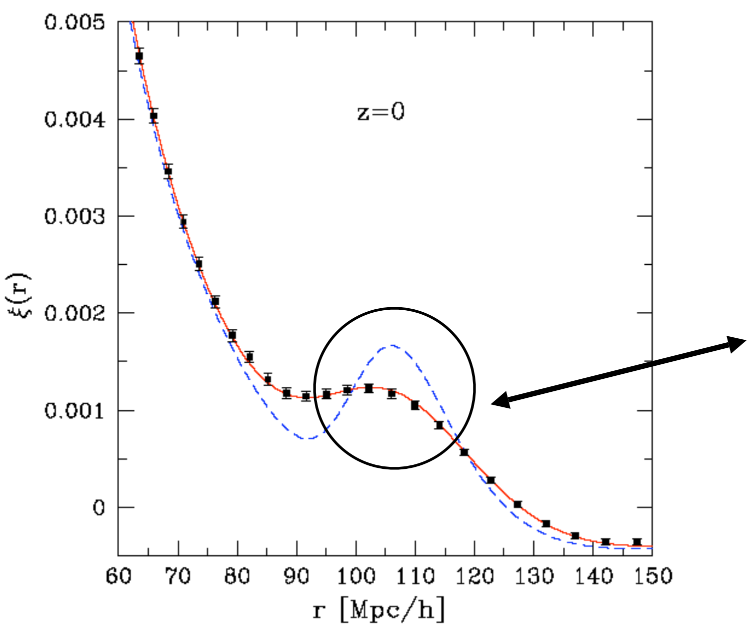
$$P(k,z) = G_\delta^2(k,z) \times P_0(k) + P_{\text{Mode Coupling}}(k,z) \longleftrightarrow \xi(r) = [\xi_{\text{linear}} \otimes G_\delta^2](r) + \xi_{\text{Mode Coupling}}(r)$$

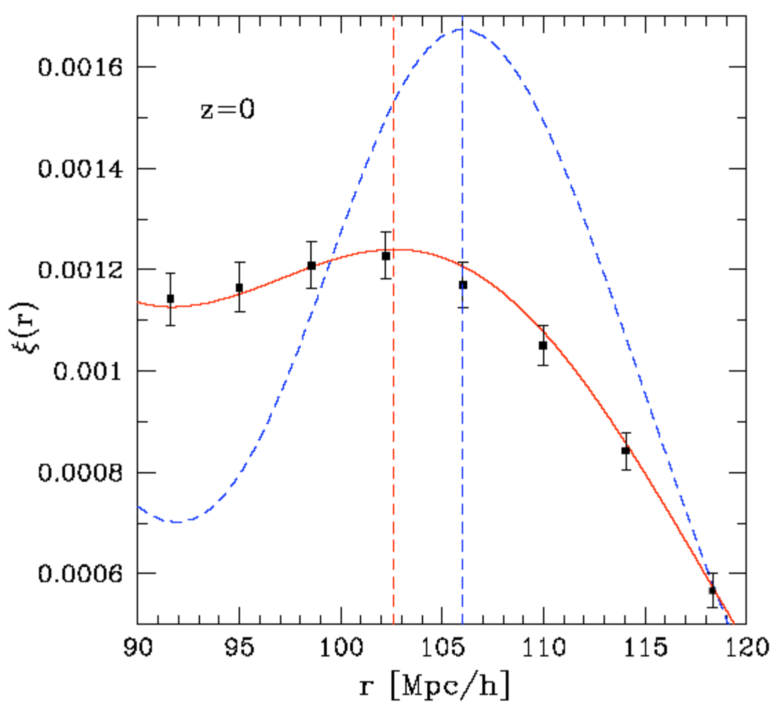




Testing Renormalized PT with N-body simulations







\# Motivates fitting formulas

\# Study systematic effects with accuracy

\# Revise Halo Model?

$$B(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3) \;\equiv\; G_\delta^2(k_1,z)P_0(k_1)\,G_\delta^2(k_2,z)P_0(k_2)\,\Gamma_\delta^{(2)}(\mathbf{k}_1,\mathbf{k}_2) + B_{\rm irreducibles}$$

Three-point propagator :

$$\Gamma_\delta^{(2)}(\mathbf{k}_1,\mathbf{k}_2) = 2F_2^{(s)}(\mathbf{k}_1,\mathbf{k}_2) + \text{nonlinear corrections}$$

* The low-k behavior can be obtained computing the one-loop diagrams

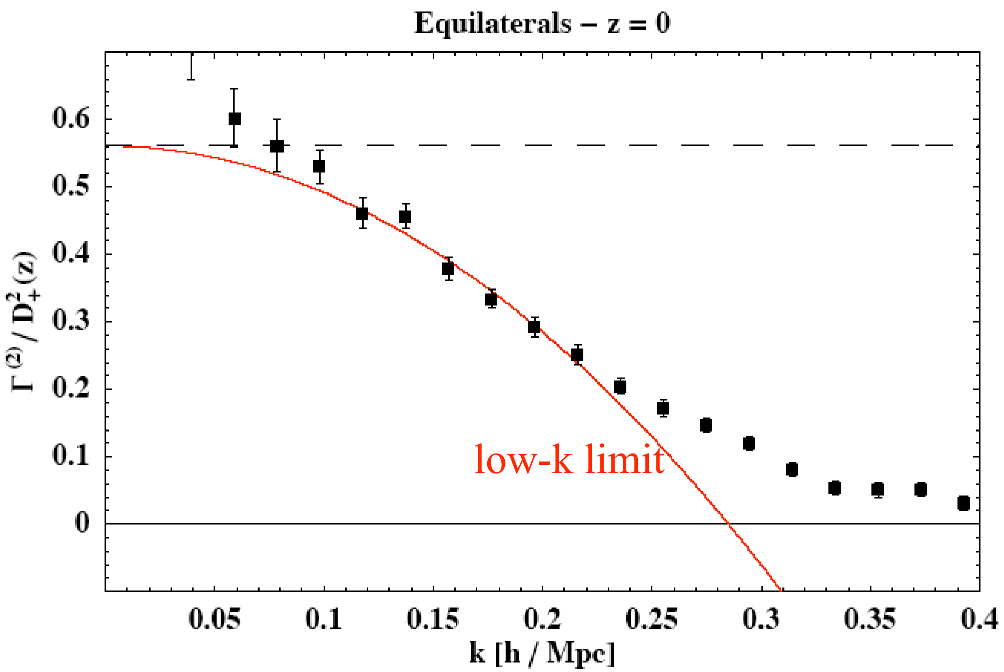
$$\frac{\Gamma^{(2)}(k,z)}{D_+^2(z)} \approx \frac{4}{7}\left(1 - \frac{1219}{7840}k^2\sigma_v^2 D_+^2\right)$$

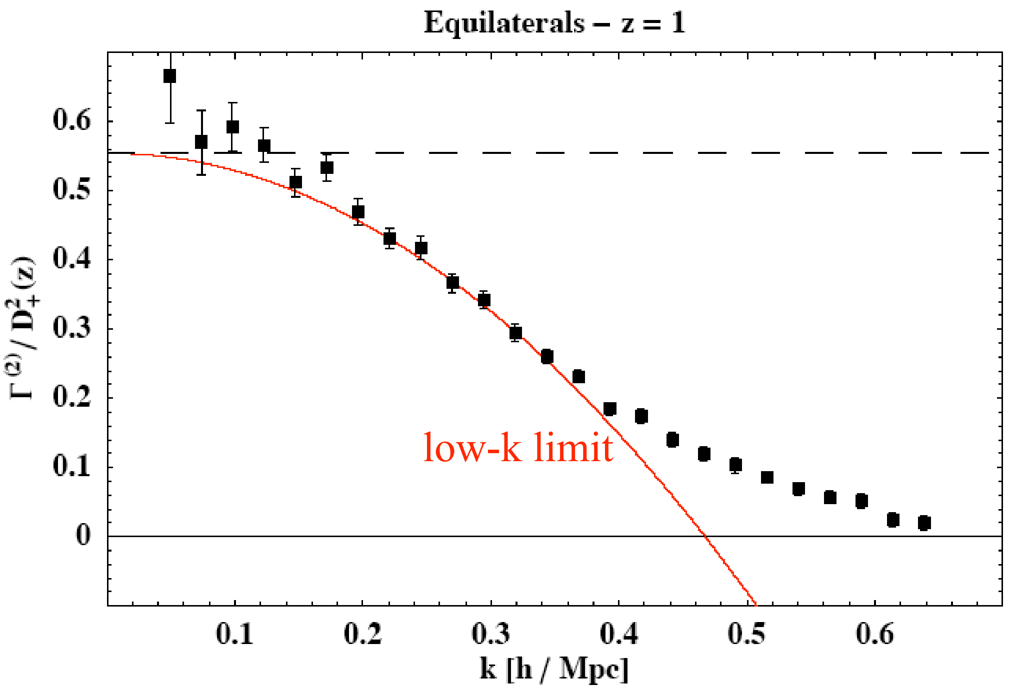
low-k limit for equilateral triangles

$$\sigma_v^2 \equiv \frac{1}{3}\int d^3q \frac{P_0(q)}{q^2}$$

* It can also be measured !

$$\Gamma_\delta^{(2)}(\mathbf{k}_1,\mathbf{k}_2)\,\delta_{\rm D}(\mathbf{k}-\mathbf{k}_{12}) = \frac{\langle \delta(\mathbf{k},z)\delta_0(-\mathbf{k}_1,z)\delta_0(-\mathbf{k}_2,z)\rangle}{P_0(k_1)P_0(k_2)}$$





Conclusions

- New formalism to study nonlinear clustering of DM ("resummed standard PT") well defined , provide physical insight , ..

-- Gives new strength for (R) Pert. Theories to play an important role in modeling observed structure accurately in forthcoming surveys (precision cosmology needs precision modeling)

-- Perfect match for BAO predicts → P(k) and $\xi(r)$ at sub-percent level at all scales of interest !

- Important to study systematic effects, step to physically motivated fitting formulae

- We could also re-sum the Bispectrum perturbative series, emergence of two-point propagator (suppression of primordial features/non gaussianites)
→ we have a clear although long path to follow

-Future steps :: inclusion of Redshift Distortions and Galaxy Bias to compare with survey data,