The Bispectrum in Renormalized Perturbation Theory

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Renormalized Cosmological Perturbation Theory
M. Crocce and R. Scoccimarro, Phys. Rev. D 73 063519 & 063520
RPT: Beyond two-point observables
F. Bernardeau, M. Crocce and R. Scoccimarro, in preparation
Overview

* Problems with standard PT
* Renormalized PT approach:

> Power Spectrum and two-point Propagator (Baryon Acoustic Oscillations)

> Bispectrum and three-point Propagator (work in progress)

analytic and N-body results

Conclusions
Standard perturbation theory expands the density contrast in terms of the linear solution,

\[ P(k, z) = D_+(z) P_0(k) + P_{1\text{loop}}(k, z) + P_{2\text{loop}}(k, z) + \ldots \]

\[ P_{1\text{loop}} \sim O(P_{\text{lin}} \dot{\Delta}_{\text{lin}}) \]

\[ P_{2\text{loop}} \sim O(P_{\text{lin}} \Delta_{\text{lin}}^2) \]

\[ \Delta_{\text{lin}} \equiv 4\pi k^3 P_{\text{lin}} \]

This expansion is valid at large scales where fluctuations are small, but it brakes down when approaching the nonlinear regime where \( \Delta_{\text{lin}} \gtrsim 1 \).

One needs to sum up all orders to get meaningful answers!!
RPT and two-point statistics: Power Spectrum

\[ P(k, z) = D^2_+(z)P_0(k) + P_{13}(k, z) + P_{22}(k, z) + \ldots \]

\[ P_{22}(k, z) \equiv 2 \int [F_2^{(s)}(k - q, q)]^2 P_{\text{lin}}(|k - q|, z) P_{\text{lin}}(q, z) d^3q \]

\[ P_{13}(k, z) \equiv D^2_+(z) P_0(k) 6 \int F_3^{(s)}(k, q, -q) P_{\text{lin}}(q, z) d^3q \]

all orders can be systematically incorporated

\[ P(k, z) = D^2_+(z) \left[ 1 + 6 \int F_3^{(s)} P_{\text{lin}} d^3q + \ldots \right] P_0(k) + P_{\text{irreducibles}}(k, z) \]

\[ P(k, z) = G^2_\delta(k, z) \times P_0(k) + P_{\text{Mode Coupling}}(k, z) \]

final density / vel divergence

G is the nonlinear propagator (or two-point propagator)

\[ G_{ab}(k, \eta) \delta_D(k - k') \equiv \left\langle \frac{\delta \Psi_a(k, \eta)}{\delta \phi_b(k')} \right\rangle \]

Initial conditions
In reality we do this diagrammatically keeping all sub-leading time dependencies (i.e. nonlinear corrections involve integrals of momentum and time)

\[ G_\delta(k, z) : \]

\[ P(k, z) : \]
\[ P(k, z) = G_\delta^2(k, z) \times P_0(k) + P_{\text{Mode Coupling}}(k, z) \]

\[ G_\delta(k, z) \text{ is well defined and has physical meaning} \]

* The propagator is a measure of the memory to the initial conditions

\[ \begin{align*}
\text{At large scales} & \text{ it reduces to the usual growth factor in linear theory (i.e. memory is “preserved”): } G_\delta(k \to 0) \to D_+ \\
\text{At smaller scales} & \text{ it receives nonlinear corrections due to mode coupling that drive it to zero (the final field “does not remember” the initial distribution): } G_\delta(k \to \infty) \to 0
\end{align*} \]

* The rest of the diagrams (still an infinite number) can be thought of as the effect of Mode Coupling. The propagator can be re-summed in them as well

# They measure generation of structure at small scales

# They dominate in a narrow range of scales drastically improving convergence. Now small scale large amplitude fluctuations are exponentially damped

\[ P_{\text{MC}}(k, z) \sim \mathcal{O}([G^2(k', z') P_0(k')]) \]
Example: Zel'dovich Approximation (particles moving in straight lines according to their primordial gravitational forces)

* In this approx. is possible to compute the nonlinear propagator and mode coupling power exactly

\[ G_\delta^{ZA} = a \exp(-k^2 \sigma^2_\delta / 2) \]

\[ \sigma^2_\delta = (1/3) \int d^3q P_L(q, \alpha) / q^2 \]

**ZA Nonlinear Power Spectrum in standard PT expansion**

\[ P(k, z) = [D_+(z)]^2 P_0(k) + P^{(1)}(k, z) + \cdots \]

depends on \[ D^2_+(z) P_0(k) \]

**ZA Nonlinear Power Spectrum in RPT expansion**

\[ P(k, z) = G_\delta^2(k, z) \times P_0(k) + P_{\text{Loop}}(k, z) + \cdots \]

depends on \[ G^2(k, z) P_0(k) \]
RPT and three-point statistics: Bispectrum

\[ B(k_1, k_2, k_3) \equiv D^4_+(z) P_0(k_1) P_0(k_2) 2 F_2^{(s)}(k_1, k_2) + \underbrace{B_{222} + B^I_{321} + B^H_{321} + B_{411}}_{\text{one-loop corrections}} + \text{permutations} \]

\[ B_{321}^{II} \equiv 6 D^4_+(z) P_0(k_1) P_0(k_2) F_2^{(s)}(k_1, k_2) \int P_{\text{lin}}(q, z) F_3^{(s)}(k_1, q, -q) d^3q \quad \rightarrow \quad \text{renormalises } D_+ \]

\[ B_{411} \equiv 12 D^4_+(z) P_0(k_1, z) P_0(k_2, z) \int P_{\text{lin}}(q, z) F_4^{(s)}(q, -q, -k_2, -k_3) d^3q \quad \rightarrow \quad \text{renormalises } F_2^{(s)} \]

\[ B_{222}, B_{321} \rightarrow \text{one loop irreducible} \]

\[ B(k_1, k_2, k_3) \equiv G_\delta^2(k_1, z) P_0(k_1) G_\delta^2(k_2, z) P_0(k_2) \left( 2 F_2^{(s)}(k_1, k_2) + 12 \int P_{\text{lin}} F_4^{(s)} d^3q + \ldots \right) + B_{\text{irreducibles}} \]

three-point propagator

\[ \Gamma^{(2)}_{abc}(k_1, k_2, z) \delta_D(k - k_1 - k_2) \equiv \left\langle \frac{\delta^2 \Psi_a(k, z)}{\delta \phi_b(k_1) \delta \phi_c(k_2)} \right\rangle \]
What else can be said about propagators? (as they became key ingredients)

Two-point propagator:

* For Gaussian init cond it can be shown that

\[ G_\delta = \langle \delta(k, z) \delta_0(-k, z) \rangle / P_0(k) \]

Thus it can be measured from N-body!

The departure from linear evolution at large scales is well described by the one-loop diagram (first nonlinear correction)

low-k limit:

\[ G_\delta(k, z) \approx D_+(z) \left( 1 - \frac{61}{210} k^2 \sigma_v^2 D_+^2 \right) \]
The nonlinear regime \(\rightarrow\) large-k limit

We were able to re-sum the dominant contribution to all orders!

\[
G_\delta(k, z) \simeq D_+(z) \exp \left( -\frac{1}{2} k^2 \sigma_v^2 (D_+(z) - 1)^2 \right)
\]

\[
\sigma_v^2 = \frac{1}{3} \int d^3 q \frac{P_0(q)}{q^2}
\]

To recover the two-point propagator for all scales we match this asymptotic result with to the low-k (one-loop correction) expression by

* regarding the one-loop propagator as the power series expansion of a Gaussian

- must decay monotonically as k increases for fixed time
- must decay monotonically as time increases for fixed k
Comparison between RPT and N-Body Simulations
(z = 0, 2, 5)

- The RPT predictions match simulations, even into the nonlinear regime, for density and velocity fields, without introducing any free parameters.
Not only of academic interest is application to Baryon Acoustic Oscillations (astro-ph/0704.2783)

RPT is a perfect match for BAO because it can describe accurately the nonlinear scales where the acoustic signature extends to

Provided with the prescription for the two-point propagator we computed $P_{\text{Mode Coupling}}$ "up to two loops" (only one irreducible contribution at each order)

$$P(k, z) = G_0^2(k, z) \times P_0(k) + P_{\text{Mode Coupling}}(k, z)$$

$$\xi(r) = [\xi_{\text{linear}} \otimes G_0^2](r) + \xi_{\text{Mode Coupling}}(r)$$
Testing Renormalized PT with N-body simulations

# Motivates fitting formulas

# Study systematic effects with accuracy

# Revise Halo Model?
Three-point propagator:

\[ \Gamma^{(2)}(k_1, k_2) = 2F_2^{(s)}(k_1, k_2) + \text{nonlinear corrections} \]

*The low-k behavior can be obtained computing the one-loop diagrams*

\[
\frac{\Gamma^{(2)}(k, z)}{D^2_+(z)} \approx \frac{4}{7} \left( 1 - \frac{1219}{7840} k^2 \sigma_v^2 D^2_+ \right) \quad \text{low-k limit for equilateral triangles}
\]

\[ \sigma_v^2 \equiv \frac{1}{3} \int d^3q \frac{P_0(q)}{q^2} \]

*It can also be measured!*

\[ \Gamma^{(2)}(k_1, k_2) \delta_D(k - k_{12}) = \frac{\langle \delta(k, z) \delta_0(-k_1, z) \delta_0(-k_2, z) \rangle}{P_0(k_1)P_0(k_2)} \]
Conclusions

- New formalism to study nonlinear clustering of DM (“resummed standard PT”) well defined, provide physical insight, ..

- Gives new strength for (R) Pert. Theories to play an important role in modeling observed structure accurately in forthcoming surveys (precision cosmology needs precision modeling)

- Perfect match for BAO predicts \( P(k) \) and \( \xi(r) \) at sub-percent level at all scales of interest!

- Important to study systematic effects, step to physically motivated fitting formulae

- We could also re-sum the Bispectrum perturbative series, emergence of two-point propagator (suppression of primordial features/non gaussianites) we have a clear although long path to follow

Future steps :: inclusion of Redshift Distortions and Galaxy Bias to compare with survey data,