Towards a Universal Halo Mass Function for $f_{NL}$ Cosmologies

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In preparation
Halo mass function

Lots of interest in using halo counts as a cosmological probe (SZ, optical,...).

- Mass function can be computed precisely (~5%) and robustly for standard cosmology (Jenkins et al. 01, Warren et al. 03)

- \( \frac{dN}{dM} \) appears universal — i.e. \( f(\sigma) \) — for standard cosmologies

\[
\sigma^2(M, z) = \frac{1}{2\pi^2} \int_0^{\infty} k^2 P(k) W^2(k, M) dk
\]

- Abundance of rare objects is exponentially sensitive to cosmological parameters, power spectra (Holder & Haiman 01) and we could hope for 5% on \( \sigma_8 \) and \( \Omega_m \) (probably optimistic)
Non-gaussianity and growth of structure

- Simplest inflationary models predict small but nonzero non-gaussianity. For $\Phi \rightarrow \Phi + f_{NL} (\Phi^2 - \langle \Phi^2 \rangle)$, expect $f_{NL} \sim O(1)$ (e.g. Maldacena 2003).

- Current WMAP constraints: $f_{NL} \approx O(100)$

- The abundance of rare peaks can be strongly affected by this sort of skewness $\Rightarrow$ Can we constrain $f_{NL}$ with $dN/dM$?

- So far, qualitative estimates have been made using (extended) Press-Schechter, but PS mass function is off by an order of magnitude for cluster-sized halos (Verde et al. 01, Scoccimarro 04, Sefusatti et al. 06).

- Simulations are required to properly address the effect of $f_{NL}$ on $dN/dM$. 
Generating ICs for N-body simulations with $f_{NL} \neq 0$

1. Generate a realization of $\Phi$ as Gaussian field with a (nearly) scale invariant power spectra

2. Add the quadratic term in real space

3. Apply transfer function in Fourier space

4. Use Zeldovich approximation to generate the particle data

Caveat: finite volume effect could be tricky due to mode-coupling but it seems to be unimportant
Large scale structures depend on $f_{NL}$

- Same initial conditions, different $f_{NL}$
- Slice through a box in a simulation $N_{\text{part}}=512^3$, $L=800$ Mpc/h

$\begin{align*}
\text{L} = 375 \text{ Mpc/h} \\
\text{M} = 80 \text{ Mpc/h}
\end{align*}$

- Under-dense region evolution decrease with $f_{NL}$
- Over-dense region evolution increase with $f_{NL}$

- $f_{NL} = -3000$
- $f_{NL} = -300$
- $f_{NL} = 0$
- $f_{NL} = +300$
- $f_{NL} = +3000$
Looking at individual clusters

Most massive cluster in our simulation
- For small enough $f_{NL}$, same peaks arise, with different heights (implying different masses)
- Can we extend to any cluster?

- $f_{NL}=+3000$
  - $M=1.2 \times 10^{16} \, M_{\odot}$

- $f_{NL}=0$
  - $M=5.1 \times 10^{15} \, M_{\odot}$

- $f_{NL}=-300$
  - $M=4.3 \times 10^{15} \, M_{\odot}$
Halo matching procedure

Starting from a cluster catalog

For all the particles in a given halo of mass \( M_0 \) in a \( f_{\text{NL}} = 0 \) we can determine which group they end up in for \( f_{\text{NL}} \neq 0 \) and look at the mass distribution of those groups

We define, \( M_f \), the matching mass to \( M_0 \) as the mode of this distribution

There is noise in this mapping because the cluster catalog itself is noisy
Building the $P(M_f|M_0)$ distribution

$f_{NL} = 300$
Statistical approach to this mapping for $f_{NL}>0$

$P(M_f \mid M_0)$ binned in $M_0$

$1 < \frac{M(f_{NL}=0)}{10^{14} \ [h^{-1}M_\odot]} < 5$
Statistical approach to this mapping for $f_{NL} < 0$

$P(M_f | M_0)$ binned in $M_0$

$1 < M(f_{NL} = 0)/10^{14} \ [h^{-1}M_\odot] < 5$

Massive clusters split into smaller clusters
Towards a fitting function

- If the mapping $M_0 \rightarrow M_f$ is described by a PDF $dP/dM_f(M_0)$, then the non-gaussian mass function is a convolution over the (known) gaussian mass function

$$\frac{dN}{dM} = \int \frac{dP(M_f|M_0)}{dM_f} \frac{dN}{dM_0} dM_0$$

(e.g. Jenkins)

- We thus aim at fitting the mean and rms of $\Delta(\log M)(z)$
- The simplest thing to do is to consider a... Gaussian...
- We’d expect the mean of the PDF to be shifted by $\Delta(\log M) \propto f_{NL}$
- We find that a good fit is given by

$$\frac{\bar{M}_f}{M_0} - 1 = 6 \times 10^{-5} f_{NL} \sigma_8 \sigma(M_0, z)^{-2}$$

$$\sigma \left( \frac{\bar{M}_f}{M_0} - 1 \right) = 0.012 (f_{NL} \sigma_8)^{0.4} \sigma(M_0, z)^{-0.5}$$
Mass function from N-body simulation...

\[ f_{NL} = 300 \]

\[ N_{\text{part}} = 512^3, \quad L = 800 \text{ Mpc/h}, \quad z = 0 \]
The PDF appears non-Gaussian, but if we approximate it as a Gaussian with mean and variance given earlier, then we find:

\[
\frac{dN}{dM_f} = \int \frac{dP(M_f|M_0)}{dM_f} \frac{dN}{dM_0} dM_0
\]

Comparison to the fitting formula.
Shall we worry about such discrepancies?

SPT (dN/dM) simulations with Planck prior

N-body sims: 
\( \sigma(f_{NL})=87 \)  
(unmarg \( \sigma(f_{NL})=10 \))

PDF->EPS formalism: 
\( \sigma(f_{NL})=123 \)  
(unmarg \( \sigma(f_{NL})=10 \))

Normalized to the same \( n(z, M) \) at \( f_{NL}=0 \)

Preliminary
Can we hope to measure $f_{NL}$ with high-z halo counts?

- If our ansatz for the mapping relation is correct then it suggests that what matters is $\sigma(M,z)$ and not $M/M^*(z)$

- For example, high-z QSOs or galaxies, even though they have $M/M^*$ greater than 200 or so belong to $10^{12}$ halos and thus lead to a 2% effect for $f_{nl} \sim 300$ at $z=6$

- This is to be compared to a 25% effect at $z=1$ for a $10^{15}$ object
Comparison with previous prescriptions

\[ f_{NL} = +300 \]

\[ f_{NL} = -300 \]
Conclusions

- We propose a simple but accurate fitting formula for a generalized non-Gaussian mass function.

- The accuracy of this prescription is of order the uncertainties in the usual ΛCDM mass function, i.e. ~5%.

- Should be of direct relevance for future or current cluster count surveys like SZA, SPT, ACT, Planck.

- But still a small effect and limited by the accuracy in the mass measurement.
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