DBI in the low redshift sky

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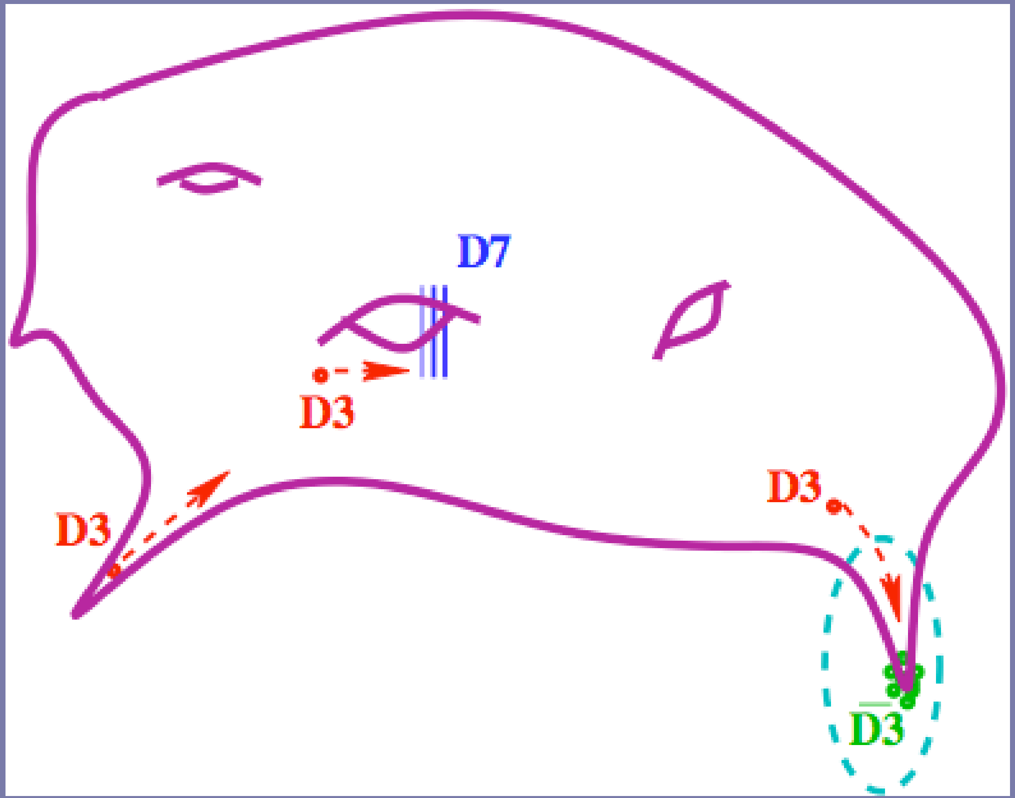
Outline

- Motivation for looking for non-Gaussianity in places other than the CMB
- Cluster number counts: deriving the non-Gaussian mass function
- Cluster number counts: the non-Gaussian mass function
- The bispectrum

Motivation: the DBI model



Motivation: the DBI model

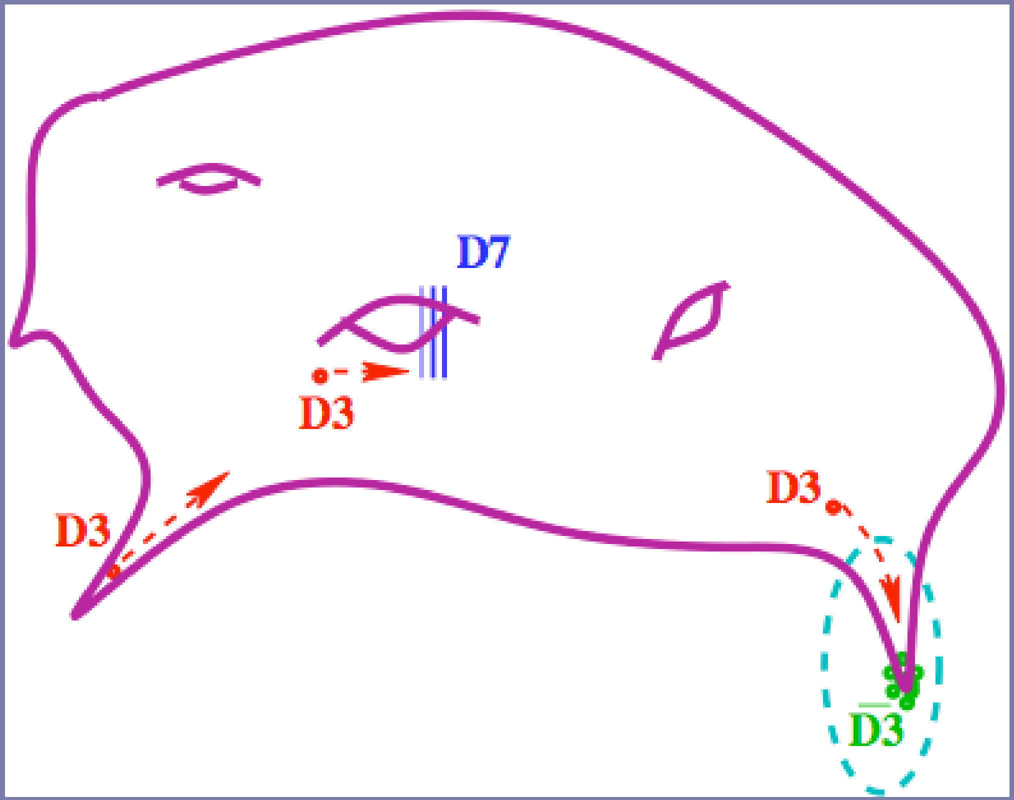
"speed limit"

$$\gamma(\phi) = \frac{1}{\sqrt{1 - \dot{\phi}^2 T^{-1}}}$$

sound speed

$$c_s = \frac{1}{\gamma}$$

slow roll equations modified $c \longrightarrow c_s$



The skewness in arbitrary sound speed models

$$\langle\zeta(\mathbf{k}_1)\zeta(\mathbf{k}_2)\zeta(\mathbf{k}_3)\rangle = (2\pi)^7\delta(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3)\frac{\mathcal{P}(K)^2}{k_1^3k_2^3k_3^3}\left(\mathcal{A}_\lambda+\mathcal{A}_c+\mathcal{A}_o+\mathcal{A}_\epsilon+\mathcal{A}_\eta+\mathcal{A}_s\right)$$

$$\langle\zeta(\mathbf{k}_1)\zeta(\mathbf{k}_2)\rangle = (2\pi)^3\delta(\mathbf{k}_1+\mathbf{k}_2)2\pi^2\mathcal{P}(k)k^{-3}$$

Chen, Huang, Kachru, Shiu (hep-th/0605045)

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primordial curvature

$$\delta(\mathbf{k}, a) \propto +\zeta(\mathbf{k})$$

Chen, Huang, Kachru, Shiu (hep-th/0605045)

The skewness in arbitrary sound speed models

model dependent functions of k_1 k_2 k_3

$$\langle \zeta(\mathbf{k}_1)\zeta(\mathbf{k}_2)\zeta(\mathbf{k}_3)\rangle = (2\pi)^7 \delta(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3) \frac{\mathcal{P}(K)^2}{k_1^3 k_2^3 k_3^3} (\mathcal{A}_\lambda + \mathcal{A}_c + \mathcal{A}_o + \mathcal{A}_\epsilon + \mathcal{A}_\eta + \mathcal{A}_s)$$

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$$\boxed{\delta(\mathbf{k}, a) \propto +\zeta(\mathbf{k})}$$

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The skewness in arbitrary sound speed models

model dependent functions of k_1 k_2 k_3

$K=k_1+k_2+k_3$

$$\langle \zeta(\mathbf{k}_1)\zeta(\mathbf{k}_2)\zeta(\mathbf{k}_3)\rangle = (2\pi)^7\delta(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3)\frac{\mathcal{P}(K)^2}{k_1^3k_2^3k_3^3}\left(\mathcal{A}_\lambda+\mathcal{A}_c+\mathcal{A}_o+\mathcal{A}_\epsilon+\mathcal{A}_\eta+\mathcal{A}_s\right)$$

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primordial curvature

$$\delta(\mathbf{k},a) \propto +\zeta(\mathbf{k})$$

Chen, Huang, Kachru, Shiu (hep-th/0605045)

The skewness in arbitrary sound speed models

$K = k_1 + k_2 + k_3$

model dependent functions of k_1 k_2 k_3

$$\langle \zeta(\mathbf{k}_1)\zeta(\mathbf{k}_2)\zeta(\mathbf{k}_3)\rangle = (2\pi)^7 \delta(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3) \frac{\mathcal{P}(K)^2}{k_1^3 k_2^3 k_3^3} (\mathcal{A}_\lambda + \mathcal{A}_c + \underbrace{\mathcal{A}_o + \mathcal{A}_\epsilon + \mathcal{A}_\eta + \mathcal{A}_s}_{})$$

proportional to slow roll parameters

Chen, Huang, Kachru, Shiu (hep-th/0605045)

The skewness in arbitrary sound speed models

K=$k_1+k_2+k_3$

model dependent functions of k_1 k_2 k_3

$$\langle\zeta(\mathbf{k}_1)\zeta(\mathbf{k}_2)\zeta(\mathbf{k}_3)\rangle = (2\pi)^7\delta(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3)\frac{\mathcal{P}(K)^2}{k_1^3k_2^3k_3^3}\left(\mathcal{A}_\lambda + \mathcal{A}_c + \underbrace{\mathcal{A}_o + \mathcal{A}_\epsilon + \mathcal{A}_\eta + \mathcal{A}_s}_{\text{proportional to slow roll parameters}}\right)$$

vanishes for DBI

proportional to slow roll parameters

Chen, Huang, Kachru, Shiu (hep-th/0605045)

The skewness in arbitrary sound speed models

$K=k_1+k_2+k_3$

model dependent functions of k_1 k_2 k_3

$$\langle\zeta(\mathbf{k}_1)\zeta(\mathbf{k}_2)\zeta(\mathbf{k}_3)\rangle = (2\pi)^7\delta(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3)\frac{\mathcal{P}(K)^2}{k_1^3k_2^3k_3^3}\left(\mathcal{A}_\lambda + \mathcal{A}_c + \underbrace{\mathcal{A}_o + \mathcal{A}_\epsilon + \mathcal{A}_\eta + \mathcal{A}_s}\right)$$

proportional to slow roll parameters

dominant term

vanishes for DBI

Chen, Huang, Kachru, Shiu (hep-th/0605045)

Our starting point

$$\langle\zeta(\mathbf{k}_1)\zeta(\mathbf{k}_2)\zeta(\mathbf{k}_3)\rangle = (2\pi)^7\delta(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3)\mathcal{P}^2(K)\frac{\mathcal{A}_c(k_1,k_2,k_3)}{k_1^3k_2^3k_3^3}$$

$$\mathcal{A}_c(k_1,k_2,k_3) \propto -\left(\frac{1}{c_s^2(K)}-1\right)$$

$$c_s(k) \propto k^\kappa$$

Our starting point

$$\langle\zeta(\mathbf{k}_1)\zeta(\mathbf{k}_2)\zeta(\mathbf{k}_3)\rangle = (2\pi)^7\delta(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3)\mathcal{P}^2(K)\frac{\mathcal{A}_c(k_1,k_2,k_3)}{k_1^3k_2^3k_3^3}$$

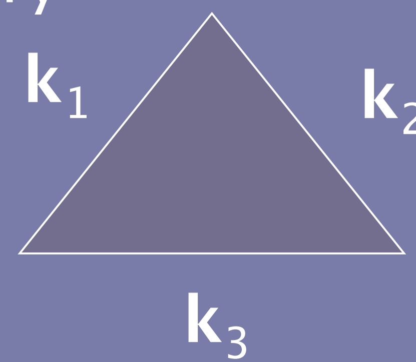
$$\mathcal{A}_c(k_1,k_2,k_3) \propto -\left(\frac{1}{c_s^2(K)}-1\right)$$

κ < 0, slow roll parameter

$$c_s(k) \propto k^{\kappa}$$

The shape?

$$\langle \zeta(\mathbf{k}_1)\zeta(\mathbf{k}_2)\zeta(\mathbf{k}_3)\rangle = (2\pi)^7 \delta(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3)\mathcal{P}^2(K)\underbrace{\frac{\mathcal{A}_c(k_1, k_2, k_3)}{k_1^3 k_2^3 k_3^3}}_{\text{maximal for equilateral geometry}}$$

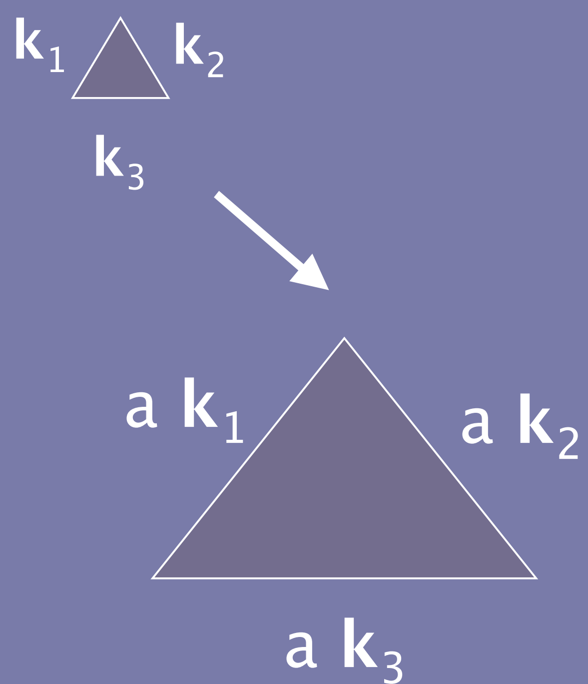


Babich, Creminelli, Zaldarriaga astro-ph/0405356

The running?

$$\langle \zeta(\mathbf{k}_1)\zeta(\mathbf{k}_2)\zeta(\mathbf{k}_3)\rangle = (2\pi)^7 \delta(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3)\mathcal{P}^2(K)\underbrace{\frac{\mathcal{A}_c(k_1,k_2,k_3)}{k_1^3 k_2^3 k_3^3}}$$

If



$$a^{-6+2(n_s-1)-2\kappa}\mathcal{P}^2(K)\frac{\mathcal{A}_c(k_1,k_2,k_3)}{k_1 k_2 k_3}$$

Babich, Creminelli, Zaldarriaga astro-ph/0405356

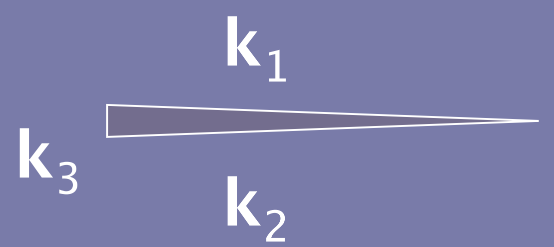
How does this compare to f_{NL} models?

$$\zeta_{NG} = \zeta_G - \frac{3}{5} f_{NL} \left(\zeta_G^2 - \langle \zeta_G^2 \rangle \right)$$

⬇

$$\langle \zeta(\mathbf{k}_1)\zeta(\mathbf{k}_2)\zeta(\mathbf{k}_3)\rangle = (2\pi)^3 \delta(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3)(2\pi^2)^2 \left[\frac{\mathcal{P}_\zeta(k_1)\mathcal{P}_\zeta(k_2)}{k_1^3 k_2^3} + (perms) \right]$$

maximal for squeezed triangles



Babich et al astro-ph/0405356

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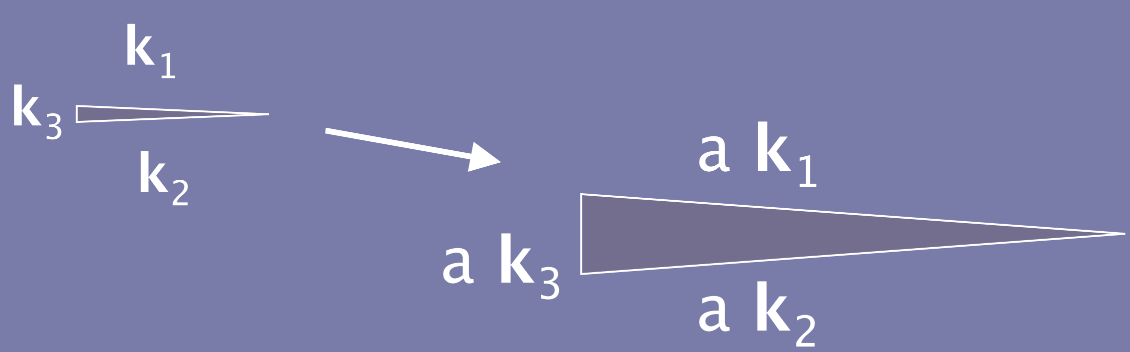


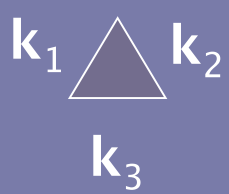
scales as

$$a^{-6+2(n_s-1)}$$

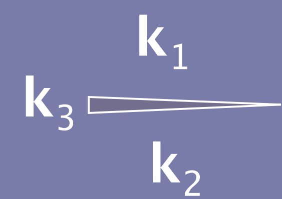
How does this compare to f_{NL} models?

geometry dependence is different

sound speed:



f_{NL}



scale dependence is different

sound speed:

$$a^{-6+2(n_s-1)-2\kappa}$$

f_{NL}

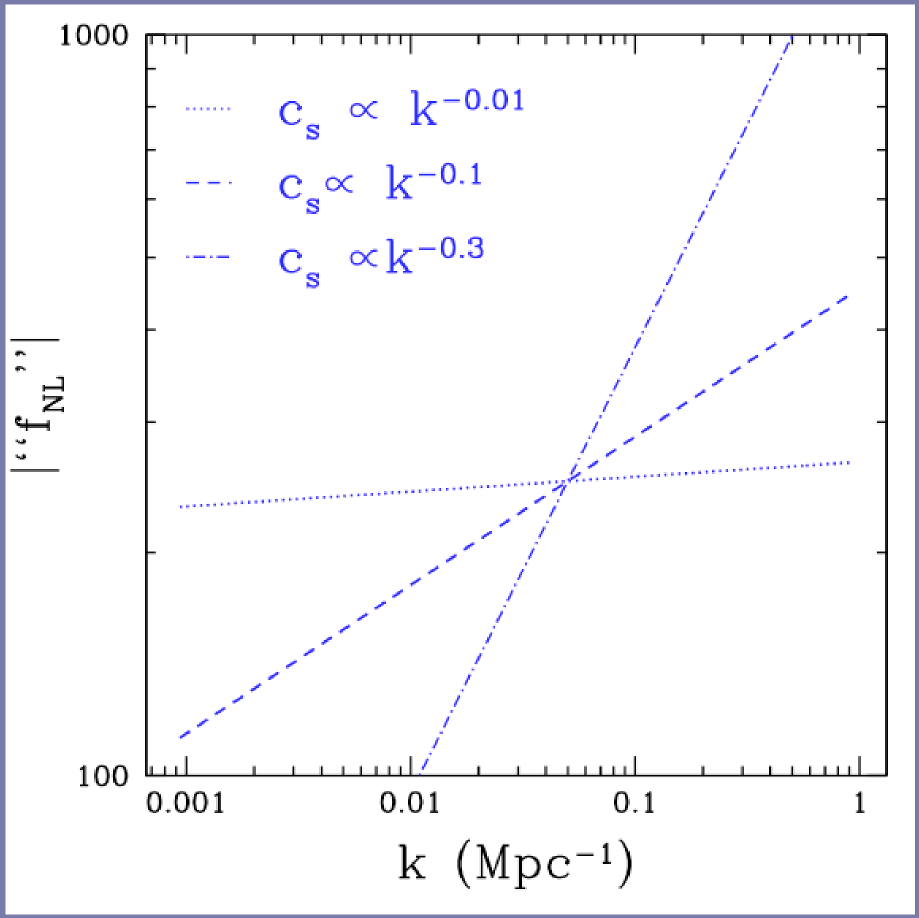
$$a^{-6+2(n_s-1)}$$

Babich et al astro-ph/0405356

Constraining non-Gaussianity

Scale dependent non-Gaussianity

$$\text{“}f_{NL}\text{”} = -\frac{35}{108}\left(\frac{1}{c_s^2} - 1\right)$$



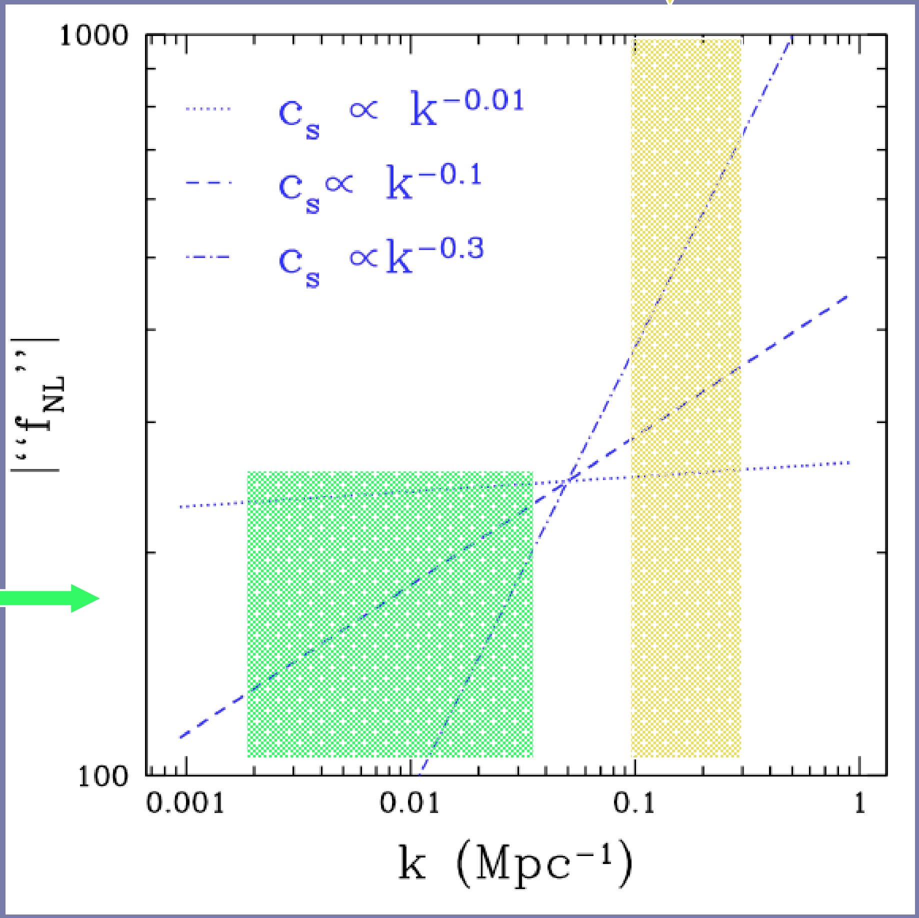
Constraining non-Gaussianity

cluster scales

Scale dependent non-Gaussianity

$$\text{“}f_{NL}\text{”} = -\frac{35}{108}\left(\frac{1}{c_s^2} - 1\right)$$

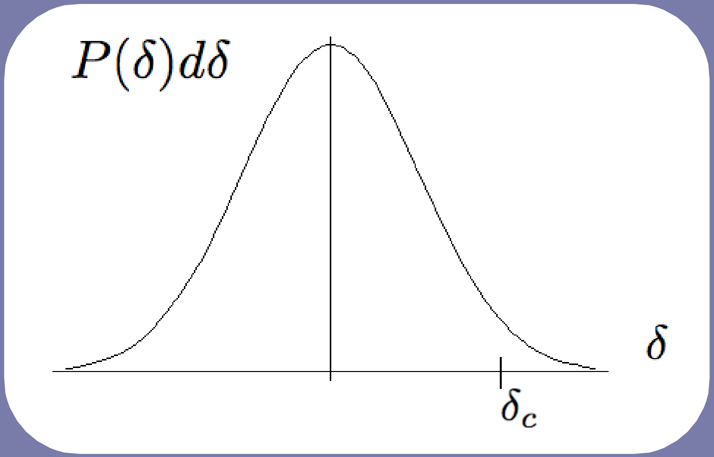
allowed by current CMB



Creminelli, Senatore, Zaldarriaga, Tegmark 2006
Scoccimarro, Sefusatti, Zaldarriaga (2004)

how do clusters know about non-Gaussianity?

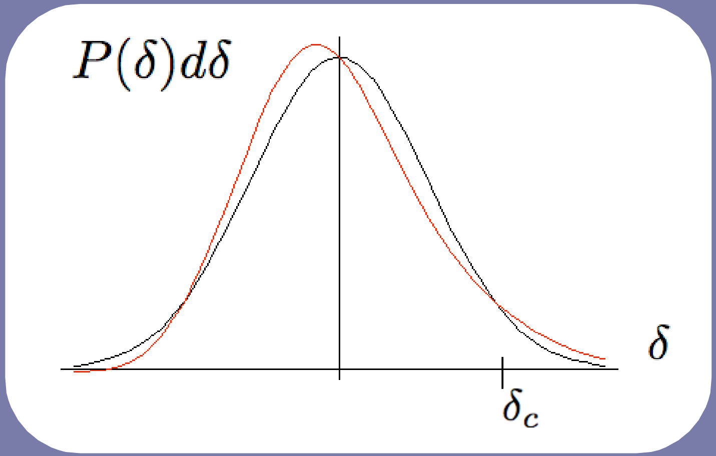
Clusters form when the (smoothed) density fluctuation is above a threshold δ_c



how do clusters know about non-Gaussianity?

positive skewness

→ more collapsed objects



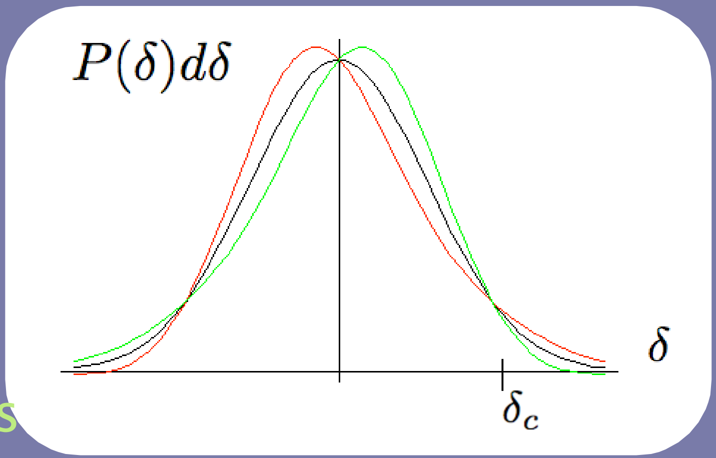
how do clusters know about non-Gaussianity?

positive skewness

→ more collapsed objects

negative skewness

→ fewer collapsed objects



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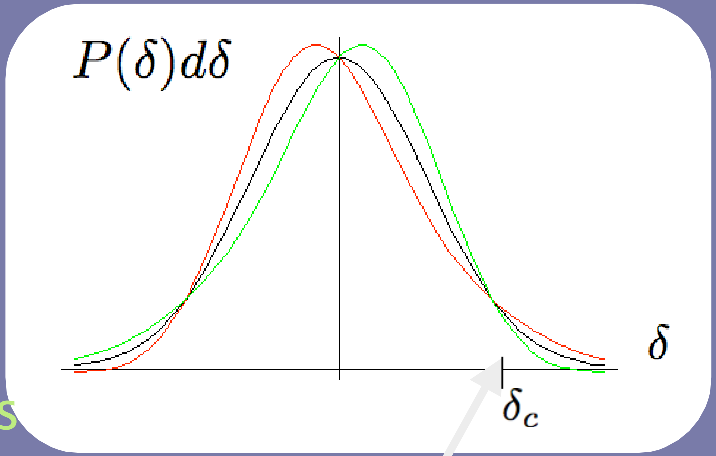


The number of clusters tells us about the tail of the probability distribution function

Cluster counts

$P(\delta_R)d\delta_R$

↓

$n(M)dM$ – number density of clusters with mass between M and M+dM

↓

dN/dz – number of clusters per redshift interval dz with mass bigger than M_{lim}

Recipe for n(M)

- Recover the probability distribution function (PDF) from $\langle \delta^3(R) \rangle$
- Follow Press-Schechter approach

$$f_{collapse}(M) = 2\int_{\delta_c}^{\infty} d\delta\, P(\delta, M) \Rightarrow \frac{dn}{dM} = -\frac{\bar{\rho}}{M}\frac{df}{dM}$$

- Use $n_{NG}(M)/n_G(M)$ where each is found from above

Press & Schechter 1974 Robinson & Baker astro-ph/9905098
Mattarese, Verde, Jimenez astro-ph/0001366

Caveats/Difficulties

- We recover an approximate p.d.f
  → limited range (in mass and redshift) of validity
- This approach will indicate the utility of cluster counts. To make precise predictions simulations will need to be done

Recovering the p.d.f.

The Edgeworth expansion

$$P(\delta)d\delta = \frac{d\delta}{\sqrt{2\pi}\sigma} e^{-\frac{\delta^2}{2\sigma^2}} \left[1 + \frac{S_3\sigma}{3!} H_3\left(\frac{\delta}{\sigma}\right) \right.$$

$$\left. + \frac{1}{2}\left(\frac{S_3\sigma}{3!}\right)^2 H_6\left(\frac{\delta}{\sigma}\right) + \frac{S_4\sigma^2}{4!} H_4\left(\frac{\delta}{\sigma}\right) + \dots \right.$$

Hermite Polynomials

$$S_n = \frac{\langle \delta^n \rangle_c}{\langle \delta^2 \rangle^{n-1}}$$

The mass function

Gaussian case (Press-Schechter)-

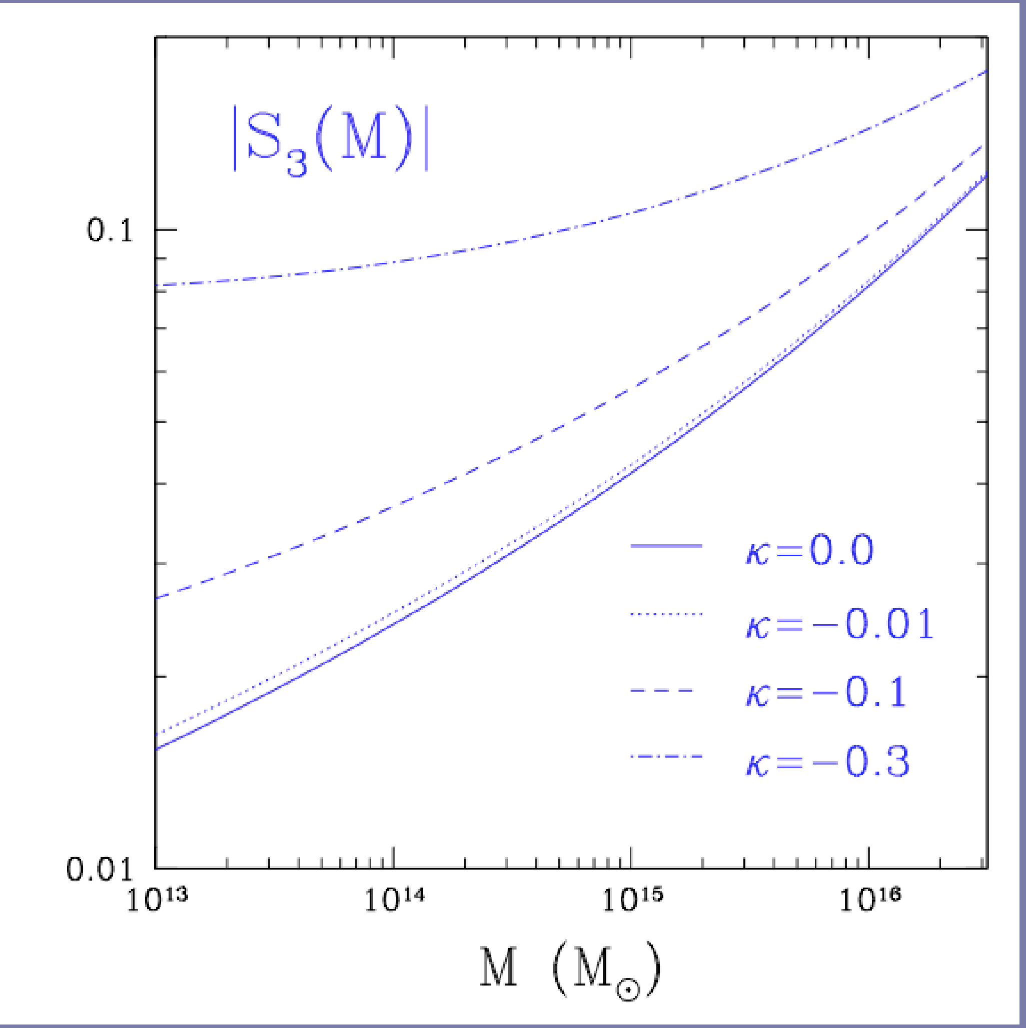
$$\frac{dn(M)}{dM} = -\sqrt{\frac{2}{\pi}}\frac{\bar{\rho}}{M}e^{-\frac{\delta_c^2}{2\sigma(M)^2}}\frac{d\ln\sigma}{dM}\frac{\delta_c}{\sigma}$$

The non-Gaussian mass function

$$\frac{dn(M)}{dM} = -\sqrt{\frac{2}{\pi}}\frac{\bar{\rho}}{M}e^{-\frac{\delta_c^2}{2\sigma(M)^2}}\left[\frac{d\ln\sigma}{dM}\left(\frac{\delta_c}{\sigma}+\frac{S_3\sigma}{6}\left(\frac{\delta_c^4}{\sigma^4}-2\frac{\delta_c^2}{\sigma^2}-1\right)\right)+\frac{1}{6}\frac{dS_3}{dM}\sigma\left(\frac{\delta_c^2}{\sigma^2}-1\right)\right]$$

δ_c (z) = collapse threshold - increases with redshift

$$S_3(M) = \frac{\langle \delta_M^3 \rangle}{\langle \delta_M^2 \rangle^2}$$



Use flat ΛCDM with

Ω_m=0.27

Ω_b=0.024/h^2

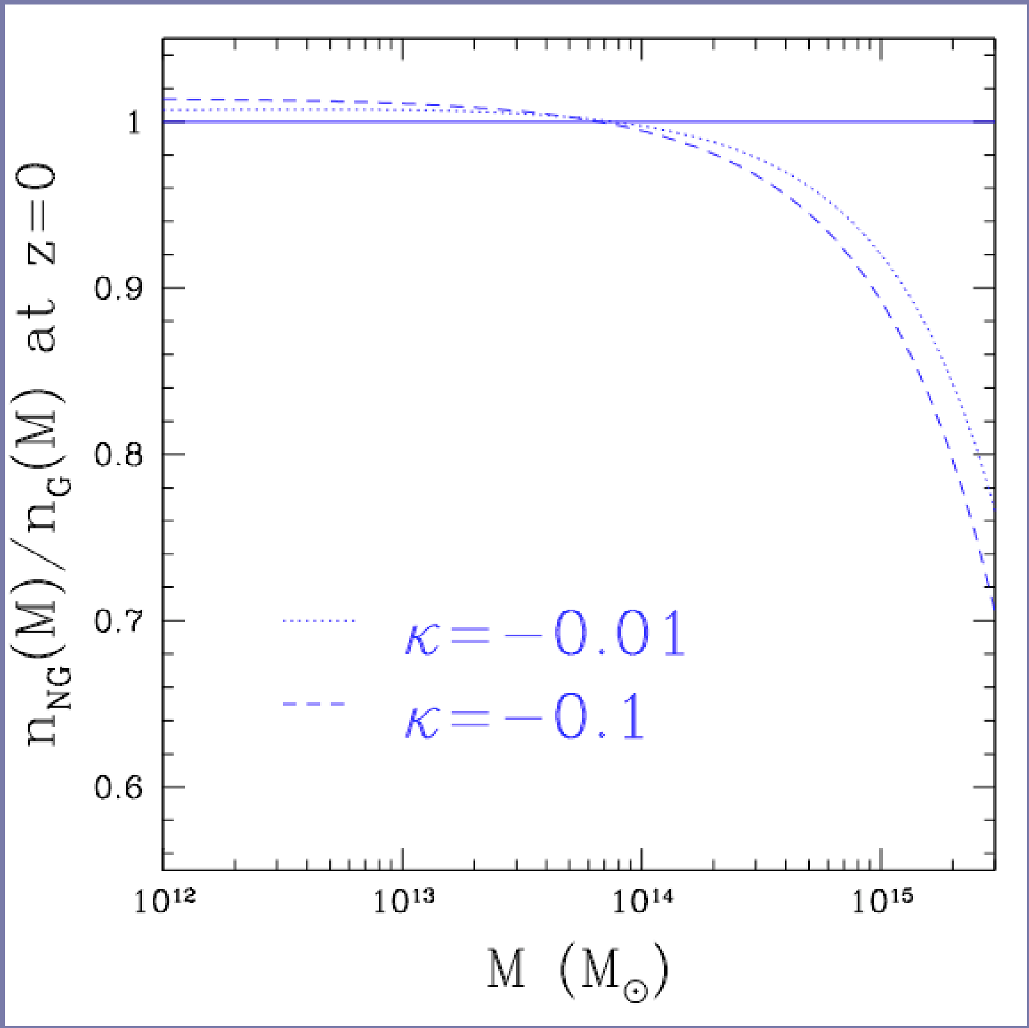
h=0.7

σ8=0.8

n_s=0.95

Smooth by Gaussian window function

The change in the mass function



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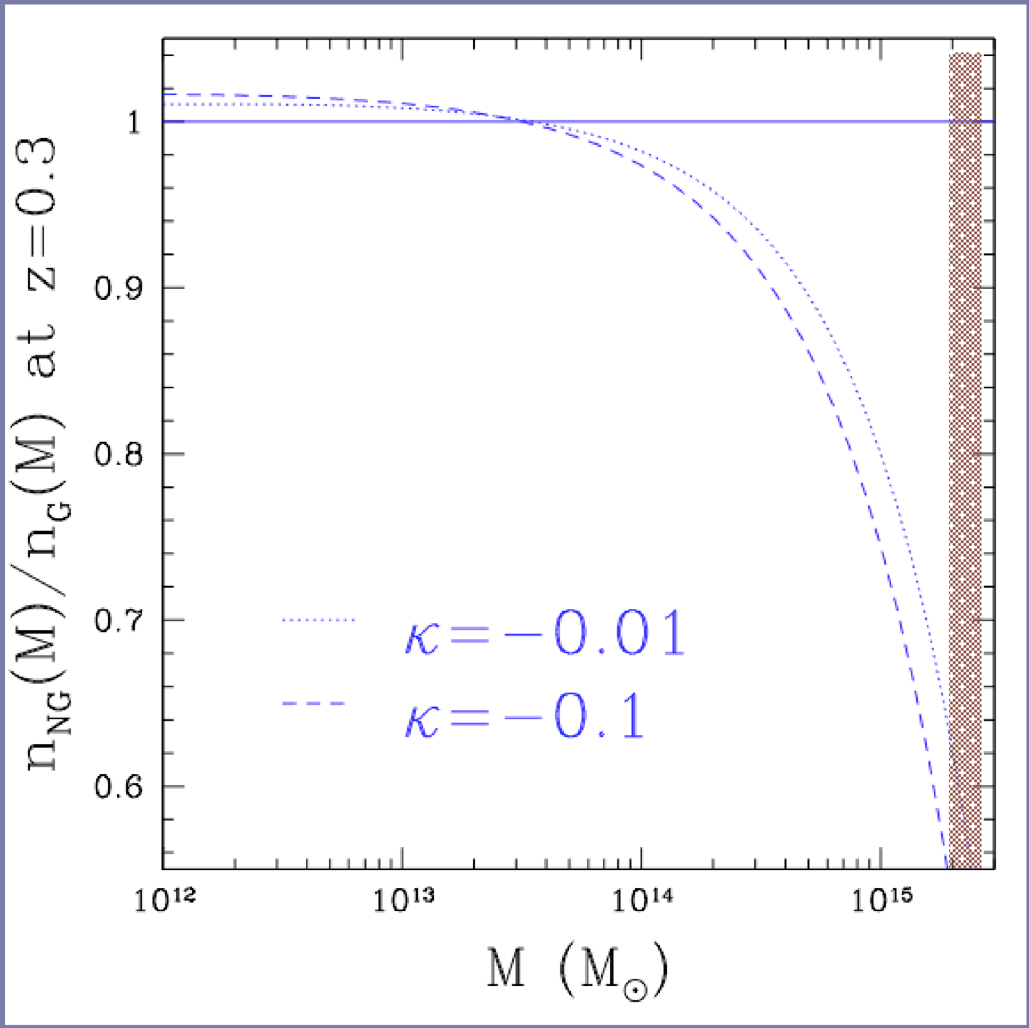
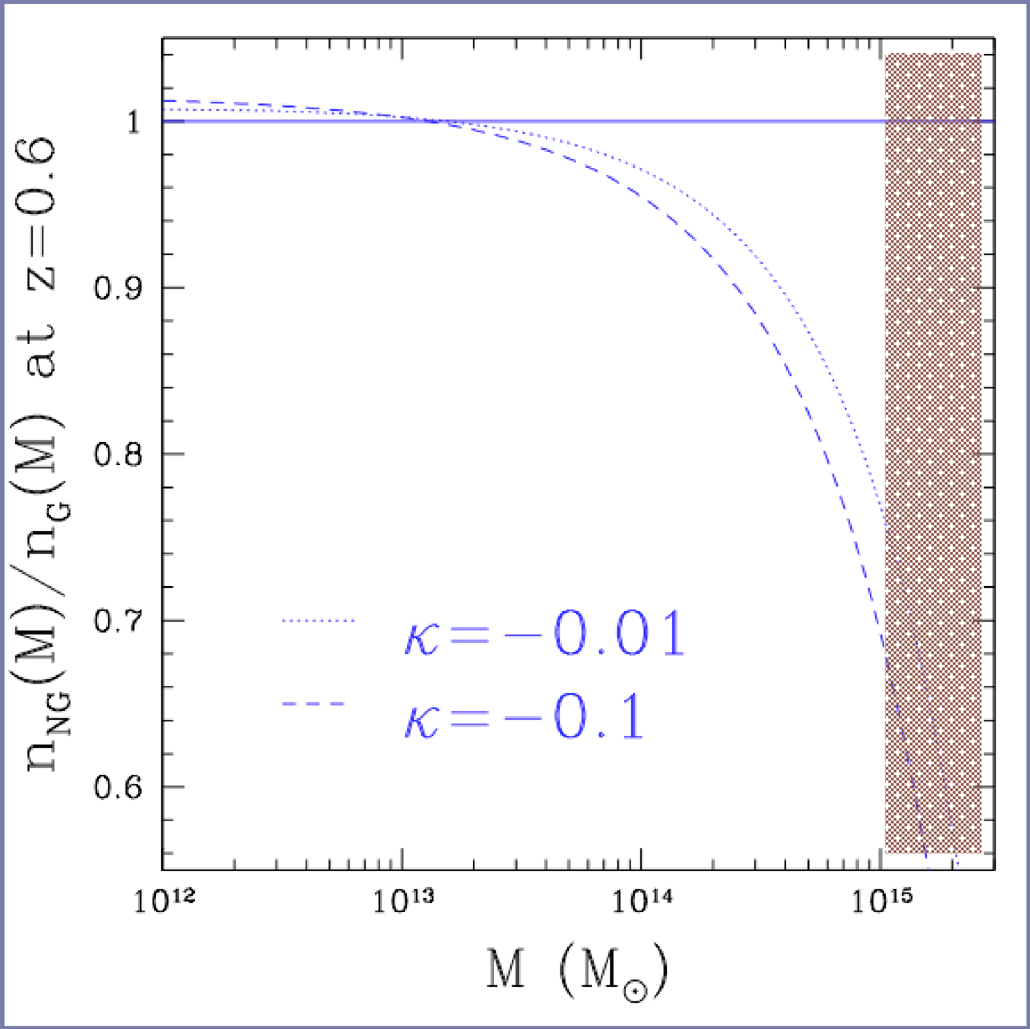


worry about expansion

The change in the mass function



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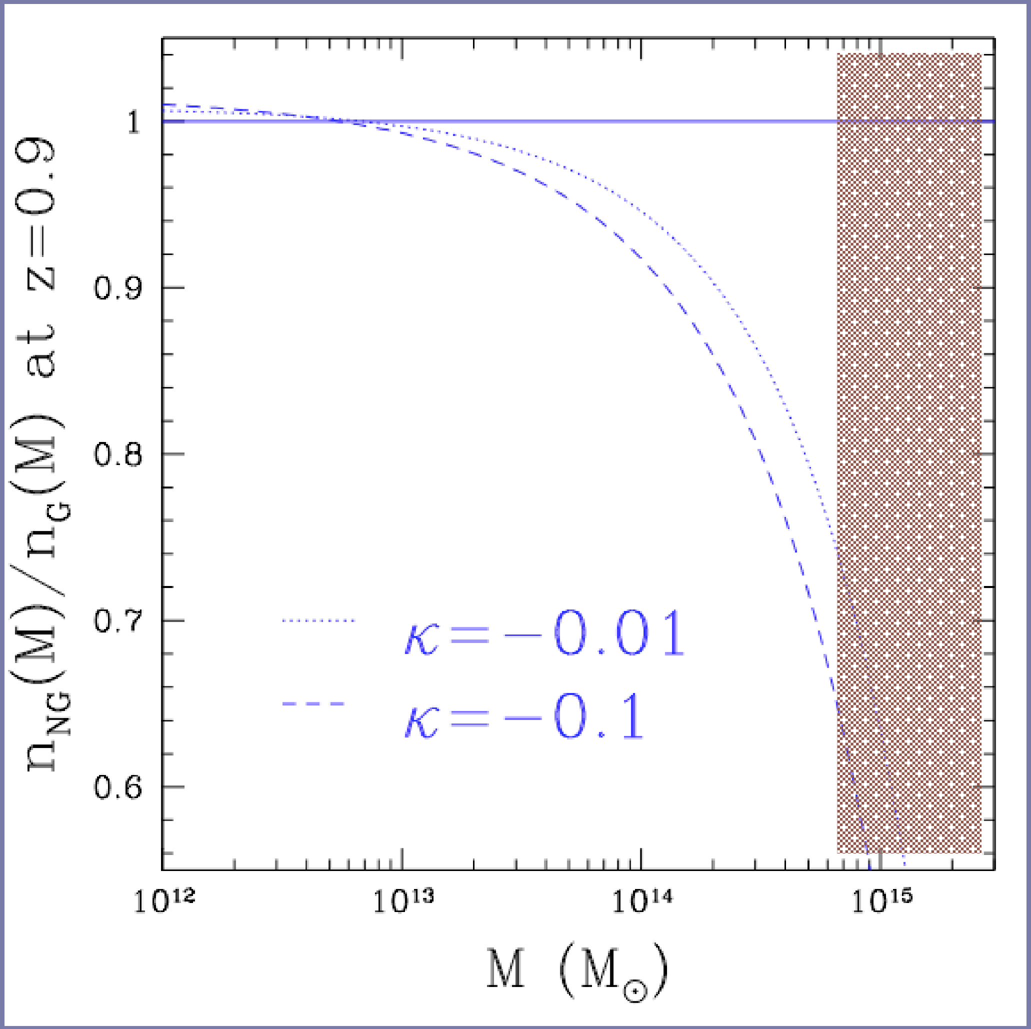
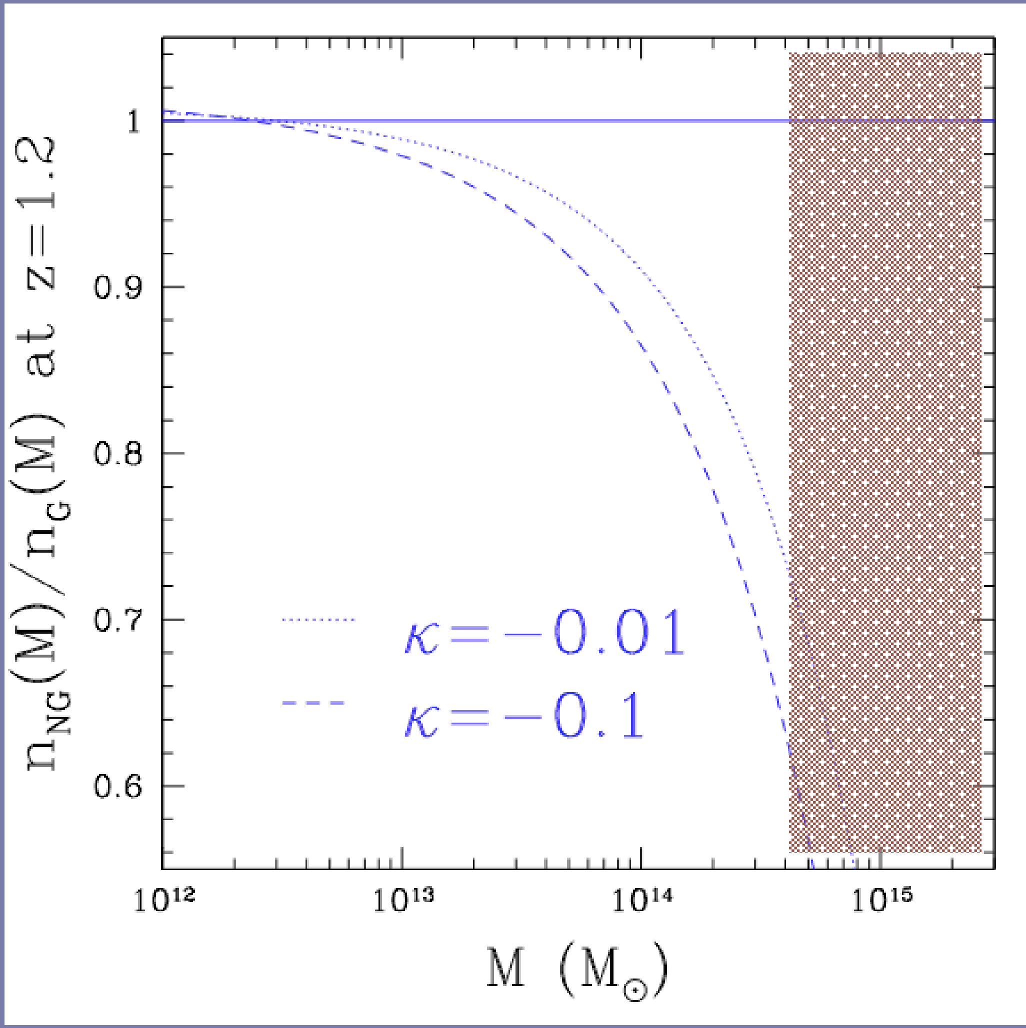


The change in the mass function

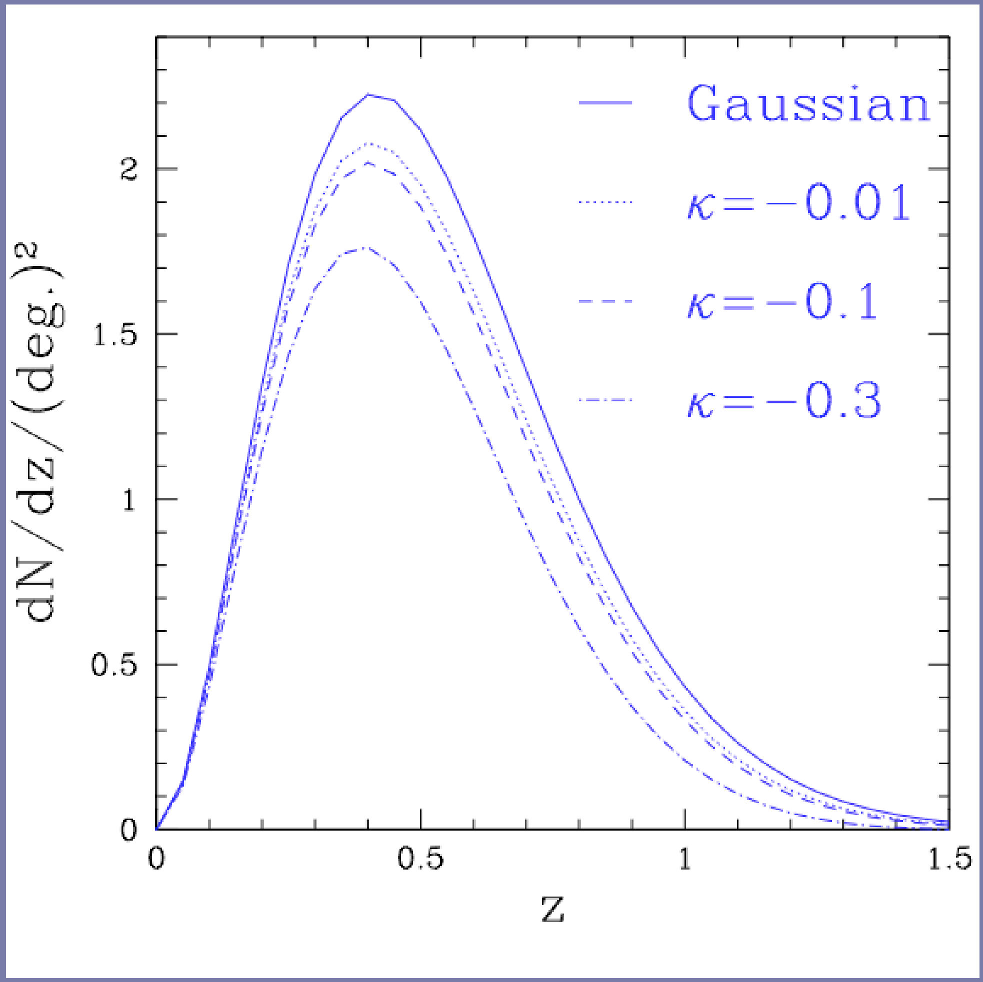


dN/dz

What we get with Press-Schechter

assuming $M_{lim}=2.5 \times 10^{14}\ M_{sun}$

(ind. of z)



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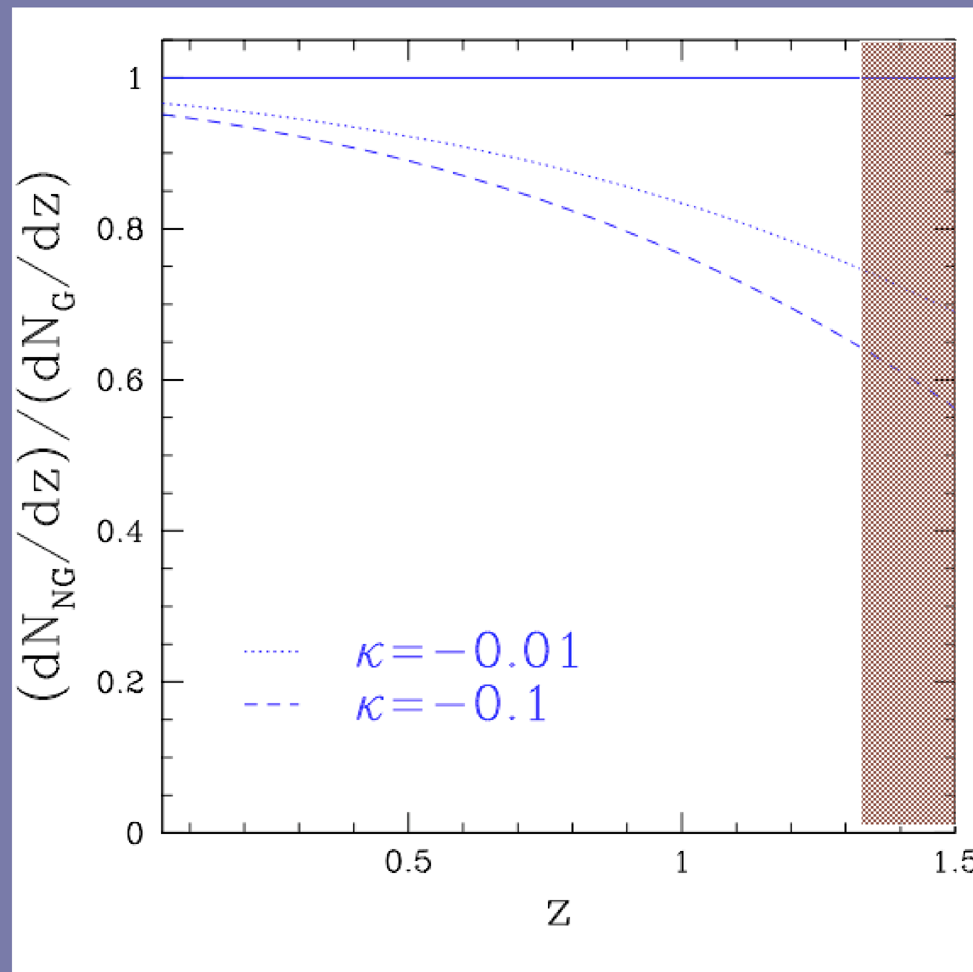

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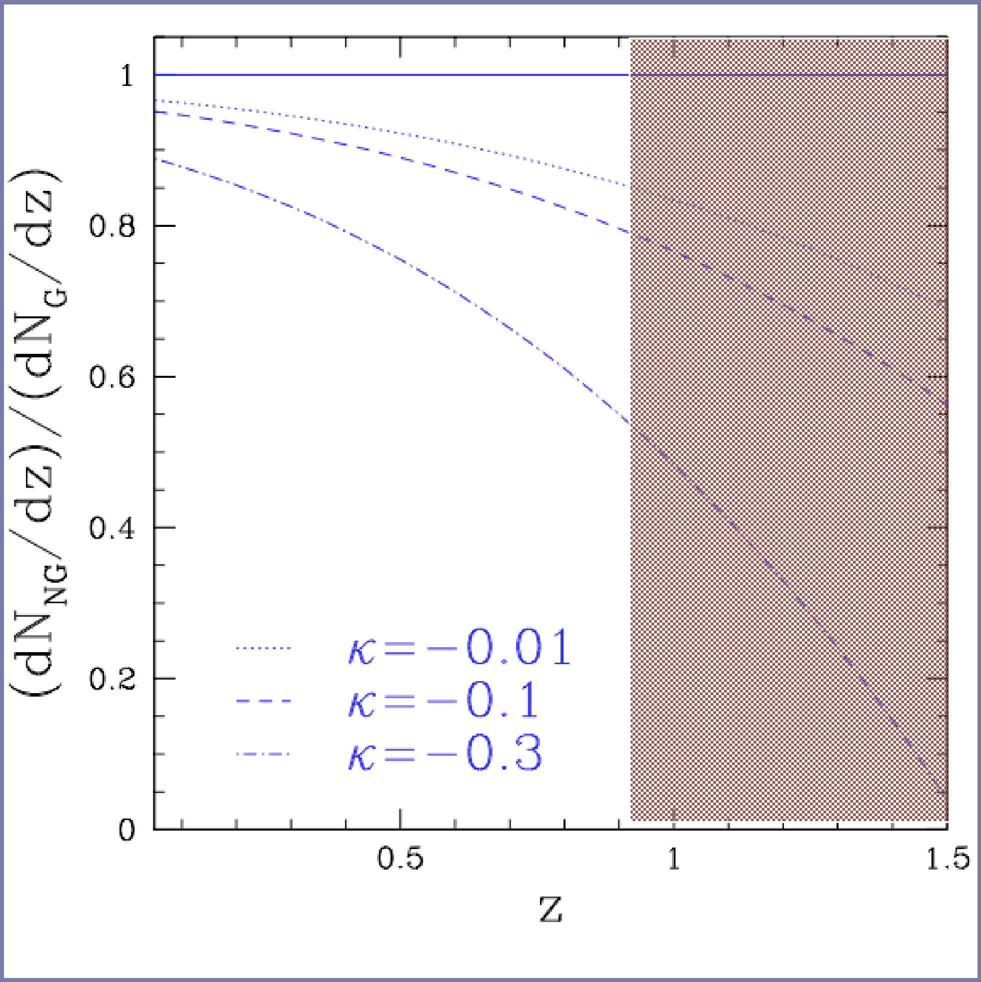
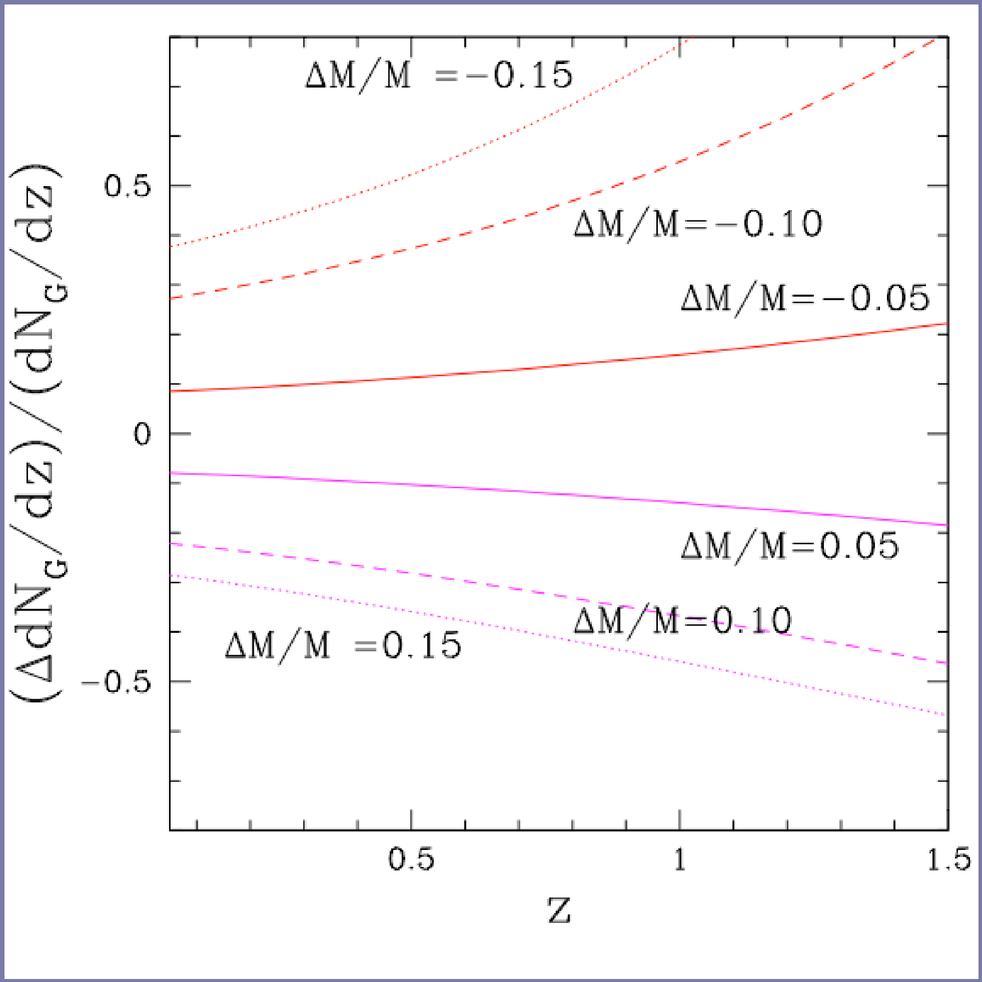


dN/dz

BUT, the mass limit isn't perfectly known

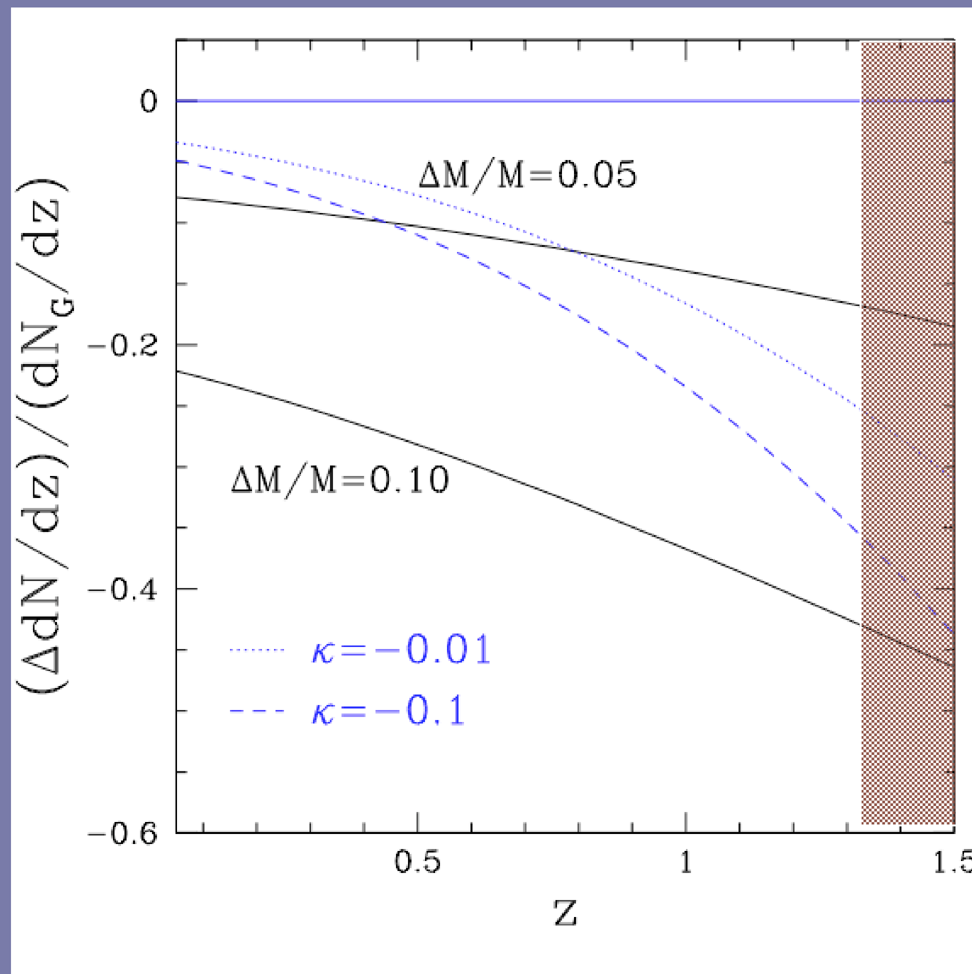
$$\Delta dN/dz = dN/dz(M_{lim} + \Delta M)$$
$$-dN/dz(M_{lim})$$



dN/dz

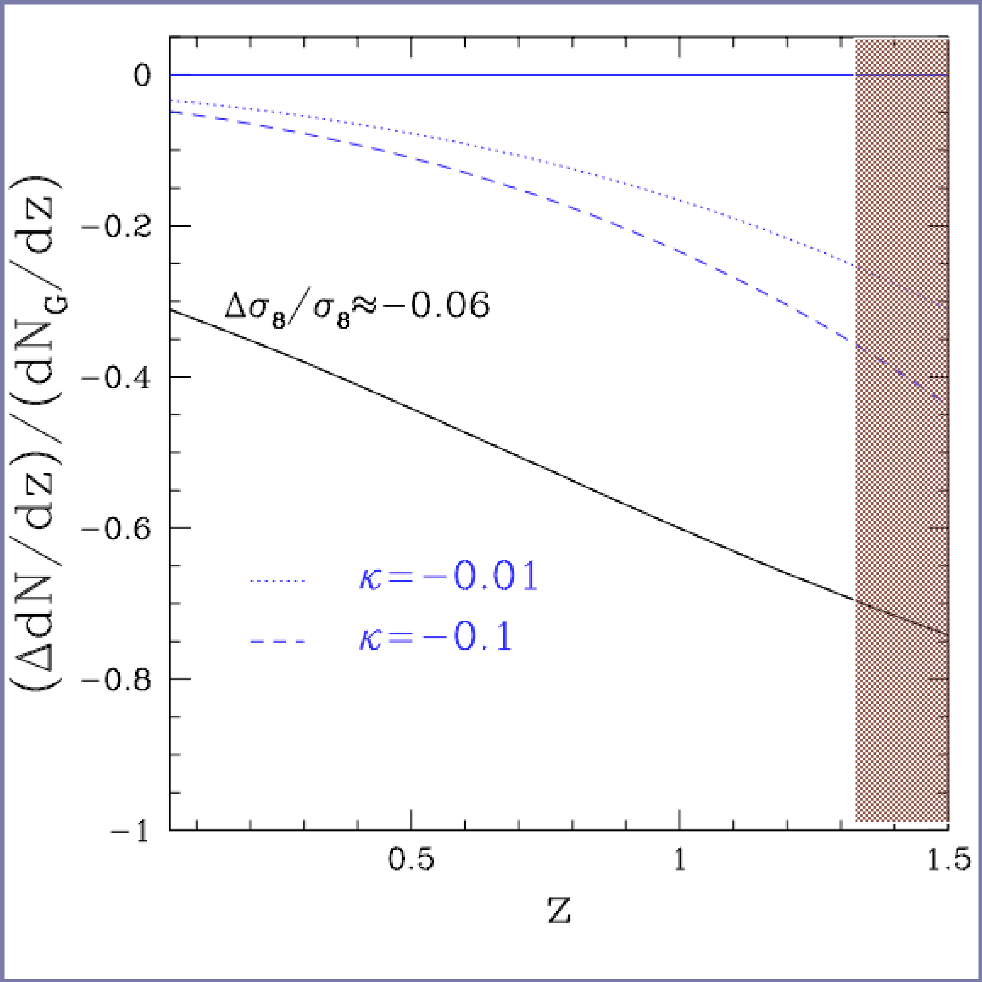
BUT, the mass limit isn't perfectly known

mass uncertainty must by small (~5%) for this to work

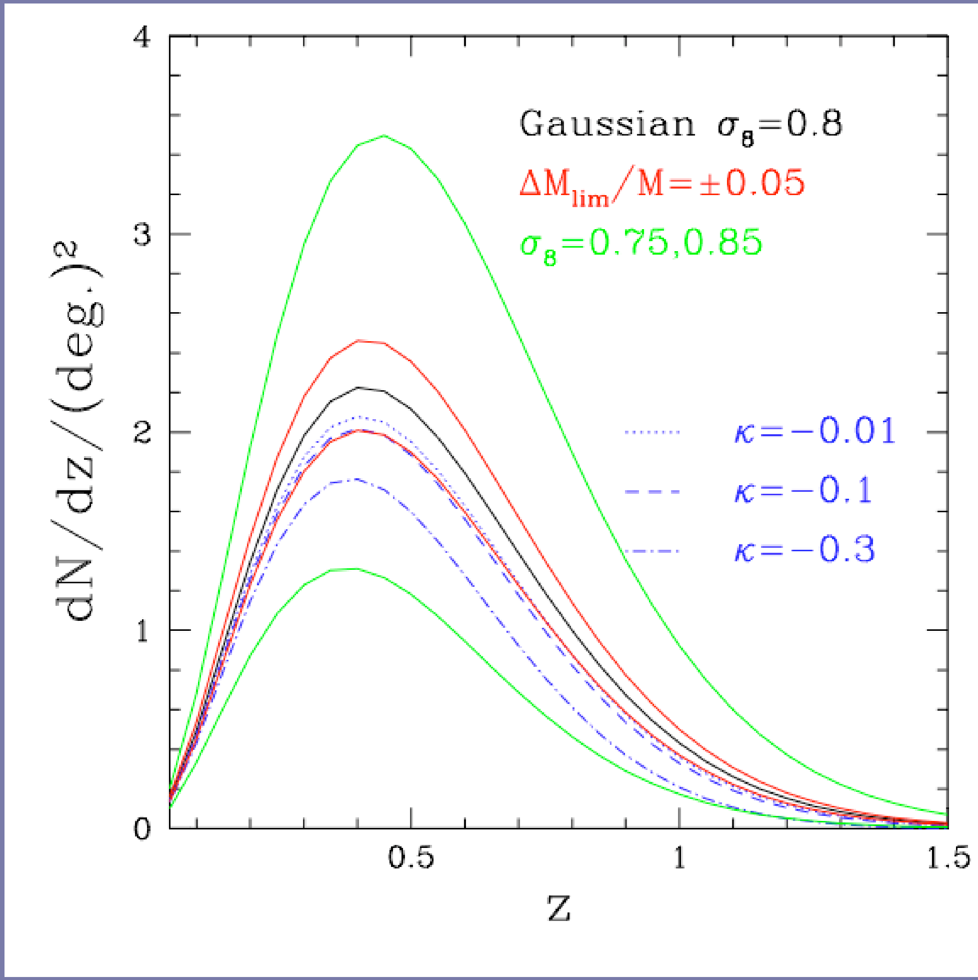


dN/dz

What about σ_8 ?



dN/dz



see also Sefusatti et al astro-ph/0609124

Other methods

The Bispectrum

In principle the bispectrum contains more information because it retains the dependence on k_1, k_2, k_3

Unlike the mass function, the bispectrum doesn’t rely on knowledge of higher order cumulants

The bispectrum avoids the issue of expanding the pd.f.

The bispectrum

$$\langle \delta(\mathbf{k}_1)\delta(\mathbf{k}_2)\delta(\mathbf{k}_3)\rangle = (2\pi)^3\delta(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3)B(k_1,k_2,k_3)$$

$$B(k_1,k_2,k_3) = B_I(k_1,k_2,k_3) + B_G(k_1,k_2,k_3)$$

due to initial non-Gaussianity

evolved non-Gaussianity due to non-linear evolution

The bispectrum

$$\langle \delta(\mathbf{k}_1)\delta(\mathbf{k}_2)\delta(\mathbf{k}_3)\rangle = (2\pi)^3\delta(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3)B(k_1,k_2,k_3)$$

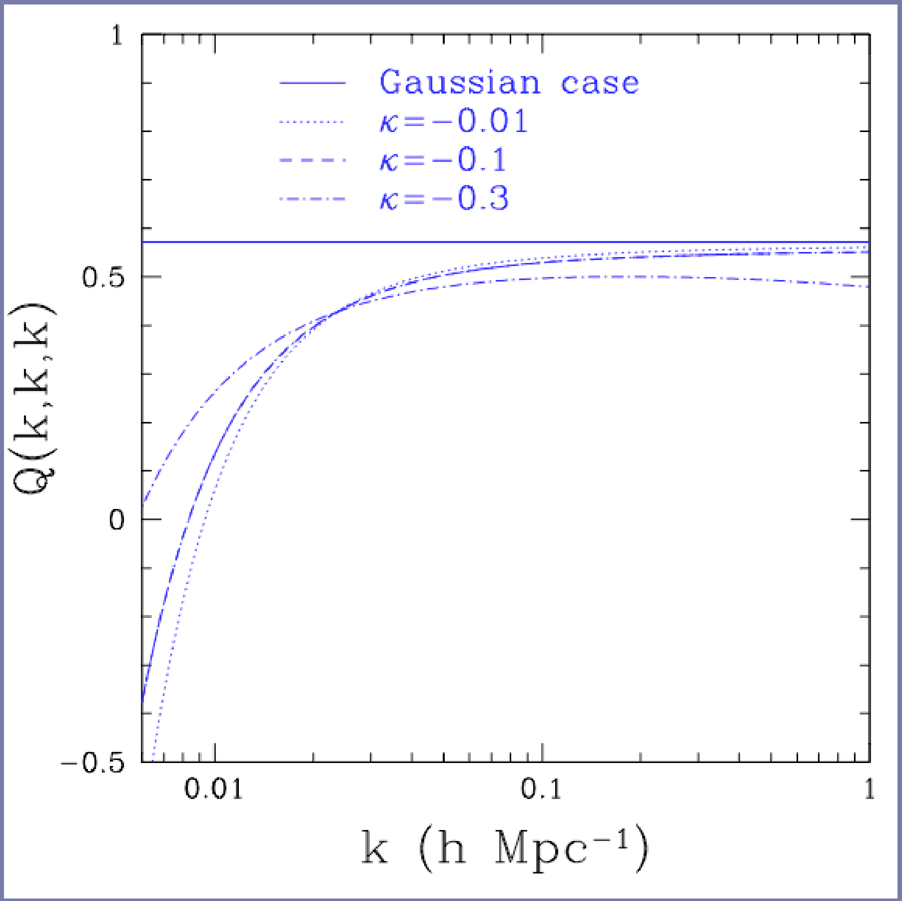
$$B(k_1,k_2,k_3) = B_I(k_1,k_2,k_3) + B_G(k_1,k_2,k_3)$$

$$Q(k_1,k_2,k_3) = \frac{B(k_1,k_2,k_3)}{P(k_1)P(k_2)+P(k_1)P(k_3)+P(k_2)P(k_3)}$$

The bispectrum

$$Q_G(k,k,k) = \frac{4}{7}$$

$$Q_I(k,k,k) \sim \frac{1}{c_s^2}\frac{1}{k^2 T(k)} \sim \frac{k^{-2\kappa}}{k^2 T(k)}$$



transfer function

$T(k/k_{eq} << 1) \sim 1$
$T(k/k_{eq} >> 1) \sim \ln(k)/k^2$

See also Sefusatti & Komatsu astro-ph/07050343

Conclusions

- Models of inflation with $c_s \neq 1$ produce scale dependent non-Gaussianty
- We have explored possibilities beyond the CMB for putting constraints on this non-Gaussianity
- Cluster number counts constrain non-Gaussianity at a different scale than CMB and can provide a cross-check to CMB constraints. Simulations will need to be done to make precise constraints.