DBI in the low redshift sky

Marilena LoVerde, Amber Miller, Sarah Shandera, Licia Verde
Columbia University & University of Pennsylvania
Outline

- Motivation for looking for non-Gaussianity in places other than the CMB
- Cluster number counts: deriving the non-Gaussian mass function
- Cluster number counts: the non-Gaussian mass function
- The bispectrum
Motivation: the DBI model
Motivation: the DBI model

“speed limit”

\[ \gamma(\phi) = \frac{1}{\sqrt{1 - \dot{\phi}^2 T^{-1}}} \]

sound speed

\[ c_s = \frac{1}{\gamma} \]

slow roll equations modified \( c \rightarrow c_s \)
The skewness in arbitrary sound speed models

\[ \langle \zeta(k_1)\zeta(k_2)\zeta(k_3) \rangle = (2\pi)^7 \delta(k_1+k_2+k_3) \frac{\mathcal{P}(K)^2}{k_1^3k_2^3k_3^3} (A_\lambda + A_c + A_o + A_\epsilon + A_\eta + A_s) \]

\[ \langle \zeta(k_1)\zeta(k_2) \rangle = (2\pi)^3 \delta(k_1+k_2) 2\pi^2 \mathcal{P}(k) k^{-3} \]

Chen, Huang, Kachru, Shiu (hep-th/0605045)
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\[ \langle \zeta(k_1)\zeta(k_2) \rangle = (2\pi)^3 \delta(k_1 + k_2) 2\pi^2 \mathcal{P}(k) k^{-3} \]

primordial curvature \[ \delta(k, a) \propto + \zeta(k) \]

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proportional to slow roll parameters

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K = k_1 + k_2 + k_3

model dependent functions of k_1 k_2 k_3

proportional to slow roll parameters

vanishes for DBI

Chen, Huang, Kachru, Shiu (hep-th/0605045)
The skewness in arbitrary sound speed models

\[ \langle \zeta(k_1)\zeta(k_2)\zeta(k_3) \rangle = (2\pi)^7 \delta(k_1+k_2+k_3) \frac{P(K)^2}{k_1^3k_2^3k_3^3} (A_\lambda - A_c + A_o + A_\epsilon + A_\eta + A_s) \]

K = k_1 + k_2 + k_3

model dependent functions of k_1 k_2 k_3

dominant term

vanishes for DBI

proportional to slow roll parameters

Chen, Huang, Kachru, Shiu (hep-th/0605045)
Our starting point

\[ \langle \zeta(k_1)\zeta(k_2)\zeta(k_3) \rangle = (2\pi)^7 \delta(k_1 + k_2 + k_3) \mathcal{P}^2(K) \frac{A_c(k_1, k_2, k_3)}{k_1^3k_2^3k_3^3} \]

\[ A_c(k_1, k_2, k_3) \propto - \left( \frac{1}{c_s^2(K)} - 1 \right) \]

\[ c_s(k) \propto k^\kappa \]
Our starting point

\[
\langle \zeta(k_1)\zeta(k_2)\zeta(k_3) \rangle = (2\pi)^7 \delta(k_1 + k_2 + k_3) P^2(K) \frac{A_c(k_1, k_2, k_3)}{k_1^3 k_2^3 k_3^3}
\]

\[
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\]

\[
c_s(k) \propto k^\kappa
\]

\(\kappa < 0,\) slow roll parameter
The shape?

\[ \langle \zeta(k_1)\zeta(k_2)\zeta(k_3) \rangle = (2\pi)^7 \delta(k_1 + k_2 + k_3) \mathcal{P}^2(K) \frac{A_c(k_1, k_2, k_3)}{k_1^3 k_2^3 k_3^3} \]

maximal for equilateral geometry

Babich, Creminelli, Zaldarriaga astro-ph/0405356
The running?

\[ \langle \zeta(k_1)\zeta(k_2)\zeta(k_3) \rangle = (2\pi)^7 \delta(k_1 + k_2 + k_3) P^2(K) \frac{A_c(k_1, k_2, k_3)}{k_1^3 k_2^3 k_3^3} \]

If \( k_1 \triangle k_2 \)

\[ a^{-6 + 2(n_s - 1) - 2\kappa} P^2(K) \frac{A_c(k_1, k_2, k_3)}{k_1 k_2 k_3} \]

Babich, Creminelli, Zaldarriaga astro-ph/0405356
How does this compare to $f_{\text{NL}}$ models?

\[ \zeta_{NG} = \zeta_G - \frac{3}{5} f_{NL} (\zeta_G^2 - \langle \zeta_G^2 \rangle) \]

\[ \langle \zeta(k_1)\zeta(k_2)\zeta(k_3) \rangle = (2\pi)^3 \delta(k_1 + k_2 + k_3) (2\pi)^2 \left[ \frac{P_\zeta(k_1)P_\zeta(k_2)}{k_1^3 k_2^3} + (\text{perms}) \right] \]

maximal for squeezed triangles

Babich et al astro-ph/0405356
How does this compare to f_{NL} models?

\[ \zeta_{NG} = \zeta_G - \frac{3}{5} f_{NL} \left( \zeta_G^2 - \langle \zeta_G^2 \rangle \right) \]

\[ \langle \zeta(k_1)\zeta(k_2)\zeta(k_3) \rangle = (2\pi)^3 \delta(k_1 + k_2 + k_3)(2\pi^2)^2 \left[ \frac{\mathcal{P}_\zeta(k_1)\mathcal{P}_\zeta(k_2)}{k_1^3k_2^3} + (perms) \right] \]

scales as \[ a^{-6+2(n_s-1)} \]
How does this compare to $f_{\text{NL}}$ models?

Geometry dependence is different

Sound speed:

$$k_1 \triangle k_2$$

Scale dependence is different

Sound speed:

$$a^{-6+2(n_s-1)-2\kappa}$$

$$f_{\text{NL}}$$

$$a^{-6+2(n_s-1)}$$

Babich et al astro-ph/0405356
Constraining non-Gaussianity

Scale dependent non-Gaussianity

\[ "f_{NL}\" = - \frac{35}{108} \left( \frac{1}{c_s^2} - 1 \right) \]

\[ c_s \propto k^{-0.01} \]
\[ c_s \propto k^{-0.1} \]
\[ c_s \propto k^{-0.3} \]

![Graph showing the relationship between \(c_s\) and \(k\) with different slopes.]
Constraining non-Gaussianity

Scale dependent non-Gaussianity

\[ f_{NL} = - \frac{35}{108} \left( \frac{1}{c_s^2} - 1 \right) \]

allowed by current CMB

Creminelli, Senatore, Zaldarriaga, Tegmark 2006
how do clusters know about non-Gaussianity?

Clusters form when the (smoothed) density fluctuation is above a threshold $\delta_c$.
how do clusters know about non-Gaussianity?

positive skewness → more collapsed objects
how do clusters know about non-Gaussianity?

positive skewness
  more collapsed objects

negative skewness
  fewer collapsed objects
how do clusters know about non-Gaussianity?

positive skewness

more collapsed objects

negative skewness

fewer collapsed objects

The number of clusters tells us about the tail of the probability distribution function
Cluster counts

\[ P(\delta_R) d\delta_R \]

\[ n(M) dM \text{ – number density of clusters with mass between } M \text{ and } M + dM \]

\[ dN/dz \text{ – number of clusters per redshift interval } dz \text{ with mass bigger than } M_{\text{lim}} \]
Recipe for $n(M)$

- Recover the probability distribution function (PDF) from $\langle \delta^3( R ) \rangle$
- Follow Press–Schechter approach

\[
f_{\text{collapse}}(M) = 2 \int_{\delta_c}^{\infty} d\delta P(\delta, M)
\]

\[
\frac{dn}{dM} = -\frac{\bar{\rho}}{M} \frac{df}{dM}
\]

- Use $n_{\text{NG}}(M)/n_G(M)$ where each is found from above

Press & Schechter 1974  
Robinson & Baker astro-ph/9905098  
Mattarese, Verde, Jimenez astro-ph/0001366
Caveats/Difficulties

- We recover an approximate p.d.f limited range (in mass and redshift) of validity.
- This approach will indicate the utility of cluster counts. To make precise predictions simulations will need to be done.
Recovering the p.d.f.

The Edgeworth expansion

\[
P(\delta) d\delta = \frac{d\delta}{\sqrt{2\pi\sigma}} e^{-\frac{\delta^2}{2\sigma^2}} \left[ 1 + \frac{S_3\sigma}{3!} H_3 \left( \frac{\delta}{\sigma} \right) + \frac{1}{2} \left( \frac{S_3\sigma}{3!} \right)^2 H_6 \left( \frac{\delta}{\sigma} \right) + \frac{S_4\sigma^2}{4!} H_4 \left( \frac{\delta}{\sigma} \right) + \ldots \right]
\]

Hermite Polynomials

\[
S_n = \frac{\langle \delta^n \rangle_c}{\langle \delta^2 \rangle_n^{n-1}}
\]
The mass function

Gaussian case (Press–Schechter)–

\[
\frac{dn(M)}{dM} = -\sqrt{\frac{2}{\pi}} \frac{\bar{\rho}}{M} e^{-\frac{\delta_c^2}{2\sigma(M)^2}} \frac{d\ln \sigma}{dM} \delta_c
\]

The non–Gaussian mass function

\[
\frac{dn(M)}{dM} = -\sqrt{\frac{2}{\pi}} \frac{\bar{\rho}}{M} e^{-\frac{\delta_c^2}{2\sigma(M)^2}} \left[ \frac{d\ln \sigma}{dM} \left( \frac{\delta_c}{\sigma} + \frac{S_3 \sigma}{6} \left( \frac{\delta_c^4}{\sigma^4} - 2\frac{\delta_c^2}{\sigma^2} - 1 \right) \right) + \frac{1}{6} \frac{dS_3}{dM} \sigma \left( \frac{\delta_c^2}{\sigma^2} - 1 \right) \right]
\]

\[\delta_c (z) = \text{collapse threshold – increases with redshift}\]
$$S_3(M) = \frac{\langle \delta^3_M \rangle}{\langle \delta^2_M \rangle^2}$$

Use flat $\Lambda$CDM with

$\Omega_m = 0.27$

$\Omega_b = 0.024/h^2$

$h = 0.7$

$\sigma_8 = 0.8$

$n_s = 0.95$

Smooth by Gaussian window function
The change in the mass function

\[ n_{NG}(M)/n_C(M) \text{ at } z=0 \]

\[ \kappa = -0.01 \]

\[ \kappa = -0.1 \]
The change in the mass function

worry about expansion
The change in the mass function
The change in the mass function
The change in the mass function
What we get with Press–Schechter assuming $M_{\text{lim}} = 2.5 \times 10^{14} \, M_{\odot}$ (ind. of $z$)
$dN/dz$

What we get with Press–Schechter assuming $M_{\text{lim}} = 2.5 \times 10^{14} \, M_{\odot}$ (ind. of $z$)
What we get with Press–Schechter assuming $M_{\text{lim}} = 2.5 \times 10^{14} \, M_{\text{sun}}$ (ind. of $z$)
BUT, the mass limit isn’t perfectly known

\[
\frac{\Delta dN/dz}{dN/dz(M_{\text{lim}} + \Delta M)} = -\frac{dN/dz(M_{\text{lim}})}{dN/dz(M_{\text{lim}} + \Delta M)}
\]
dN/dz

BUT, the mass limit isn’t perfectly known

mass uncertainty must by small (~5%) for this to work
dN/dz

What about $\sigma_8$?
\( \frac{dN}{dz} \)

Gaussian \( \sigma_8 = 0.8 \)
\( \Delta M_{\text{lim}} / M = \pm 0.05 \)
\( \sigma_8 = 0.75, 0.85 \)

\( \kappa = -0.01 \)
\( \kappa = -0.1 \)
\( \kappa = -0.3 \)

see also Sefusatti et al astro-ph/0609124
Other methods

The Bispectrum

In principle the bispectrum contains more information because it retains the dependence on $k_1$, $k_2$, $k_3$

Unlike the mass function, the bispectrum doesn’t rely on knowledge of higher order cumulants

The bispectrum avoids the issue of expanding the pd.f.
The bispectrum

\[ \langle \delta(k_1)\delta(k_2)\delta(k_3) \rangle = (2\pi)^3 \delta(k_1 + k_2 + k_3) B(k_1, k_2, k_3) \]

\[ B(k_1, k_2, k_3) = B_I(k_1, k_2, k_3) + B_G(k_1, k_2, k_3) \]

due to initial non-Gaussianity

evolved non-Gaussianity due to non-linear evolution
The bispectrum

\[ \langle \delta(k_1)\delta(k_2)\delta(k_3) \rangle = (2\pi)^3 \delta(k_1 + k_2 + k_3)B(k_1, k_2, k_3) \]

\[ B(k_1, k_2, k_3) = B_I(k_1, k_2, k_3) + B_G(k_1, k_2, k_3) \]

\[ Q(k_1, k_2, k_3) = \frac{B(k_1, k_2, k_3)}{P(k_1)P(k_2) + P(k_1)P(k_3) + P(k_2)P(k_3)} \]
The bispectrum

\[ Q_G(k, k, k) = \frac{4}{7} \]

\[ Q_I(k, k, k) \sim \frac{1}{c_s^2} \frac{1}{k^2 T(k)} \sim \frac{k^{-2\kappa}}{k^2 T(k)} \]

Transfer function

\[ T(k/k_{eq} << 1) \sim 1 \]
\[ T(k/k_{eq} >> 1) \sim \ln(k)/k^2 \]

See also Sefusatti & Komatsu astro-ph/07050343
Conclusions

- Models of inflation with $c_s \neq 1$ produce scale dependent non-Gaussianity.
- We have explored possibilities beyond the CMB for putting constraints on this non-Gaussianity.
- Cluster number counts constrain non-Gaussianity at a different scale than CMB and can provide a cross-check to CMB constraints. Simulations will need to be done to make precise constraints.