Non-Gaussianity in Large-Scale Structure: Theory

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Impact of Non-Gaussianity in LSS: Sources

- primordial (e.g. from inflation)

- super-horizon dynamics (post-inflation)

- sub-horizon dynamics: new methods (RPT, RG)

- bias: going from dark matter to galaxies

- redshift distortions (in galaxy redshift surveys)

- large-distance modifications of gravity can also lead to additional NG

NG also affect the power spectrum ...

- Same physics that induces a bispectrum leads e.g. to modification of acoustic oscillation features in two-point statistics

- Nonlinear galaxy bias leads to both NG and scale-dependence in the power spectrum

- Similarly, a scale dependence in the power spectrum is induced in models with large-distance modifications of gravity (where the Poisson equation is nonlinear)

- power spectrum covariance matrix: beat coupling in a finite survey results from NG contributions to the covariance.

A new approach to model clustering (Renormalized Perturbation Theory)

In standard perturbation theory (PT), the power spectrum is

$$P(k,z) = D_+^2\, P_0(k) + P_{1\mathrm{loop}}(k,z) + P_{2\mathrm{loops}}(k,z) + \dots$$

where D is the growth factor, P0 the initial (post-recombination) spectrum and,

$$P_{1\mathrm{loop}} \sim \mathcal{O}(P_{\mathrm{lin}}\,\Delta_{\mathrm{lin}}), \qquad P_{2\mathrm{loops}} \sim \mathcal{O}(P_{\mathrm{lin}}\,\Delta_{\mathrm{lin}}^2), \qquad \Delta_{\mathrm{lin}} = 4\pi k^3 P_{\mathrm{lin}}$$

Approaching nonlinear scales, truncation at finite order in PT is not meaningful, as neglected higher-order contributions are at least equally important.

In renormalized perturbation theory (RPT), one the main insights is that if we write the growth factor as,

$$D_+(z) = \frac{\langle \delta_{\rm lin}(z)\, \delta_0 \rangle}{\langle \delta_0\, \delta_0 \rangle},$$

Then a whole set of nonlinear contributions to the power spectrum just renormalize the growth factor to the "propagator"

$$D_+(z) \longrightarrow G(k,z) = \frac{\langle \delta(z)\, \delta_0 \rangle}{\langle \delta_0\, \delta_0 \rangle},$$

The asymptotics of the propagator are easy to understand,

$$G(0,z) = D_+(z), \qquad G(k,z) \to 0 \qquad \text{as} \quad k \to \infty.$$

It is this last property that is impossible to capture by truncating PT at finite order.

Non linear effects in the power spectrum can be divided (exactly) into two classes,

- those that are proportional to the initial power at same k.
- those that create power at k even if there was no power to begin with (mode-coupling)

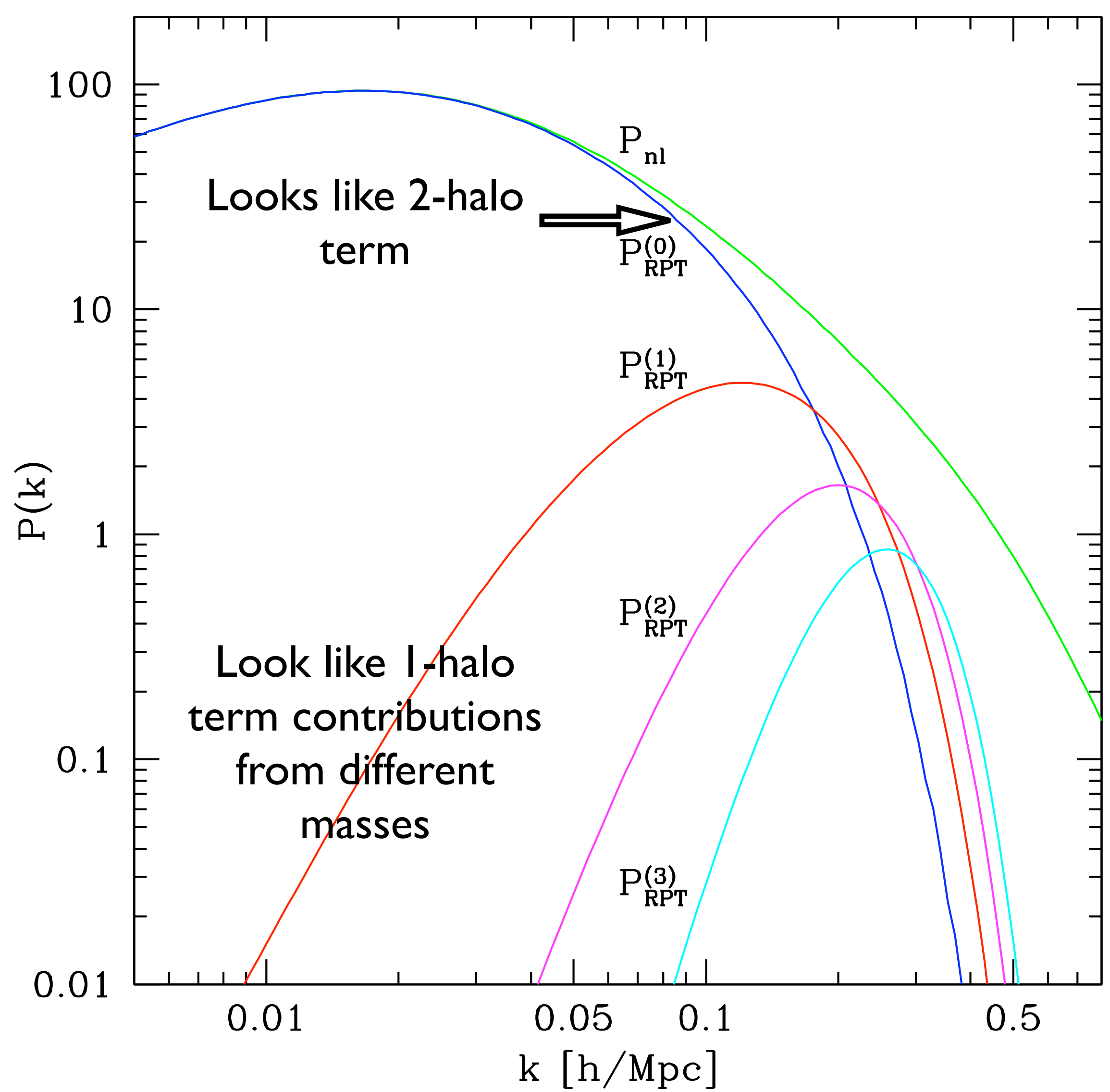
The density power spectrum can be written as,

$$P(k) = P_{\rm linear}(k)\, G_\delta(k)^2 + P_{\rm mode-coupling}(k)$$

the two-pt correlation function,

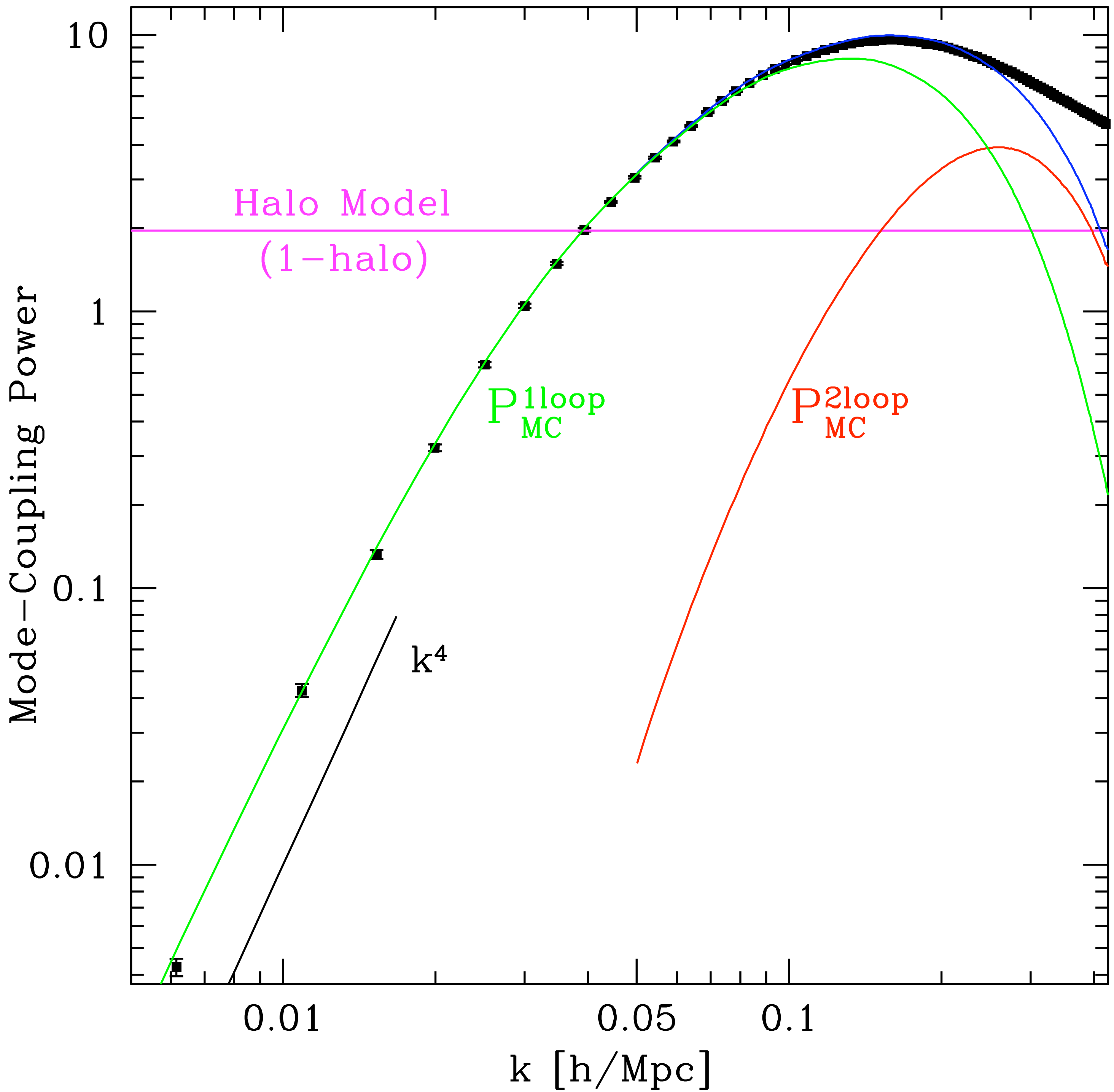
$$\xi(r) = [\xi_{\rm linear} \otimes G_\delta^2](r) + \xi_{\rm mode-coupling}(r)$$

Power Spectrum in RPT (schematic)

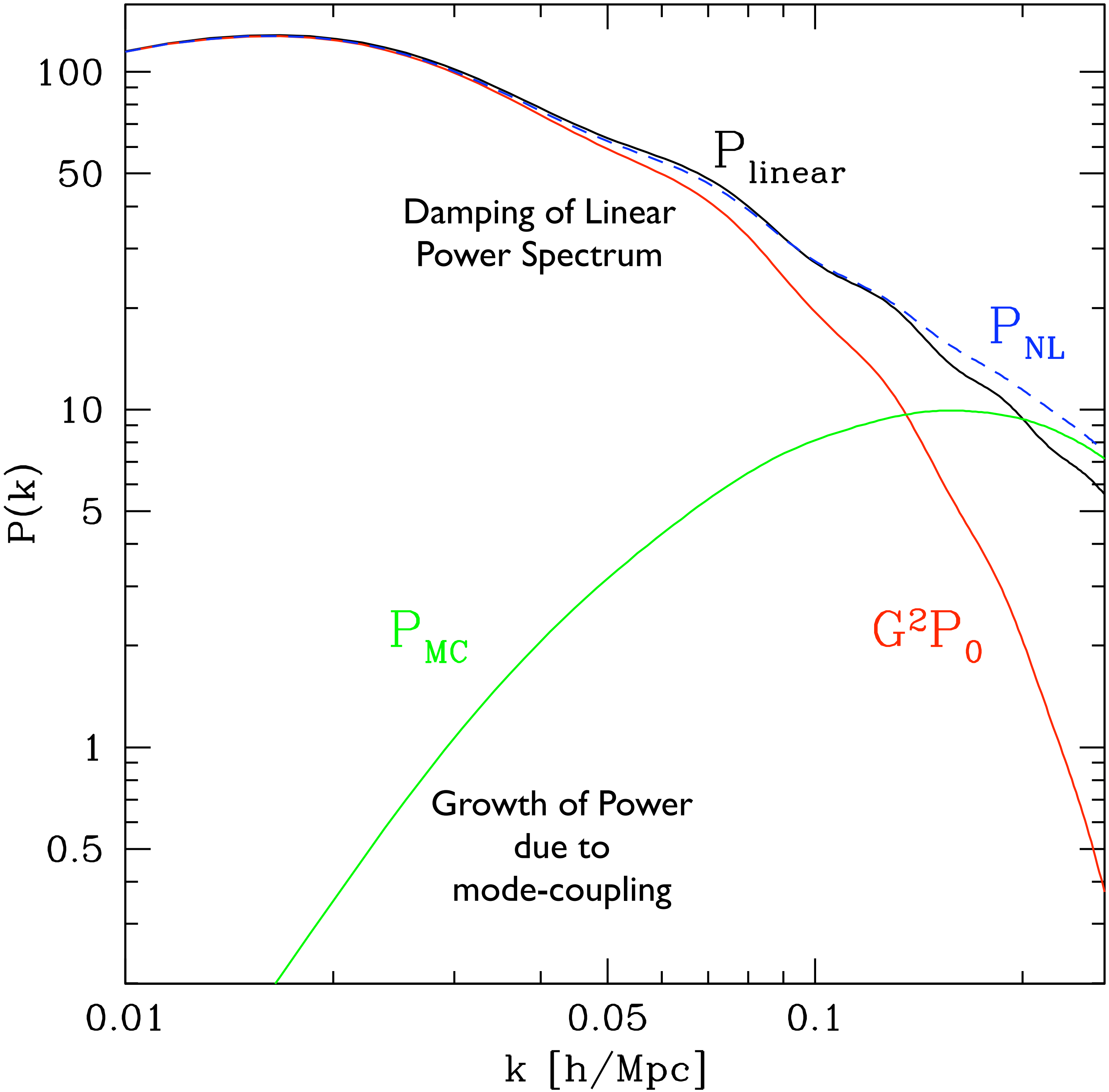


This is a well-behaved perturbation theory
(not an expansion in amplitude of fluctuations)

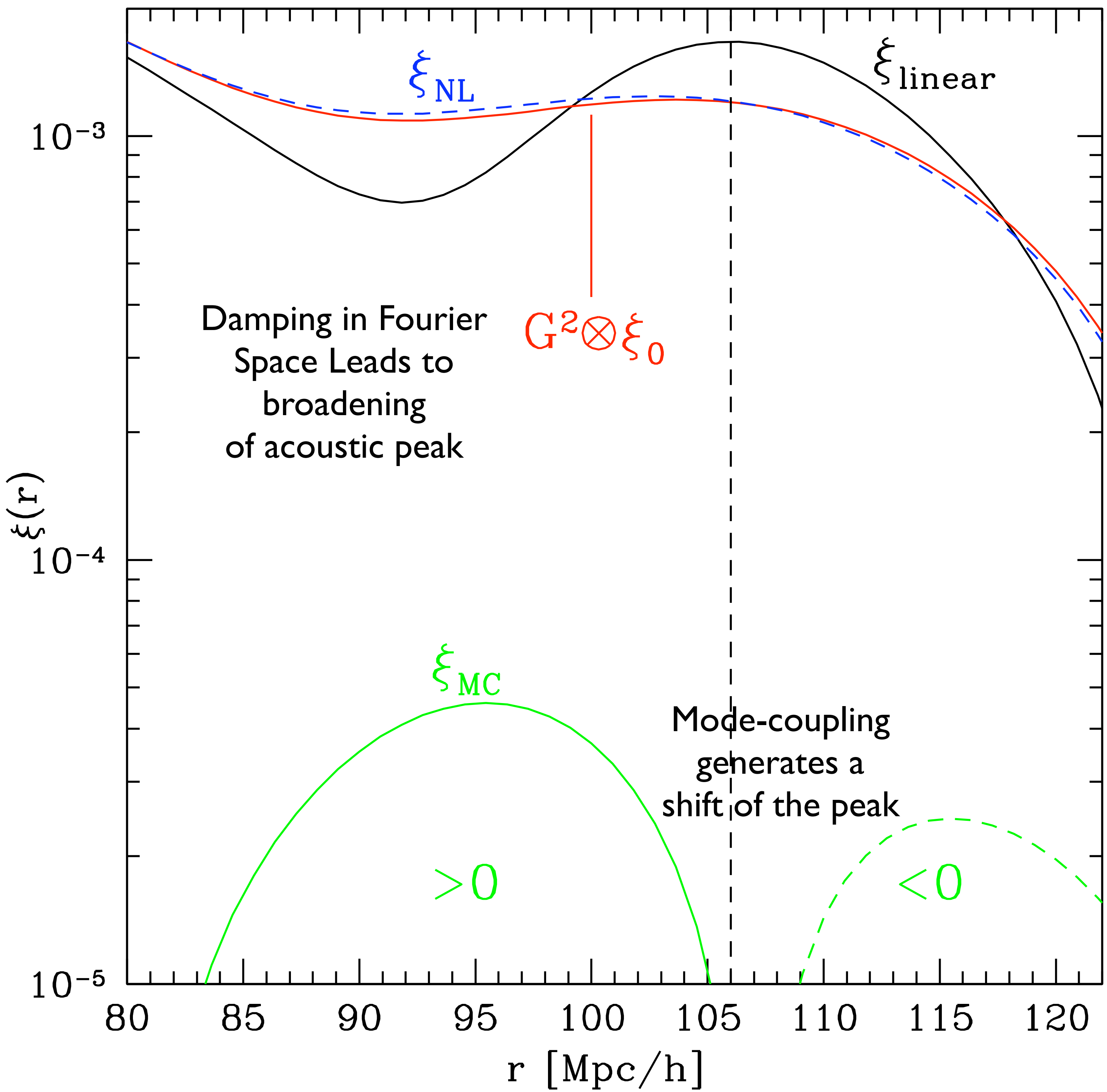
Power generated by mode-coupling, $P_{\mathrm{MC}} \equiv P - G^2\, P_0$



The Power Spectrum in RPT



The Two-Point Function in RPT



So far we have discussed dark matter...

Now, let's discuss galaxies, in particular dark matter halos (which are easier to model, and can be populated with galaxies to yield a given galaxy sample).

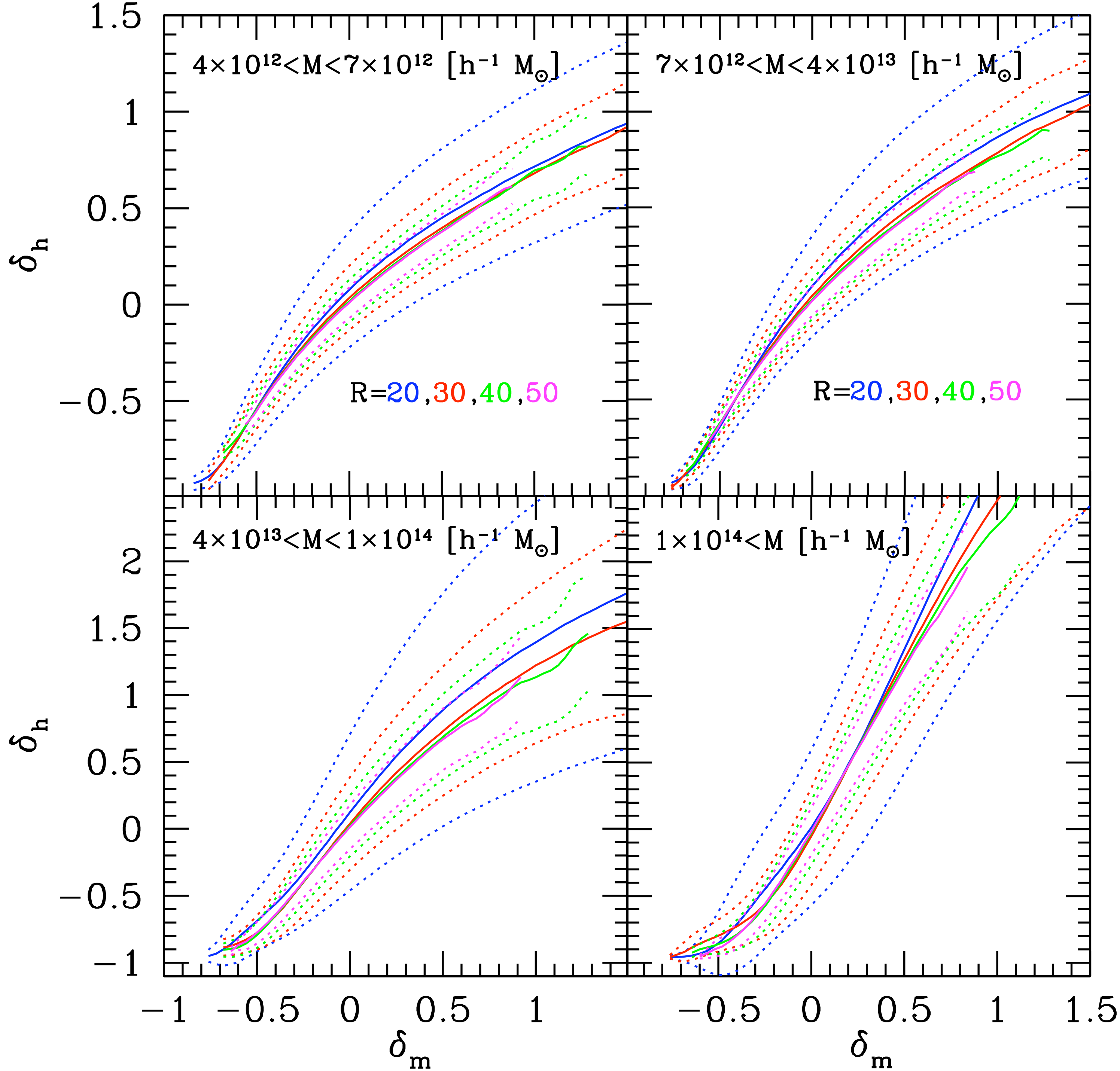
We are interested in the scale dependence of the bias.

The first thing we want to test is the validity of local bias. This says that at large scales the relationship between galaxies (or halos) and dark matter can be approximated by scale-independent bias coefficients,

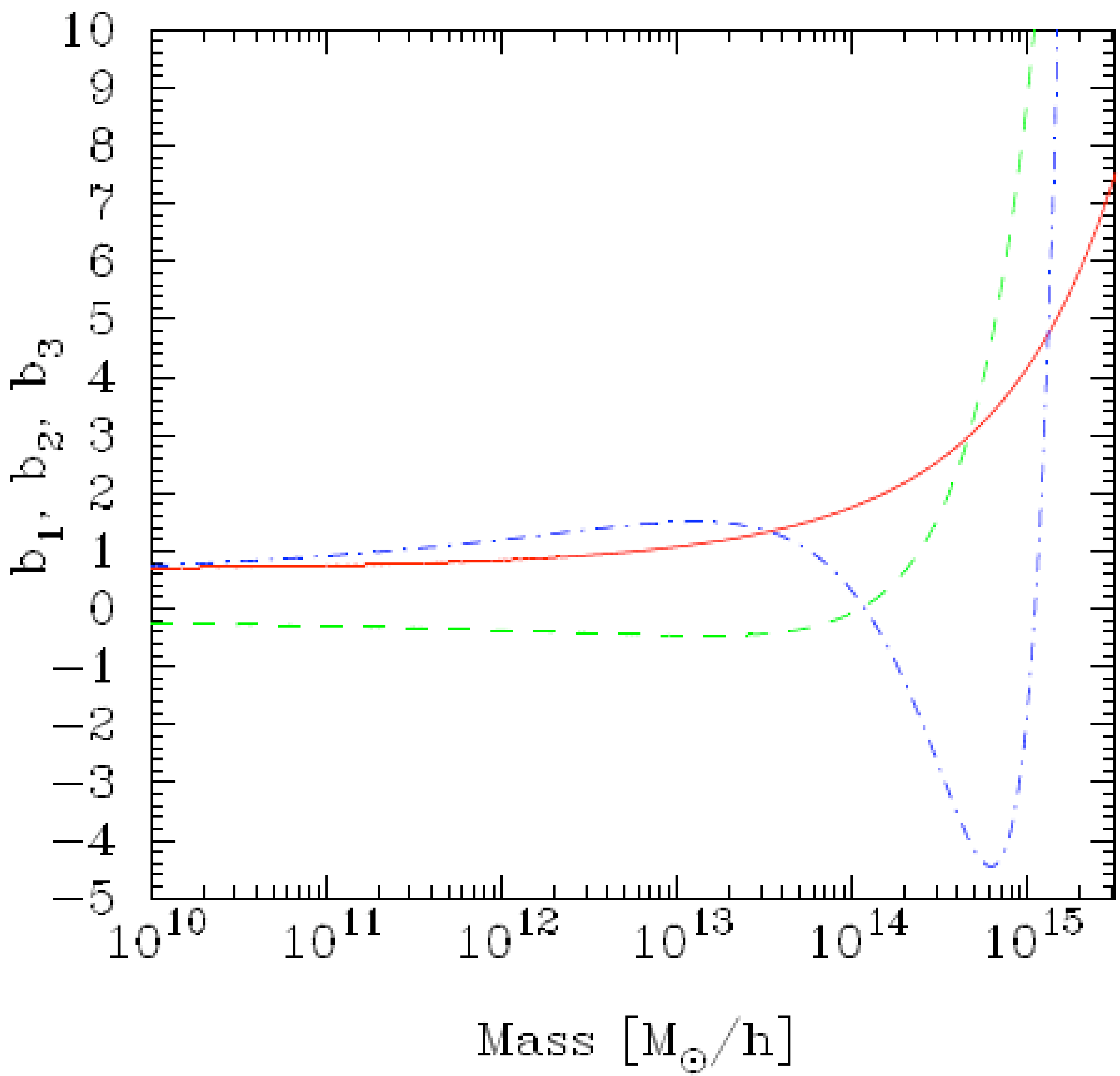
$$\delta_g \approx b_1 \delta + \frac{b_2}{2!}\delta^2 + \frac{b_3}{3!}\delta^3$$

Fry and Gaztanaga(1993)

Local Bias at large scales

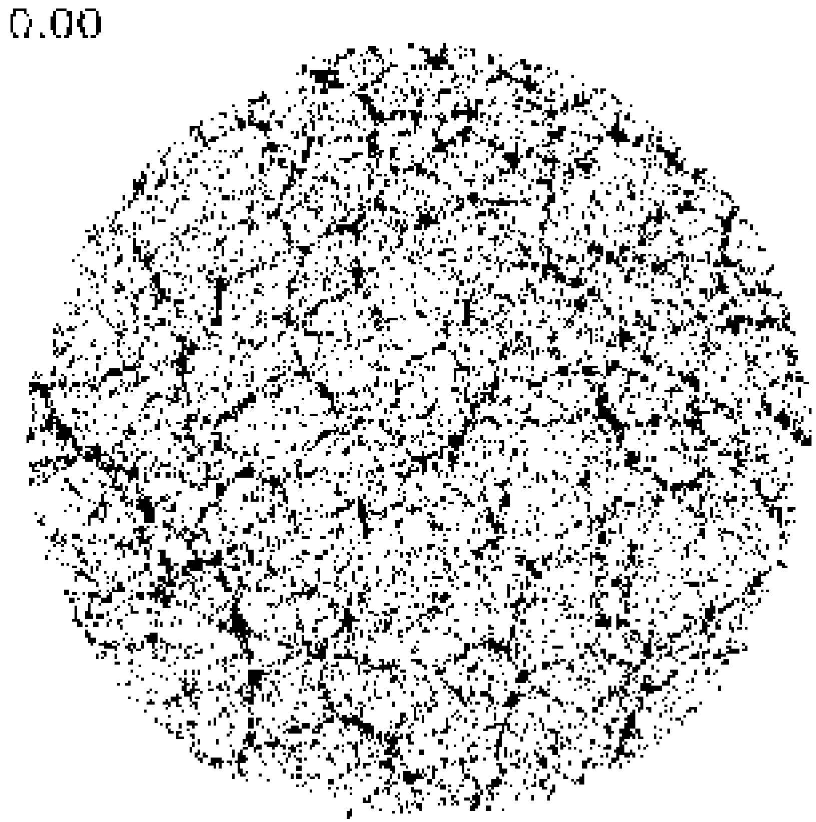


Halo Bias is non-linear: predictions from ST mass function



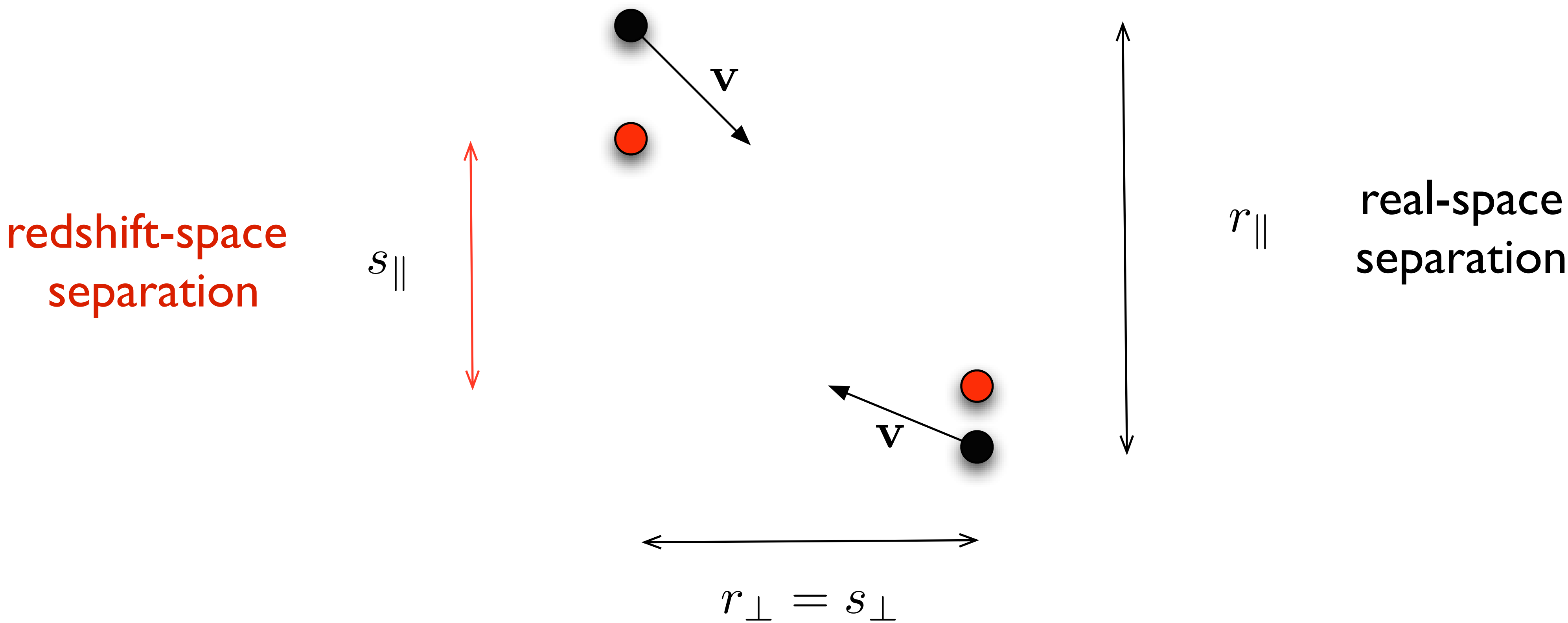
Scoccimarro, Sheth, Hui and Jain (2001)

Redshift-space distortions:



Two basic effects: "squashing" at large scales (increases clustering), and "dispersion" at small scales (decreases clustering)

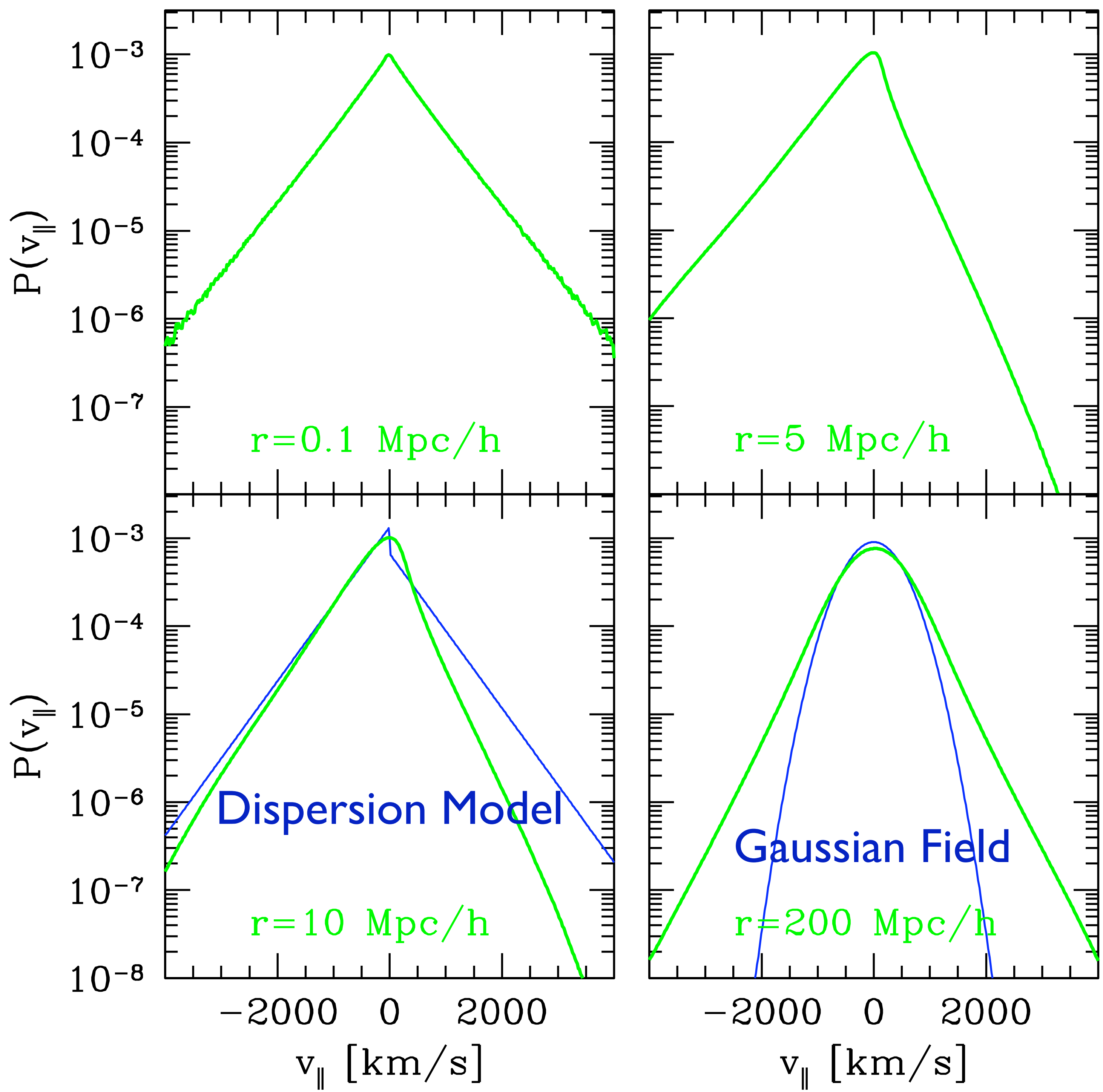
An exact relationship between real and redshift-space clustering:



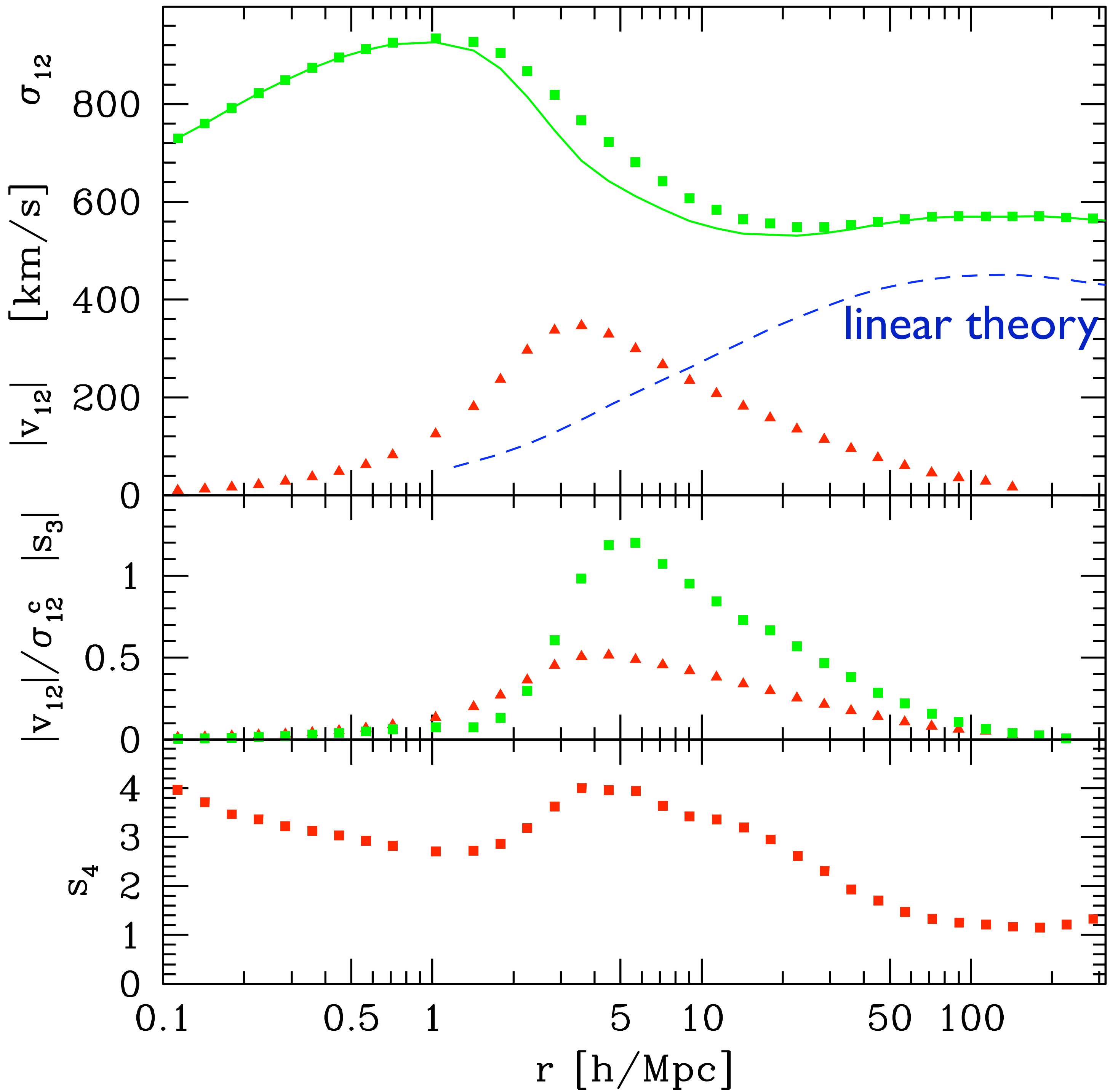
$$1 + \xi_s(s_{\parallel}, s_{\perp}) = \int_{-\infty}^{\infty} dr_{\parallel}\ [1 + \xi(r)]\ \mathcal{P}(\underbrace{r_{\parallel} - s_{\parallel}}_{\mathbf{v}_p}, \mathbf{r}),$$

Everything is encoded in the pairwise velocities PDF.

PDF from N-body simulations



moments from N-body simulations



The failure of linear theory in reproducing the second moment at large scales is due to pair weighting,

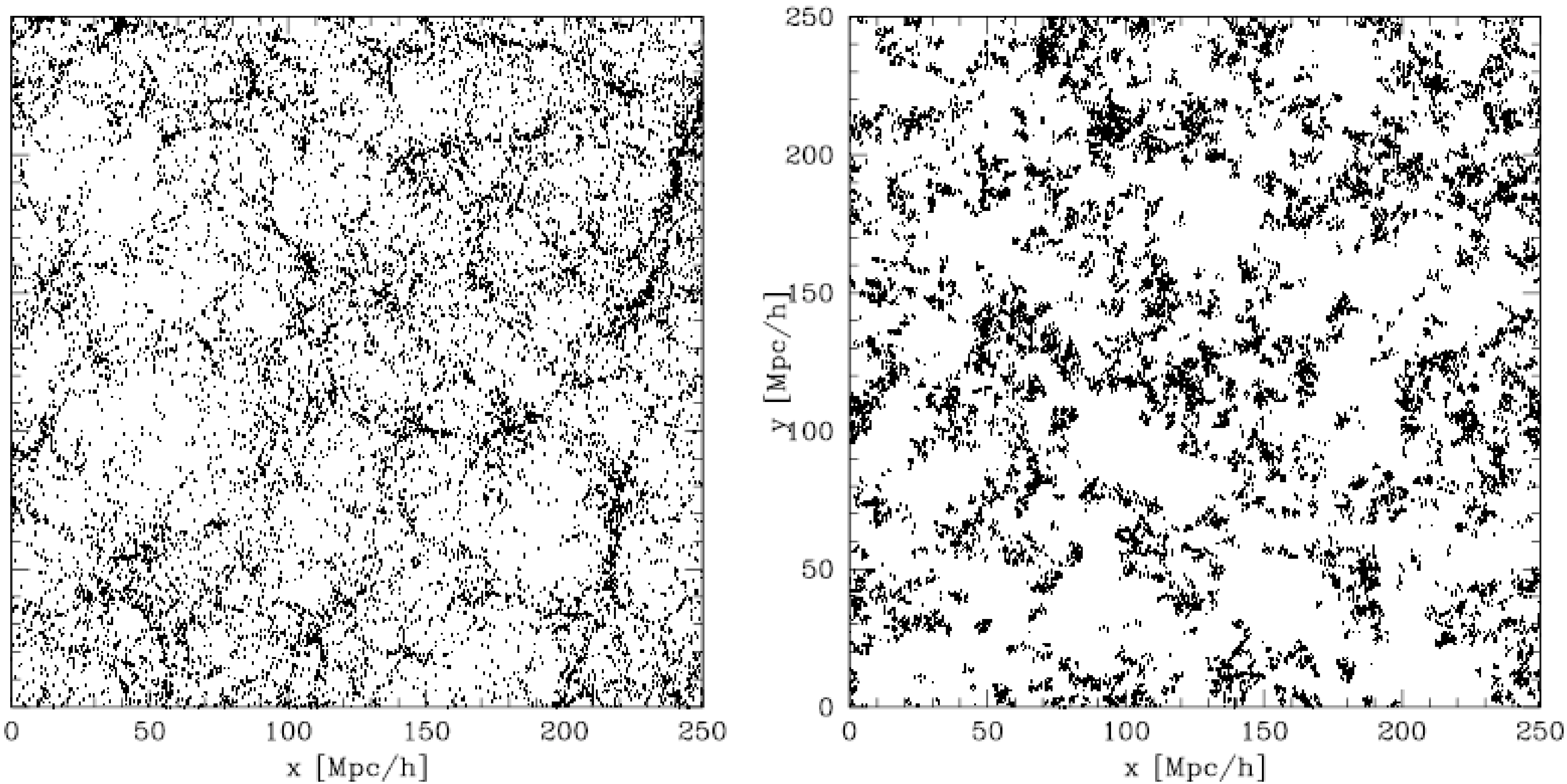
$$\sigma_{12}^2 = \frac{\langle (1+\delta_1)(1+\delta_2)(u_1-u_2)^2 \rangle}{1+\xi_{12}}$$

which at large scales reduces to,

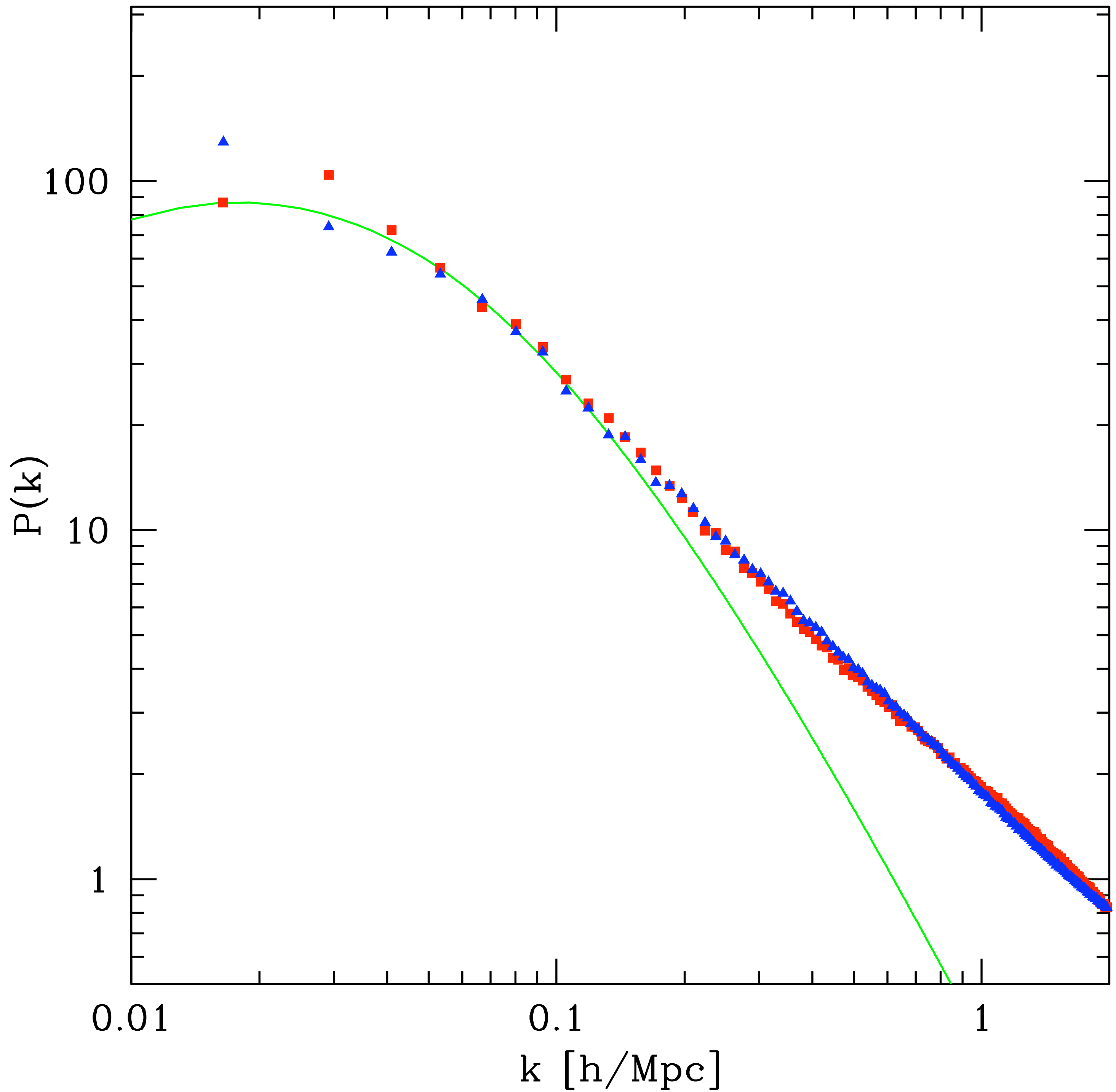
$$\sigma_{12}^2|_{\rm large\ scales} = 2\langle u^2 \rangle + 4\langle \delta u^2 \rangle$$

In addition, we have nonlinear corrections to the evolution of velocities, and these are more important than for the density.

Extracting Cosmological Information from Galaxy Clustering with Higher-Order Statistics



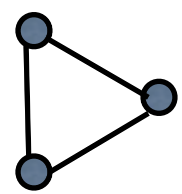
The two distributions have about the same Power Spectrum!



Three-point statistics are the lowest order measure of the shape of structures (filaments, walls, halos) generated by gravitational instability.

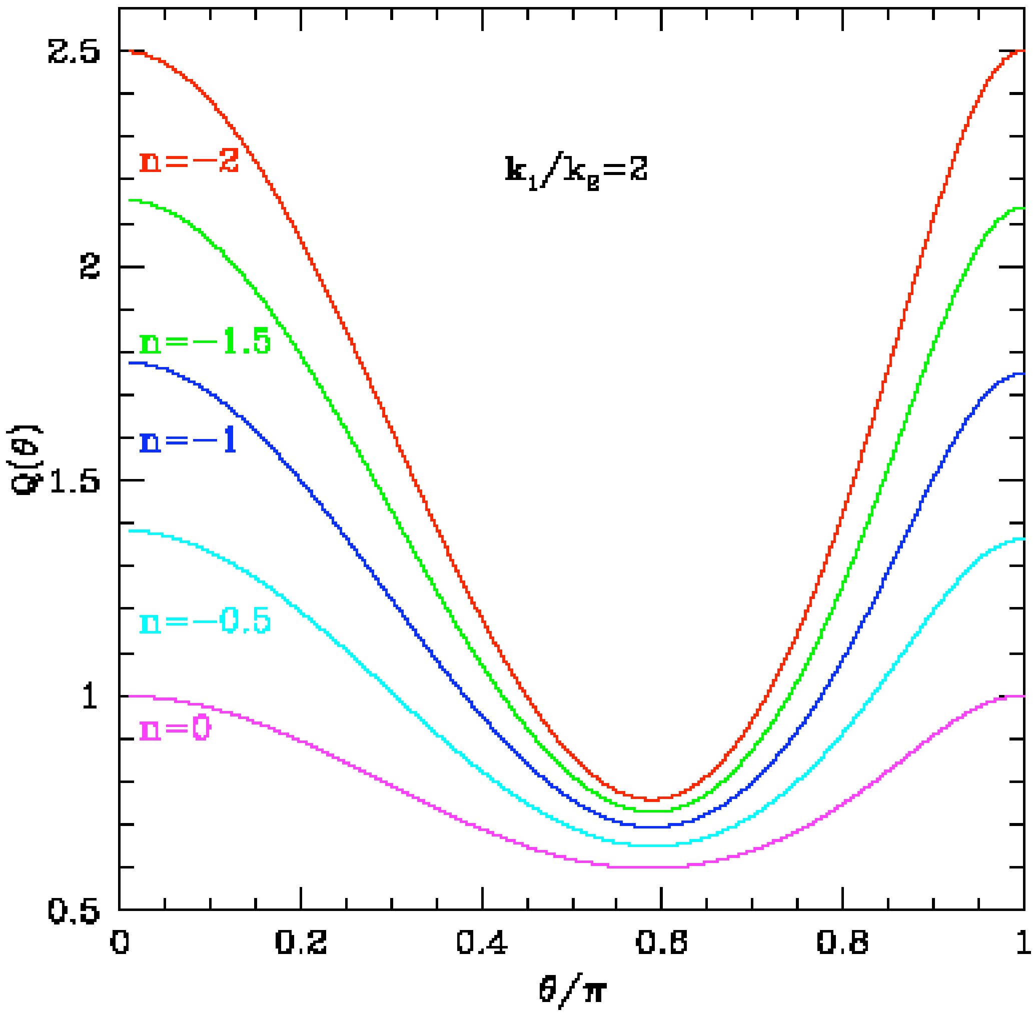
- Indeed, with two points one can only form a single shape: a line

- Three-points form a triangle, so we got different triangle shapes we can compare, for example collinear triangles with equilateral triangles,

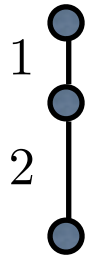


If filamentary structures are predominant, then the three-point amplitude (Q) should be larger for collinear triangles than equilateral (or isosceles).

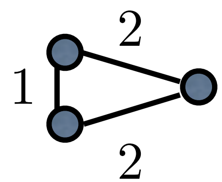
- A limitation of three-point statistics is that three points always form a plane. In order to better probe the "three-dimensional" shape of structures, one needs to go to the four-point function.



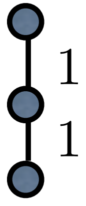
2.5
2
1.5
1
0.5
0
0.2
0.4
0.6
0.8
1
Q(θ)
θ/π
n=−2
n=−1.5
n=−1
n=−0.5
n=0
k1/k2=2



1
2

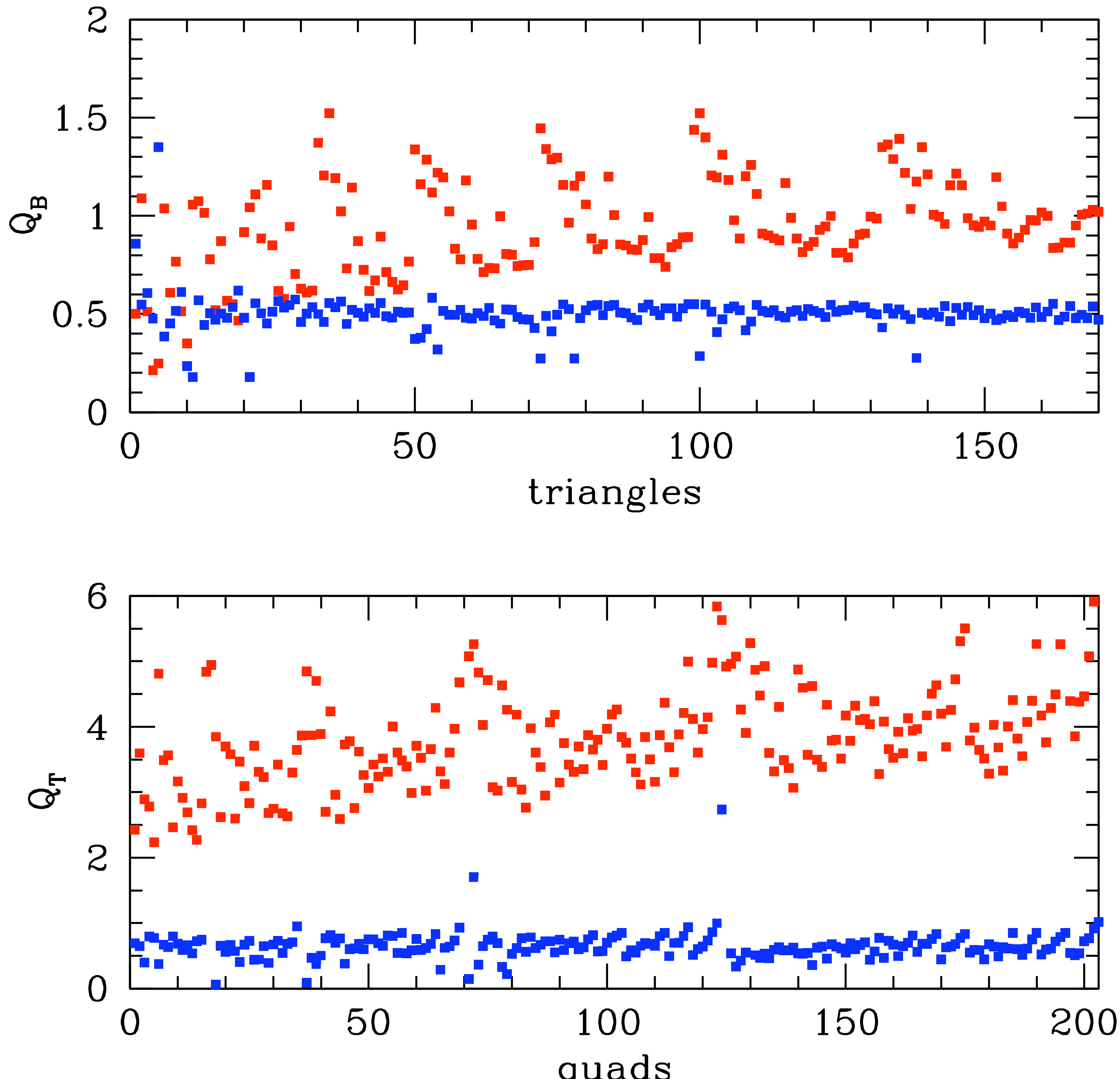


2
1
2



1
1

The two distributions can be distinguished easily by higher-order correlations!



Galaxy Bias from Large-Scale Correlations

At large scales the relationship between galaxies and dark matter can be approximated by,

$$\delta_g \approx b_1 \delta + \frac{b_2}{2!}\delta^2 + \frac{b_3}{3!}\delta^3$$

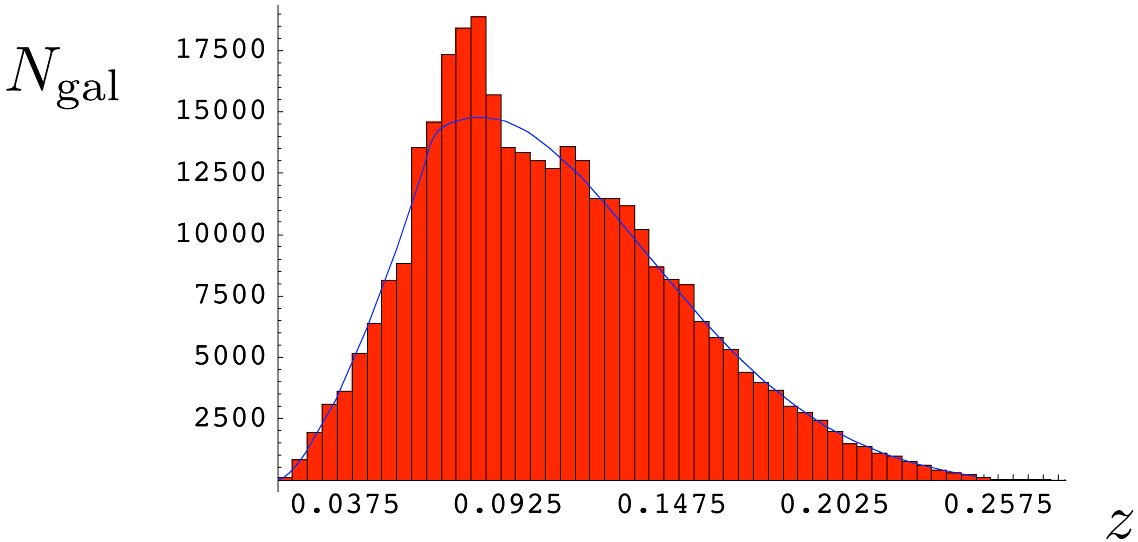
One can use higher-order correlation functions to determine the bias parameters. The three-point function (bispectrum), when suitably normalized, it depends mostly on galaxy bias,

$$Q_B = \frac{B_{123}}{P_1 P_2 + P_2 P_3 + P_3 P_1}$$

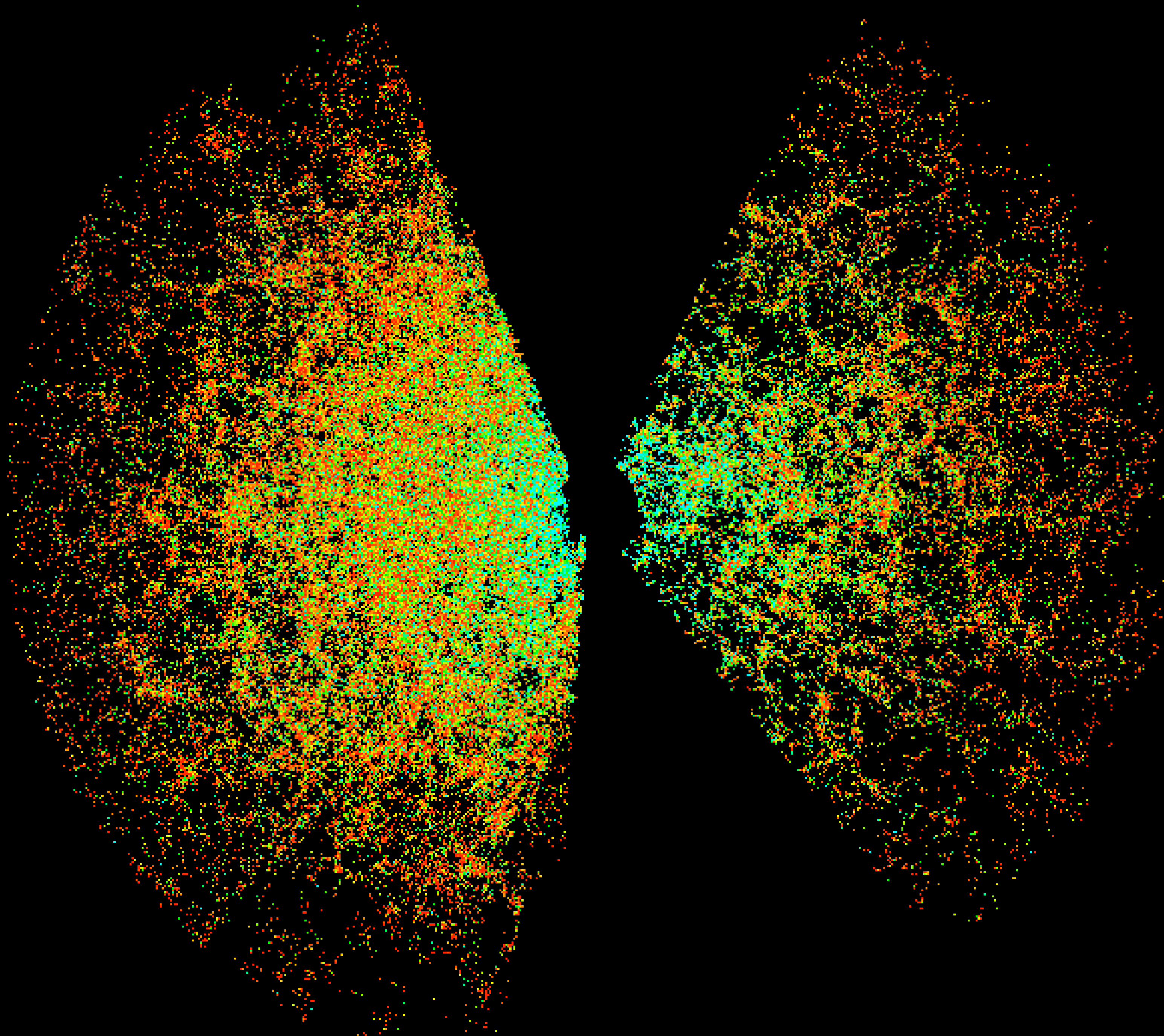
$$Q_B^g = \frac{1}{b_1} Q_B + \frac{b_2}{b_1^2}$$

The constraints on linear bias translate e.g. into constraints on sigma8 and Omega matter when combined with the measurement of the power spectrum.

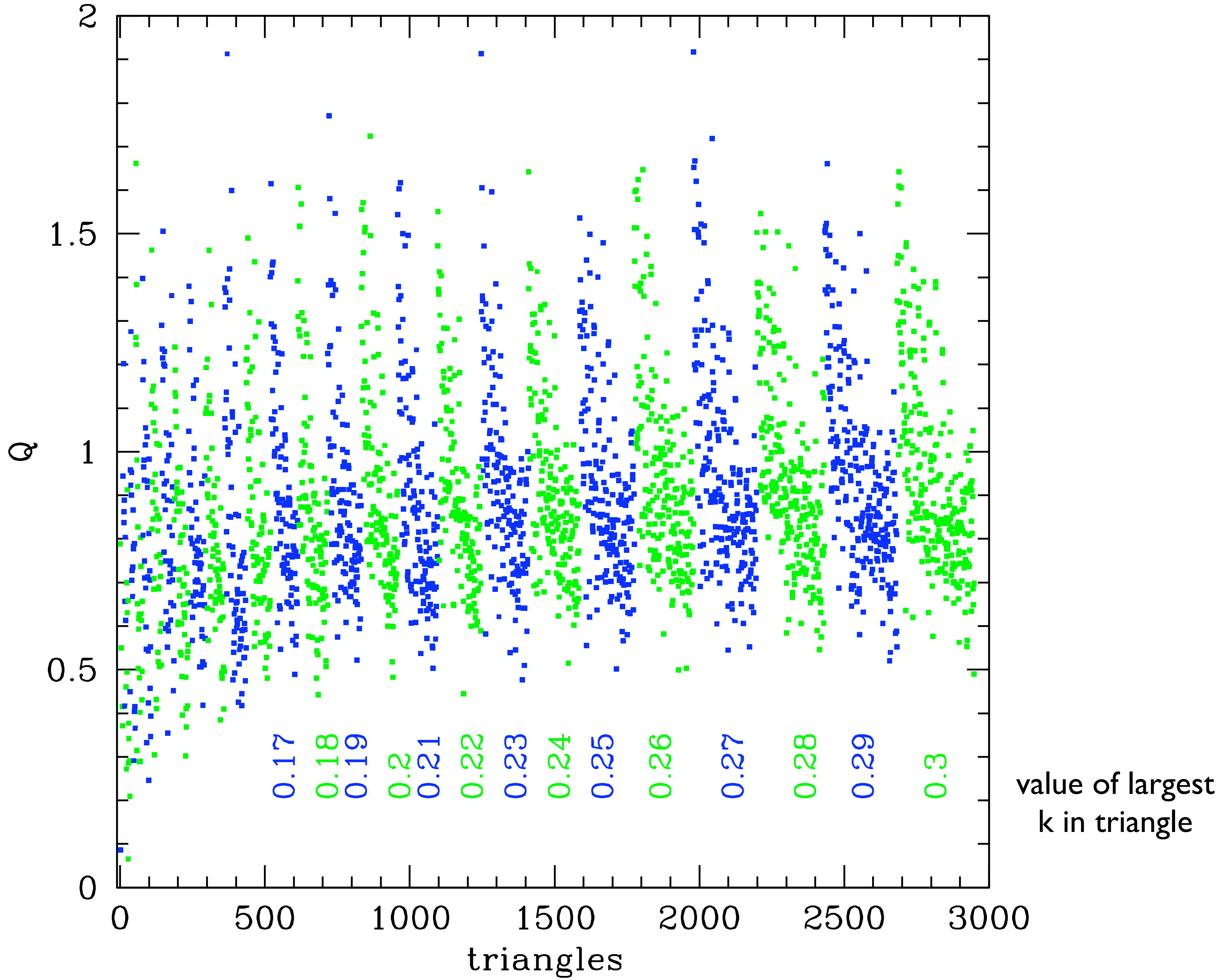
Preliminary SDSS results

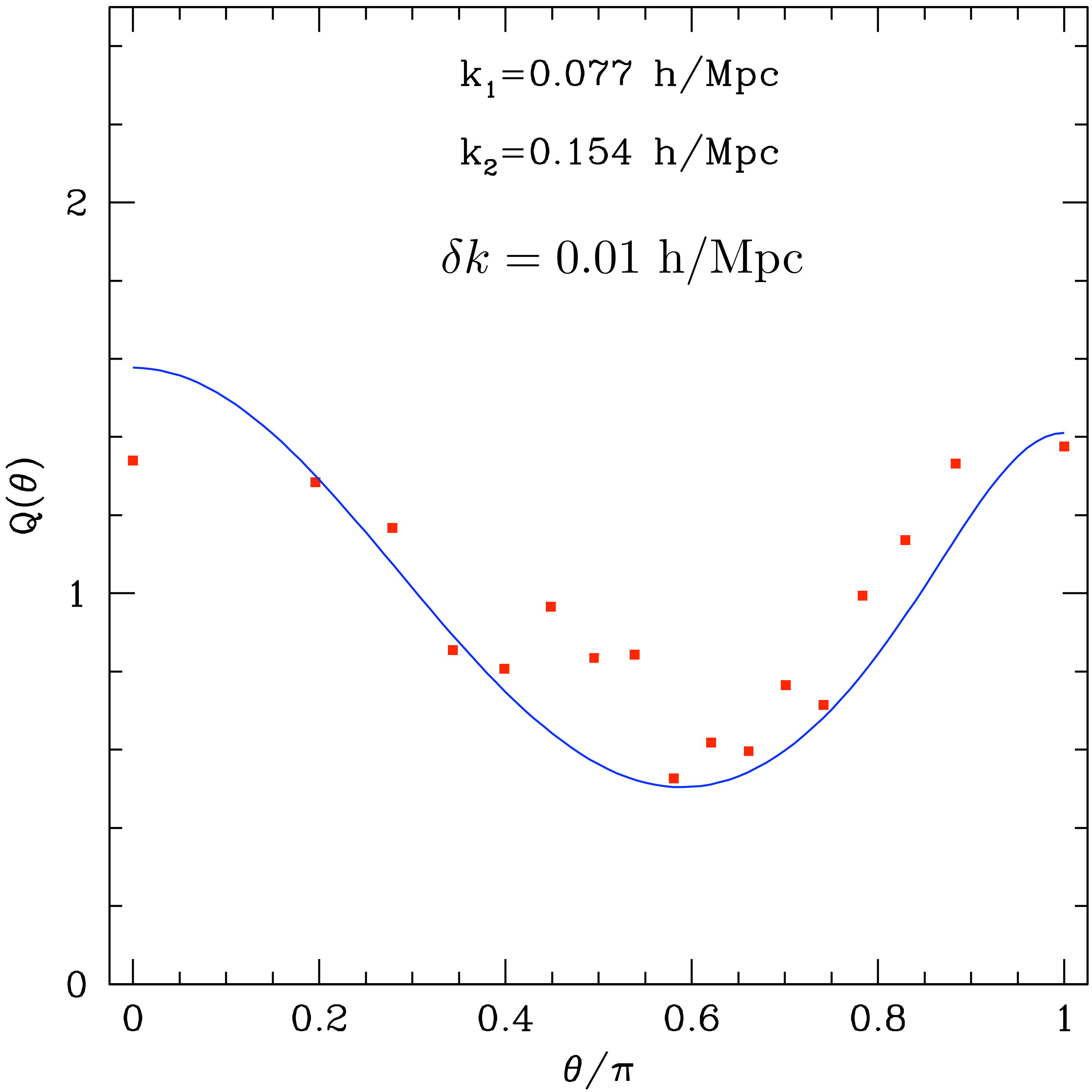


Area covered is about 5200 sq deg.

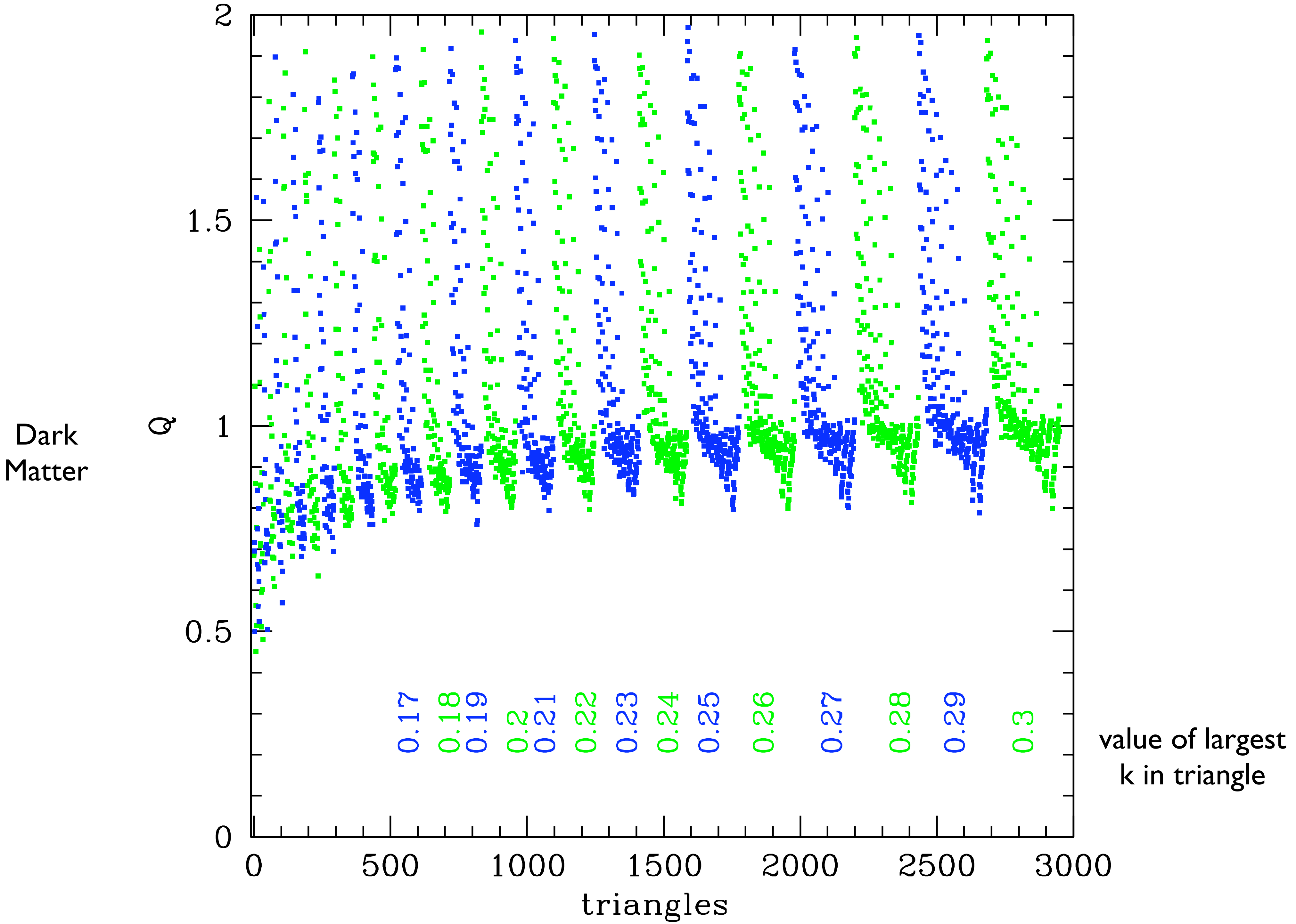


Courtesy of Michael Blanton


Q
triangles
0
0.5
1
1.5
2
500
1000
1500
2000
2500
3000
0.17
0.18
0.19
0.2
0.21
0.22
0.23
0.24
0.25
0.26
0.27
0.28
0.29
0.3
value of largest
k in triangle



k_1=0.077 h/Mpc
k_2=0.154 h/Mpc
$\delta k = 0.01$ h/Mpc
$Q(\theta)$
θ/π
0
1
2
0
0.2
0.4
0.6
0.8
1



Dark
Matter
Q
2
1.5
1
0.5
0
0
500
1000
1500
2000
2500
3000
triangles
0.17
0.18
0.19
0.2
0.21
0.22
0.23
0.24
0.25
0.26
0.27
0.28
0.29
0.3
value of largest
k in triangle

Conclusions

- Non-Gaussianity is important in large-scale structure, even at "large" scales

- The galaxy bispectrum clearly shows the dependence on triangle shape predicted by gravity from Gaussian initial conditions

- Extracting useful information from galaxy surveys requires detailed understanding of several nonlinear effects.

- NG can be used to probe fundamental physics (e.g. inflation, large-distance modifications of gravity)