The Galaxy Bispectrum:
Cosmology & primordial non-Gaussianity

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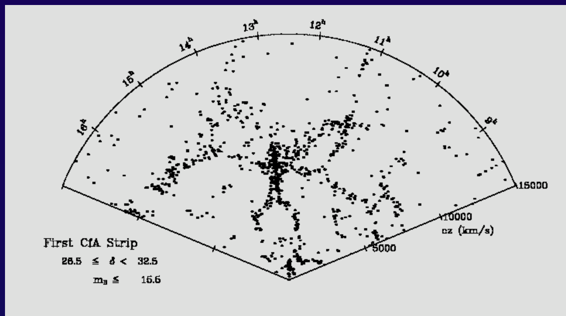
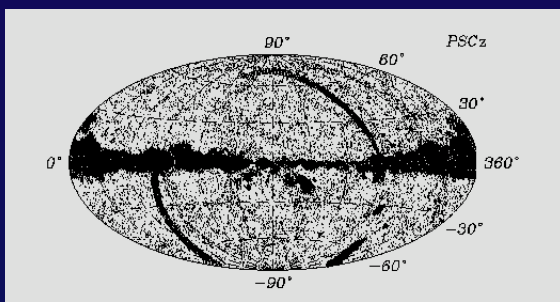
E. S. & E. Komatsu (2007)
E. S., H. M. Crocce, S. Pueblas & R. Scoccimarro (2006)
E. S. & R. Scoccimarro (2005)
R. Scoccimarro, E. S. & M. Zaldarriaga (2004)

The galaxy bispectrum: theory and observations



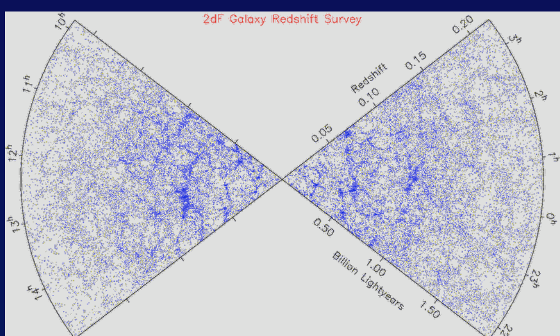
1982, **CfA Redshift Survey**
$\sim 1,000$ galaxies

$\Rightarrow$ large-scale clustering
[Baumgart & Fry (1991)]



1995, **IRAS PSCz Redshift Survey**
$\sim 15,000$ galaxies

$\Rightarrow$ galaxy bias, primordial non-Gaussianity
[Fry & Scherrer (1994), Matarrese *et al.* (1997), Verde *et al.* (2000), Feldman *et al.* (2001), Scoccimarro *et al.* (2001)]



2002 to the present, **2dFGRS** and **SDSS**
$\sim 250,000$ to $1,000,000$ galaxies

$\Rightarrow$ halo model
[Verde *et al.* (2002), Gastañaga *et al.* (2005), Nishimichi *et al.* (2006), Kulkarni *et al.* (2007)]

What about cosmological parameters? What are the prospects for primordial non-Gaussianity?

Higher order correlators as probes of non-linear physics

Contributions to the galaxy bispectrum:

- *Gravitational instability*

$$\delta_m \overset{\mathrm{PT}}{\simeq} \delta_L + \delta^{(2)} + \ldots, \qquad \Rightarrow \quad B_m(k_1,k_2,k_3) = B_G(k_1,k_2,k_3)$$

- Non-Gaussian initial conditions due to non-linearities in the *inflationary dynamics* and in the *super-horizon evolution*

$$\mathrm{N.G.\ I.C.} \qquad \Rightarrow \quad B_m(k_1,k_2,k_3) = B_I(k_1,k_2,k_3) + B_G(k_1,k_2,k_3)$$

- *Galaxy bias*

$$\delta_g(x) \simeq b_1 \delta_m(x) + \frac{b_2}{2}\delta_m^2(x)$$

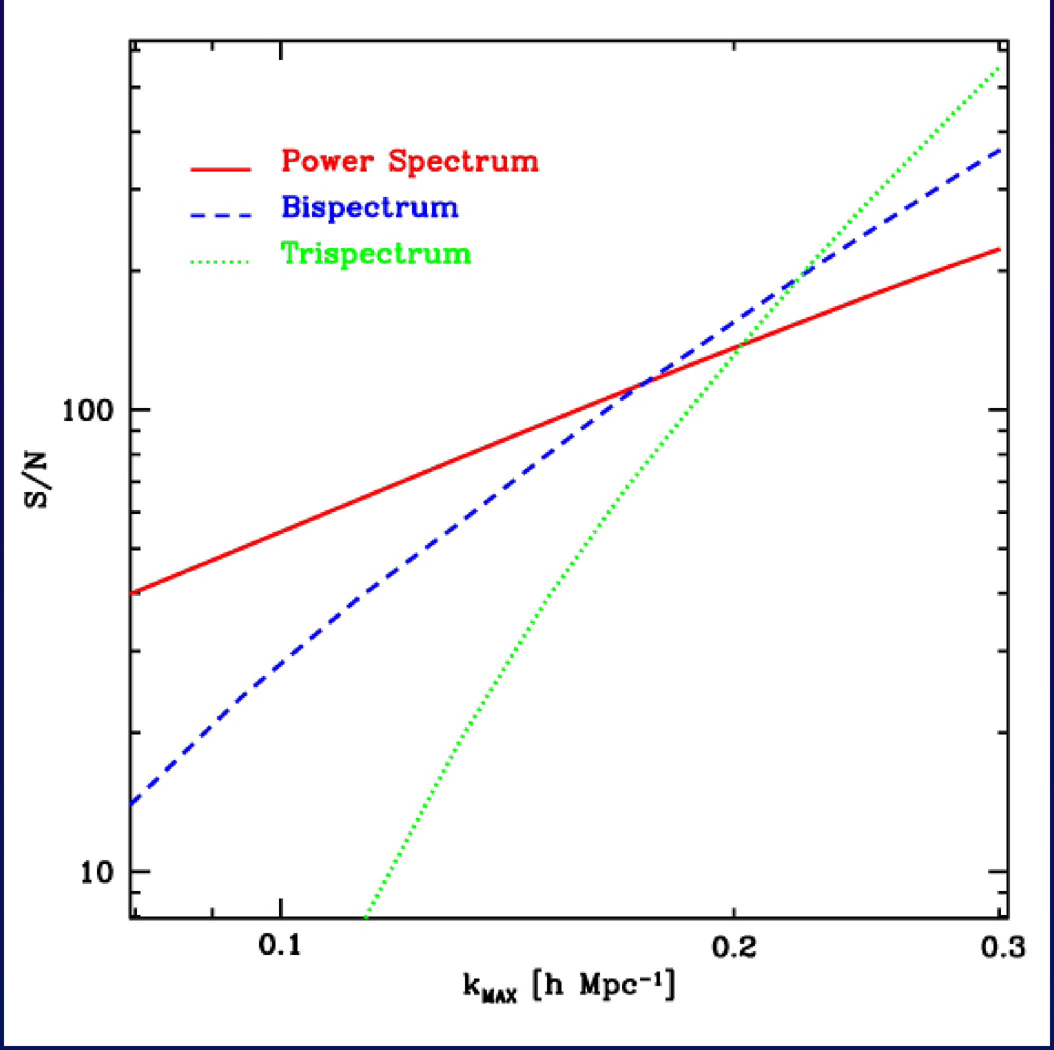
$$\Rightarrow \quad B_g(k_1,k_2,k_3) = b_1^3\, B_m(k_1,k_2,k_3) + b_2\, b_1^2\, [P(k_1)P(k_2) + \mathrm{cyc.}]$$

$$B_g(k_1,k_2,k_3) = b_1^3\ [B_G(k_1,k_2,k_3) + B_I(k_1,k_2,k_3)] + b_2\, b_1^2 \Sigma(k_1,k_2,k_3)$$

- Can we tell them apart?
- How well can we measure them?

Signal-to-noise for matter correlators: the SDSS case

Sum over all configurations up to $k_{\max}$



E.S. & R. Scoccimarro (2005)

Ideal Geometry
variance only

$V = 0.3\, h^{-3}\,\mathrm{Gpc}^3$

$\bar{n} = 0.003(\,h^{-1}\,\mathrm{Mpc})^{-3}$

$$\left(\frac{S}{N}\right)^2_P = \sum_k^{k_{\max}} \frac{P^2}{\Delta P^2}$$

$$\left(\frac{S}{N}\right)^2_B = \sum_{\mathrm{triangles}}^{k_{\max}} \frac{B^2}{\Delta B^2}$$

$$\left(\frac{S}{N}\right)^2_{\widetilde{T}} = \sum_{\mathrm{quads}}^{k_{\max}} \frac{\widetilde{T}^2}{\Delta \widetilde{T}^2}$$

$$\widetilde{T}(k_1, k_2, k_3, k_4) \sim \int d\Omega\, T(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3, \mathbf{k}_4)$$

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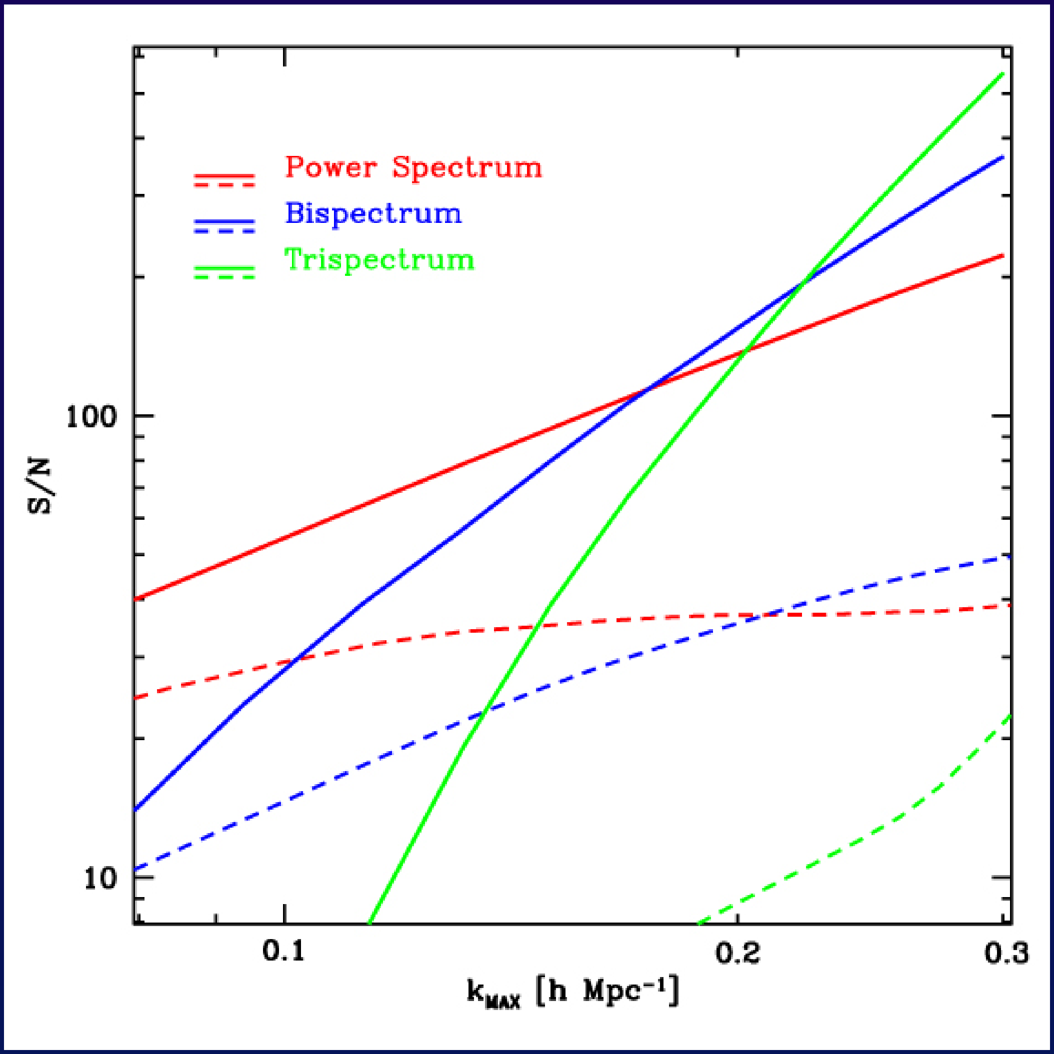

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Ideal Geometry
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SDSS geometry
+ covariance matrices
from 6000 2LPT realizations

$$\left(\frac{S}{N}\right)^2_B = \sum_{\text{triangles i,j}}^{k_{\max}} B_i\, C^{-1}_{ij}\, B_j$$

$$C_{ij} \equiv \langle \delta B_i \delta B_j \rangle$$

Cosmological parameters from a joint analysis of power spectrum and bispectrum

E. S., H. M. Crocce, S. Pueblas & R. Scoccimarro (2006)

We estimate the improvement in constraining cosmological parameters due to *adding* the bispectrum information to a likelihood analysis of the power spectrum, taking into account:

1. the survey geometry (SDSS main sample), by means of the FKP method
2. the covariance properties of power spectrum and bispectrum

$$C_{i,j} \equiv \begin{pmatrix} \langle \delta P_i \delta P_j \rangle & \langle \delta P_j \delta B_{j_1,j_2,j_3} \rangle \\ \langle \delta B_{i_1,i_2,i_3} \delta P_j \rangle & \langle \delta B_{i_1,i_2,i_3} \delta B_{j_1,j_2,j_3} \rangle \end{pmatrix}$$

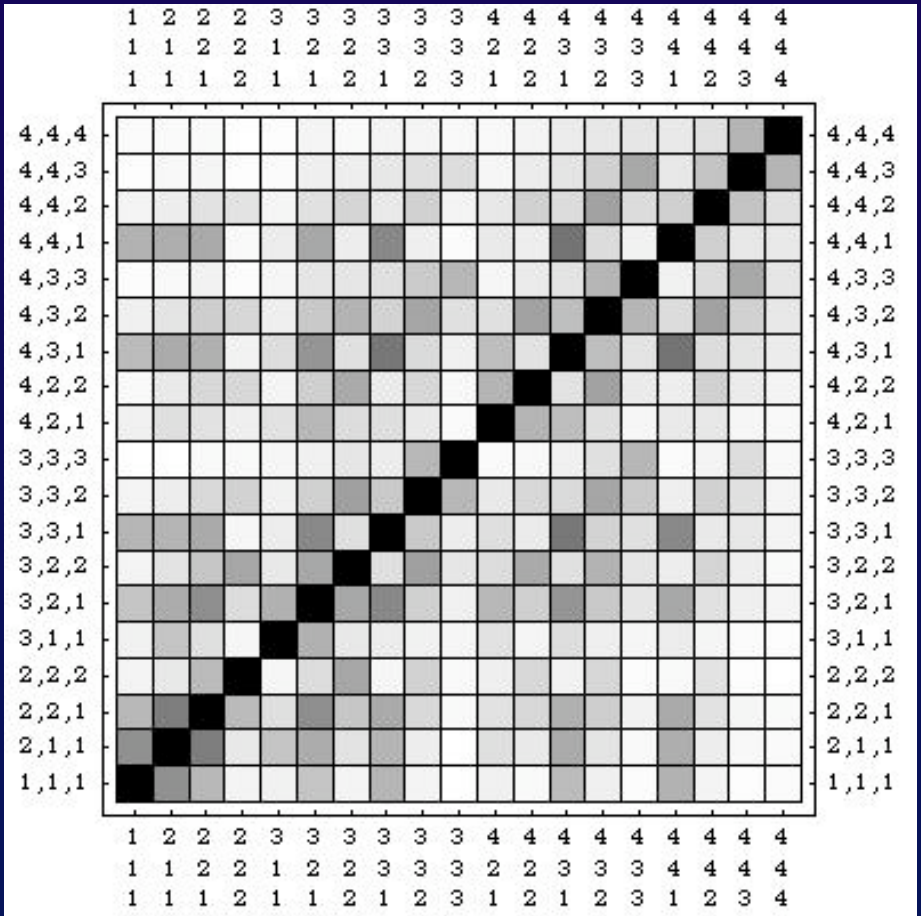
measured from 6000 redshift-space, 2LPT mock catalogues of the SDSS main sample

We assume

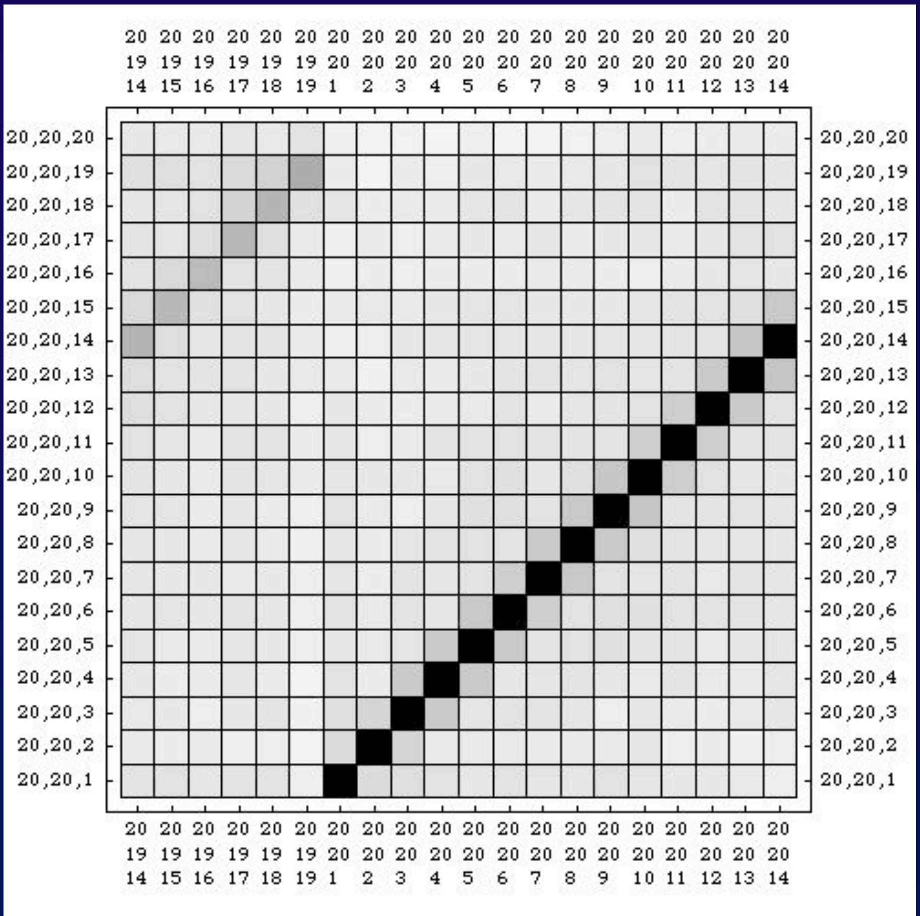
1. Gaussian initial condition
2. a Gaussian likelihood for power spectrum and bispectrum
3. the WMAP3 likelihood function

Bispectrum covariance

cross-correlation coefficients: $r_{ij} = \frac{\langle \delta B_i \delta B_j \rangle}{\sqrt{\langle \delta B_i^2 \rangle \langle \delta B_j^2 \rangle}}$



large scales triangles

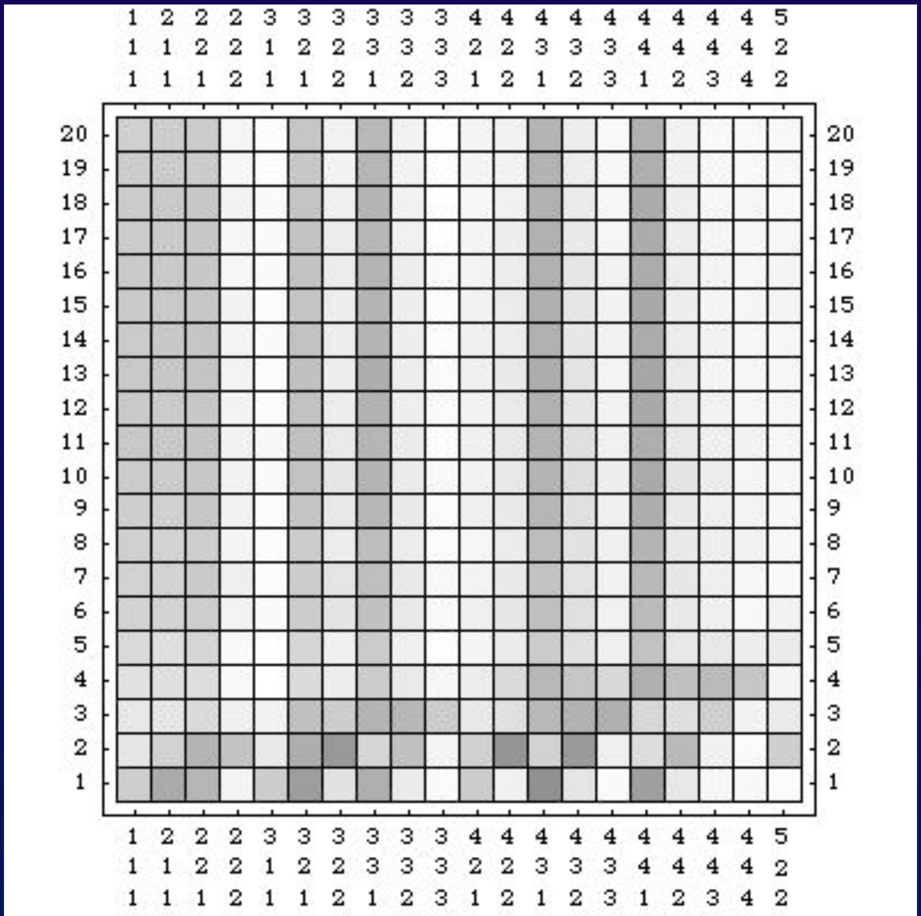


small scales triangles

$\Delta k = 0.015\, h\, \mathrm{Mpc}^{-1}$

Power spectrum - bispectrum covariance

cross-correlation coefficients: $r_{ij} = \frac{\langle \delta P_i \delta B_j \rangle}{\sqrt{\langle \delta P_i^2 \rangle \langle \delta B_j^2 \rangle}}$



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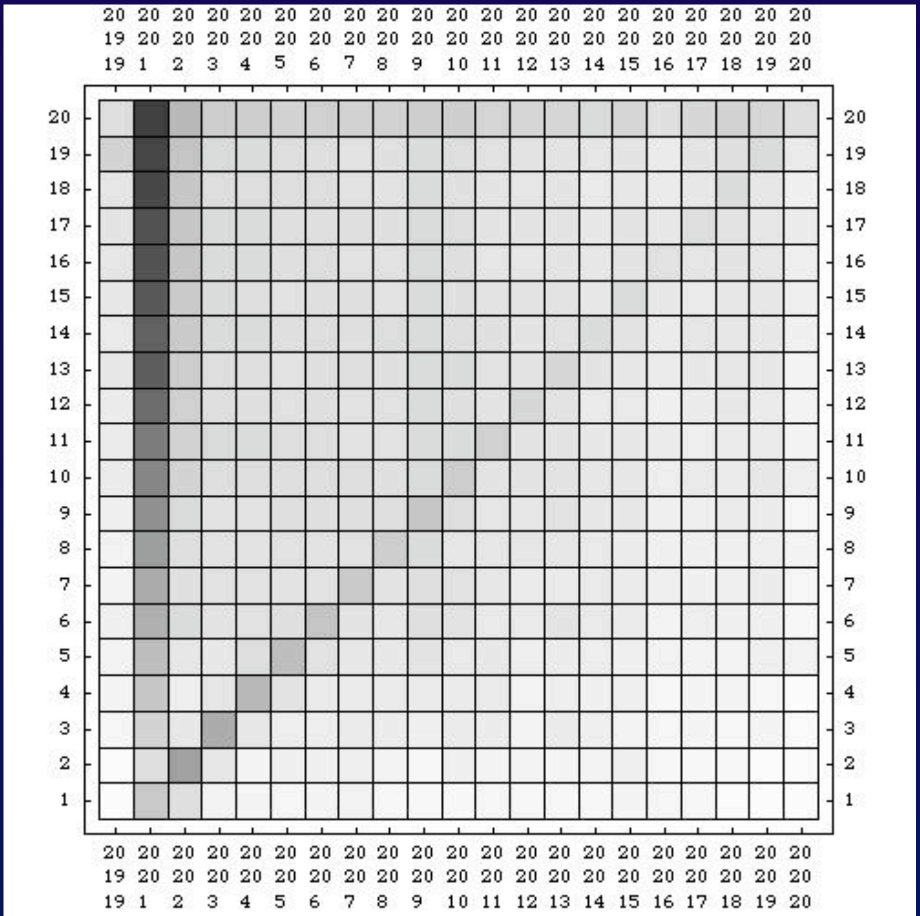

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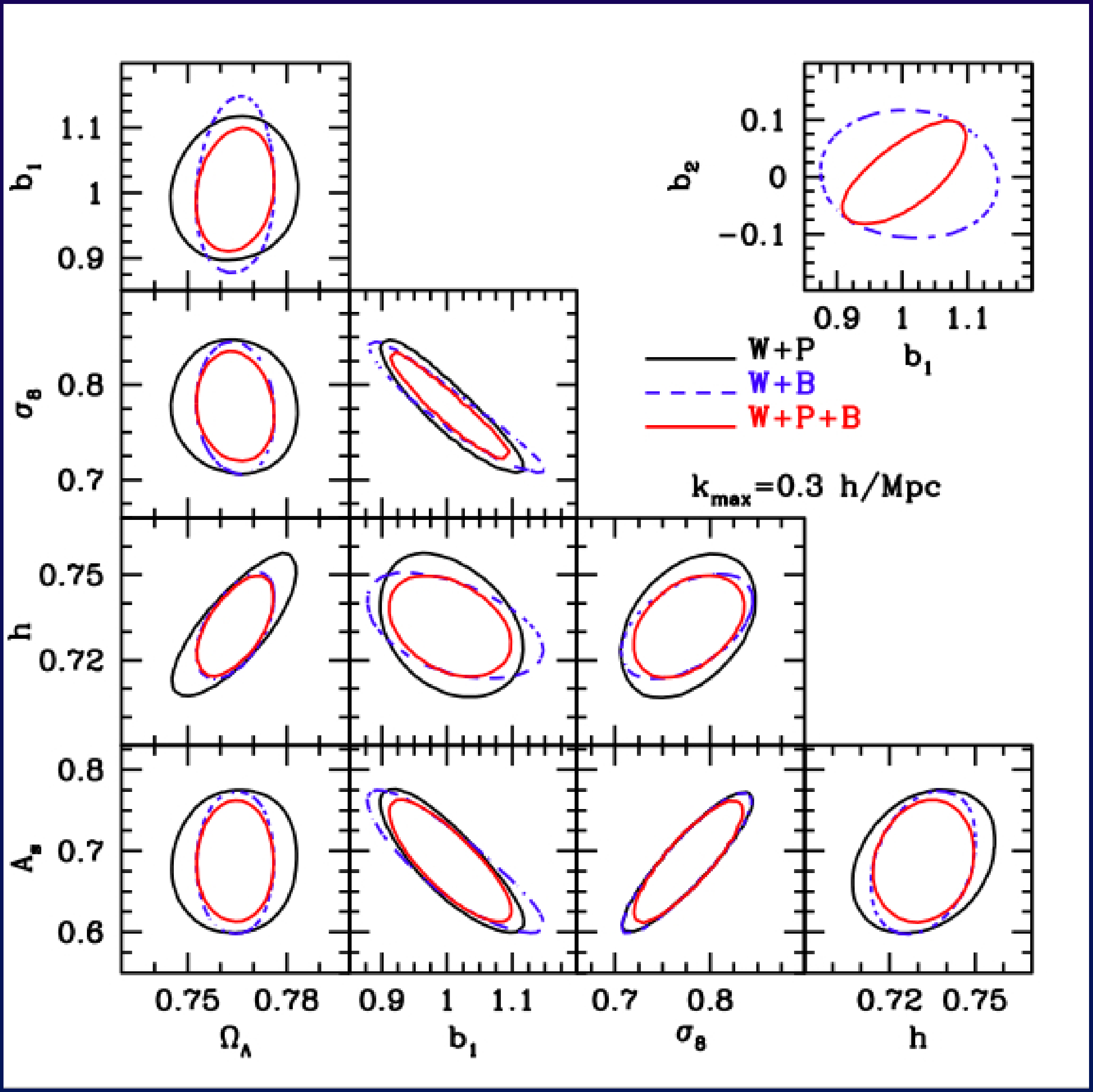
ΛCDM models forecast



all contours 95% CL

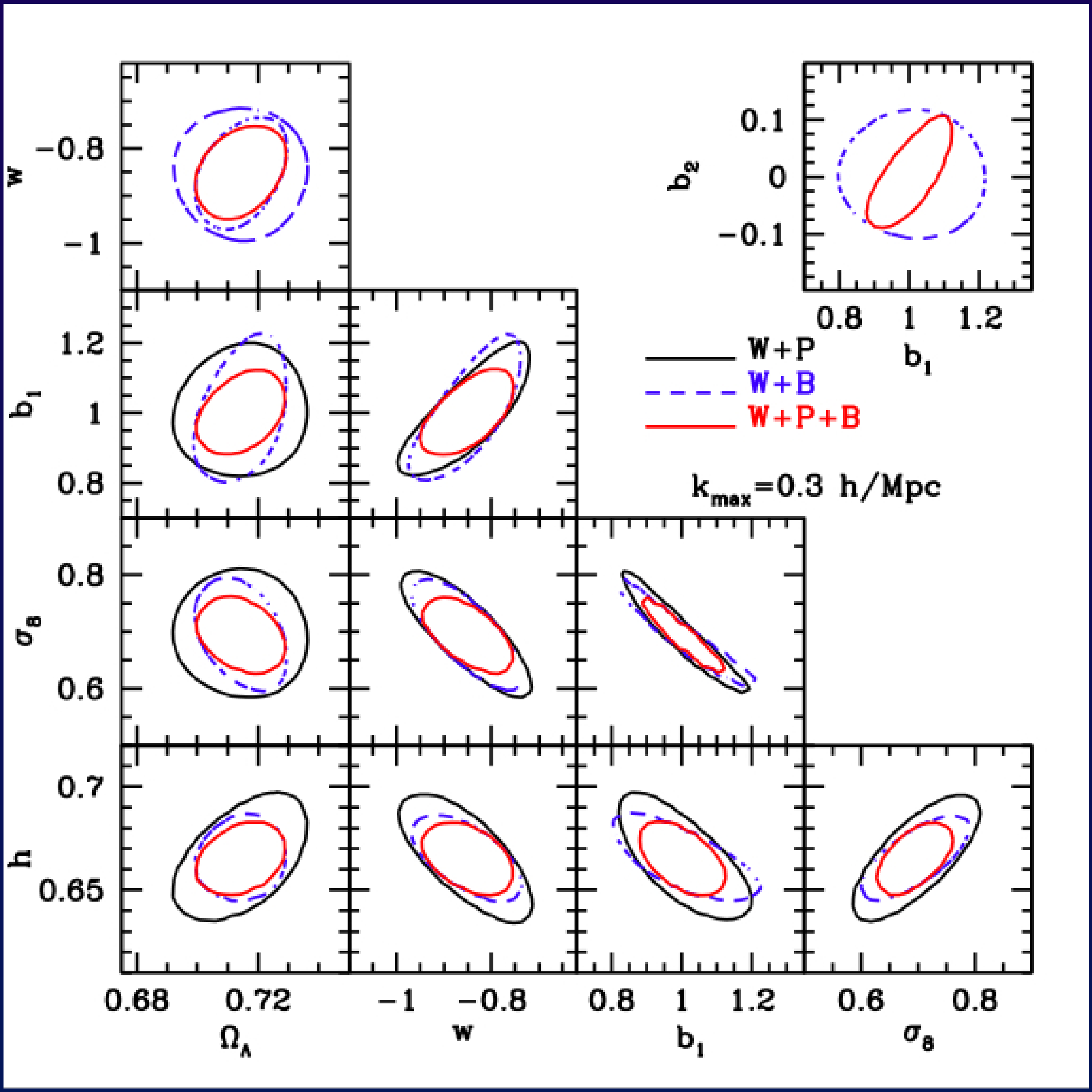
Includes WMAP3

8 parameters:
ω_d, ω_b, Ω_Λ, n_s, A_s, τ
+ b_1, b_2

derived parameters: σ_8, h

1. The bispectrum can serve as a consistency check with respect to the power spectrum analysis
2. The joint analysis can provide sensible improvements to the parameters controlling the fluctuations amplitude: σ_8, Ω_Λ, linear bias
3. Constraint on non-linear bias parameter is unaffected with respect to an analysis with fixed cosmology

wCDM models forecast



all contours 95% CL

Includes WMAP3

9 parameters:
$\omega_d,\, \omega_b,\, \Omega_\Lambda,\, n_s,\, A_s,\, \tau$
$+\, b_1,\, b_2 + w$

derived parameters: $\sigma_8,\quad h$

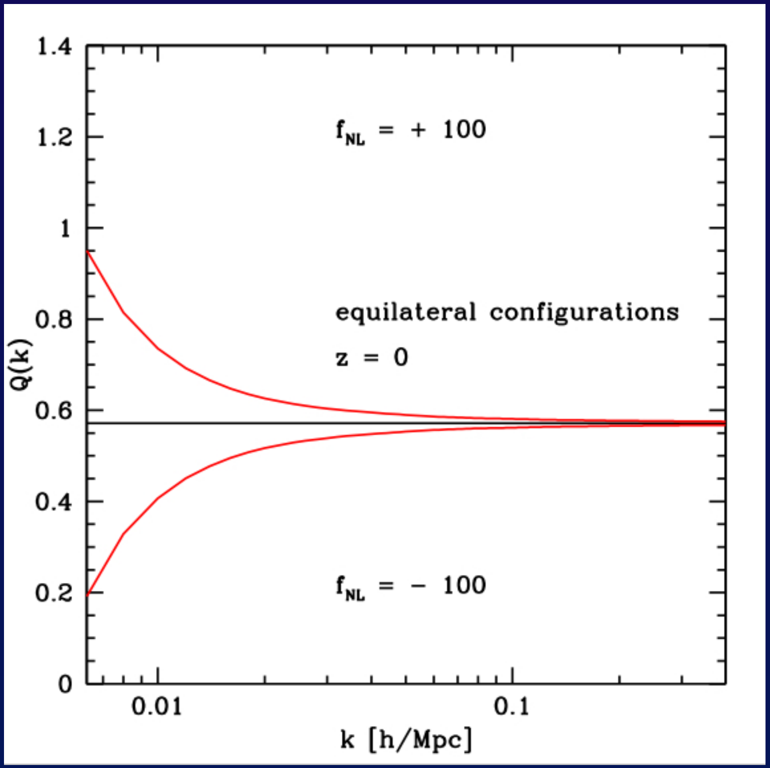
Improvements are more significant as we allow for extra parameters, as for instance the dark energy equation of state

Constraining primordial non-Gaussianity

E. S. & E. Komatsu (2007)

We consider now *Non-Gaussian* initial conditions: the *reduced* galaxy bispectrum becomes

$$Q_B^{(g)} \equiv \frac{B_g(k_1,k_2,k_3)}{P_g(k_1)P_g(k_2)+\text{cyc.}} = \frac{1}{b_1}\left[Q_G + \frac{f_{NL}}{D(z)}\widetilde{Q}_I\right] + \frac{b_2}{b_1^2}$$



We expect for SDSS main sample bispectrum:

$$\Delta f_{NL} \simeq 145 \quad (1{-}\sigma)$$

including all triangles down to $k_{\max} = 0.3\, h\, \mathrm{Mpc}^{-1}$
+ covariance + survey geometry
[R. Scoccimarro, E. S. & M. Zaldarriaga, 2004]

Current CMB constraints are of the order $\Delta f_{NL} \simeq 30$

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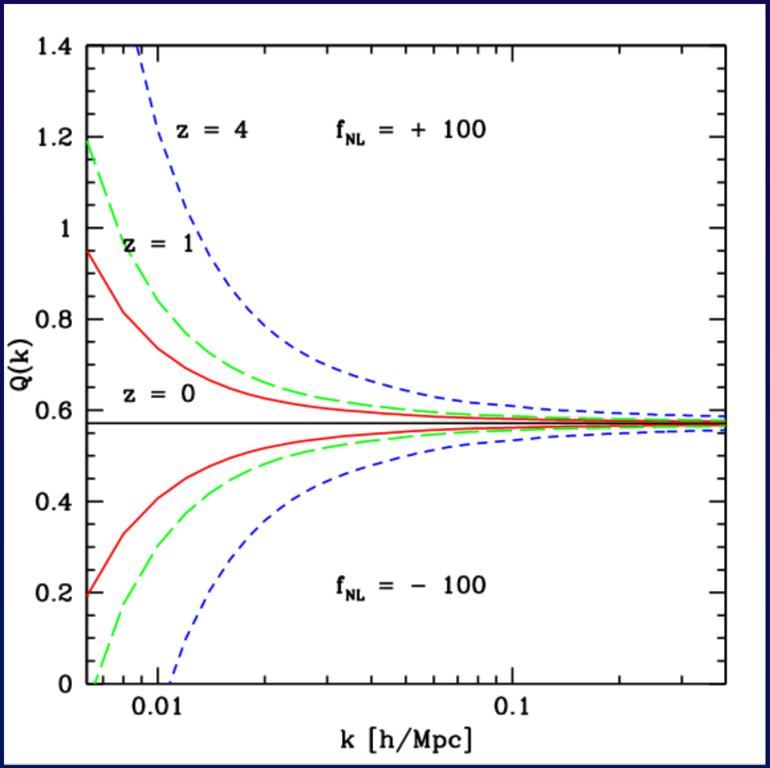


The primordial component is larger at high redshift

What can we expect from future large-volume, high-z, surveys?

High-z surveys: a simple Fisher analysis

We assume a survey to be completely specified by

1. volume, V
2. mean redshift, z
3. galaxy density, n_g

We perform a Fisher matrix analysis of the galaxy reduced bispectrum in terms of three parameters, b_1, b_2 and f_{NL}:

$$Q_B^{(g)} \equiv \frac{B_g(k_1, k_2, k_3)}{P_g(k_1)P_g(k_2) + \mathrm{cyc.}} = \frac{1}{b_1}\left[Q_G + \frac{f_{NL}}{D(z)}\widetilde{Q}_I\right] + \frac{b_2}{b_1^2}$$

taking into account:

1. shot-noise (from n_g)
2. the redshift evolution of galaxy bias, with fiducial values for the bias parameters b_1 abd b_2 obtained in the framework of the Halo Model with a given Halo Occupation Distribution

but *without* taking into account the surveys geometry and the bispectrum covariance

The shapes of primordial non-Gaussianit*ies*

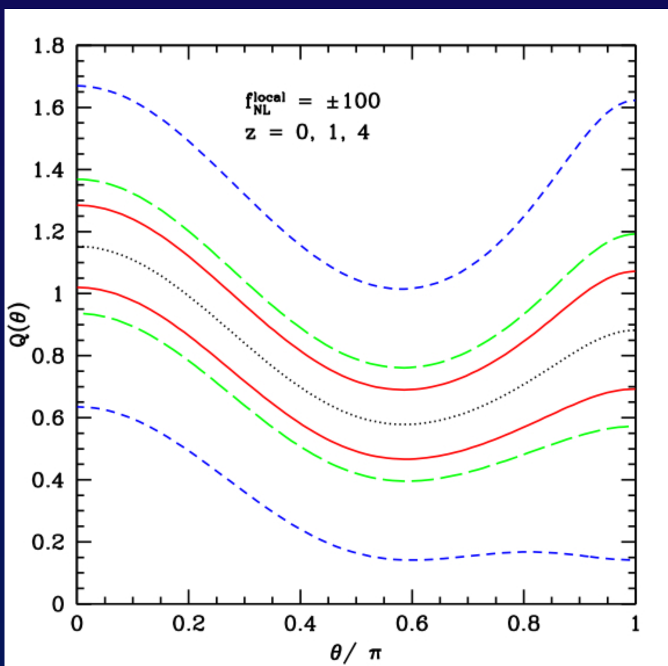
We *do not know* the dependence on the triangle shape of a possible large primordial contribution to the matter bispectrum. In general we have

$$B_\Phi(k_1,k_2,k_3) = f_{NL}F(k_1,k_2,k_3)[P_\Phi(k_1)P_\Phi(k_2) + \text{cyc.}]$$

Following Babich *et al.* (2004) and Creminelli *et al.* (2007), we consider two possibilities:

Local non-Gaussianity

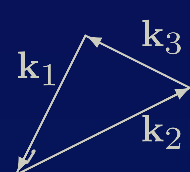
Equilateral non-Gaussianity

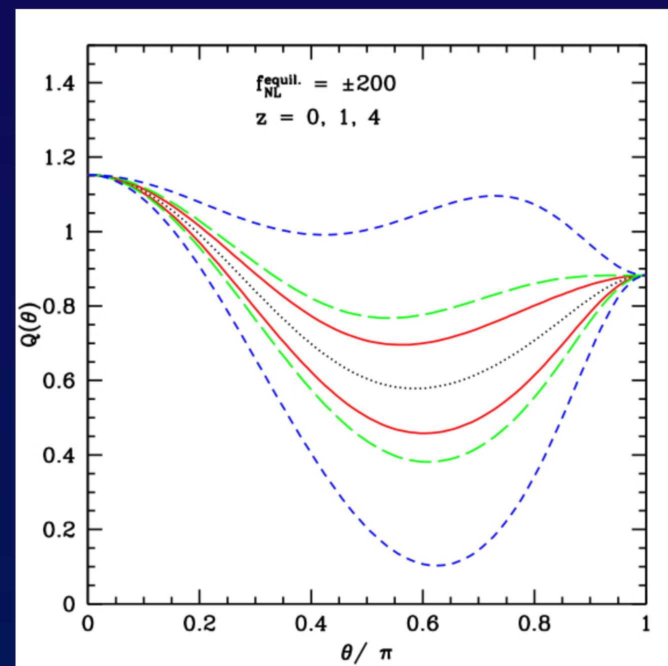


$Q_m = Q_G + Q_I$

$k_1 = 0.01\,h\,\mathrm{Mpc}^{-1}$

$k_2 = 2k_1$

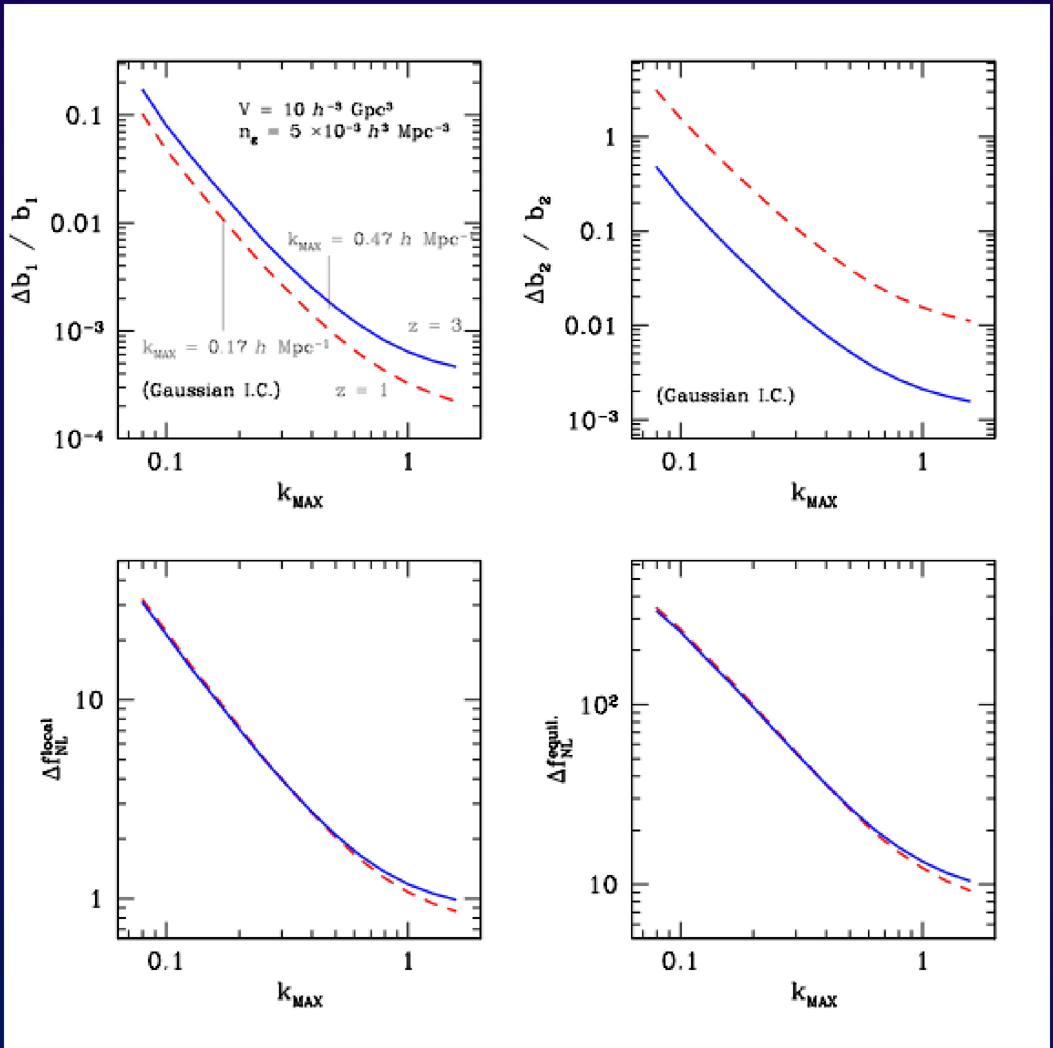




$\Delta f_{NL}^{\text{loc.}} = 34$ (WMAP3)

$\Delta f_{NL}^{\text{eq.}} = 147$ (WMAP3)

The choice of $k_{\max}$



$k_{\max}$ represents the smallest scale at which we still *trust* our model for the galaxy bispectrum

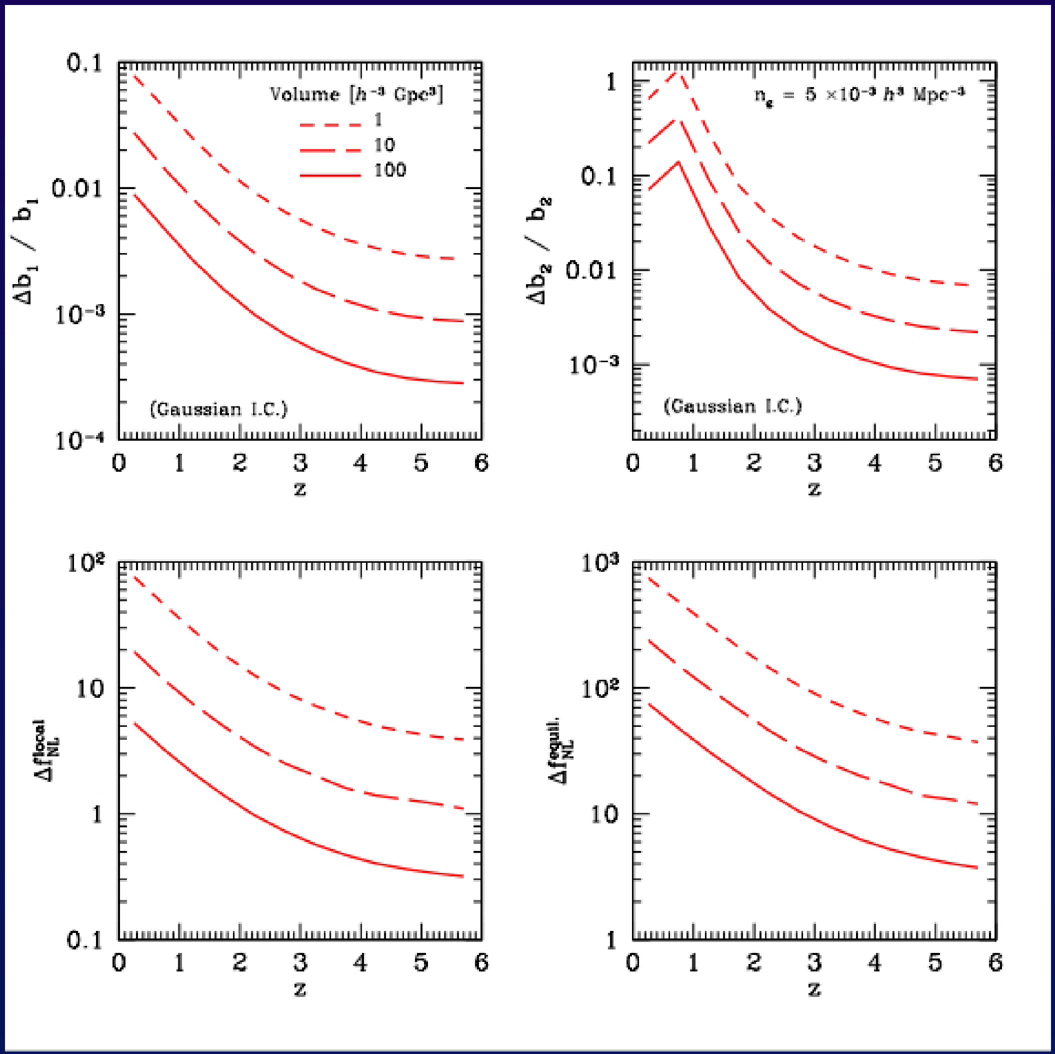
$$\left(\frac{S}{N}\right)^2_B \sim N_T \sim k^3_{\max}$$

We choose

$$k_{\max} \equiv \frac{\pi}{2R}$$

with $\sigma_m(R, z) = 0.5$

Dependence on redshift

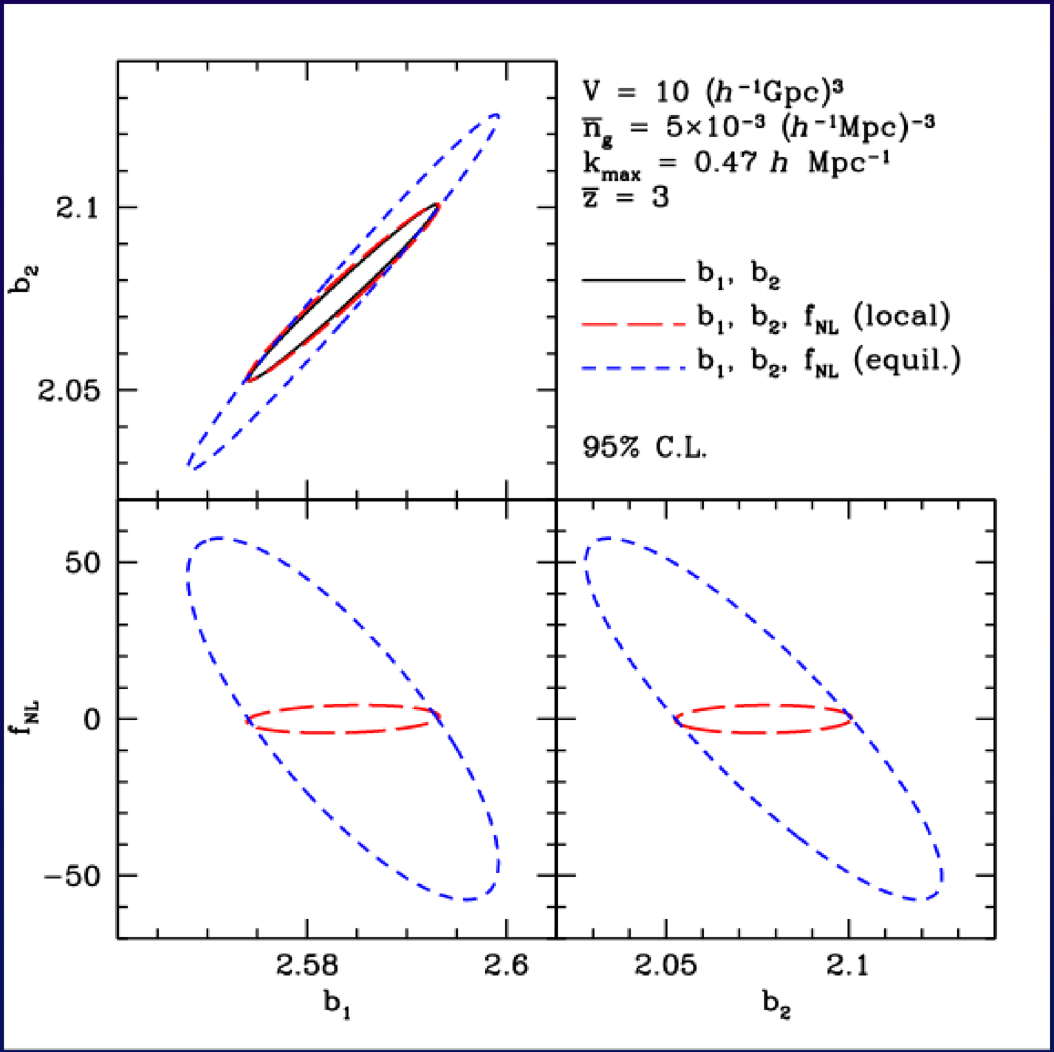


Higher redhsift allows for *larger* $k_{\max}$

$\Rightarrow$ Shot-noise is a strong limiting factor for high-z surveys:

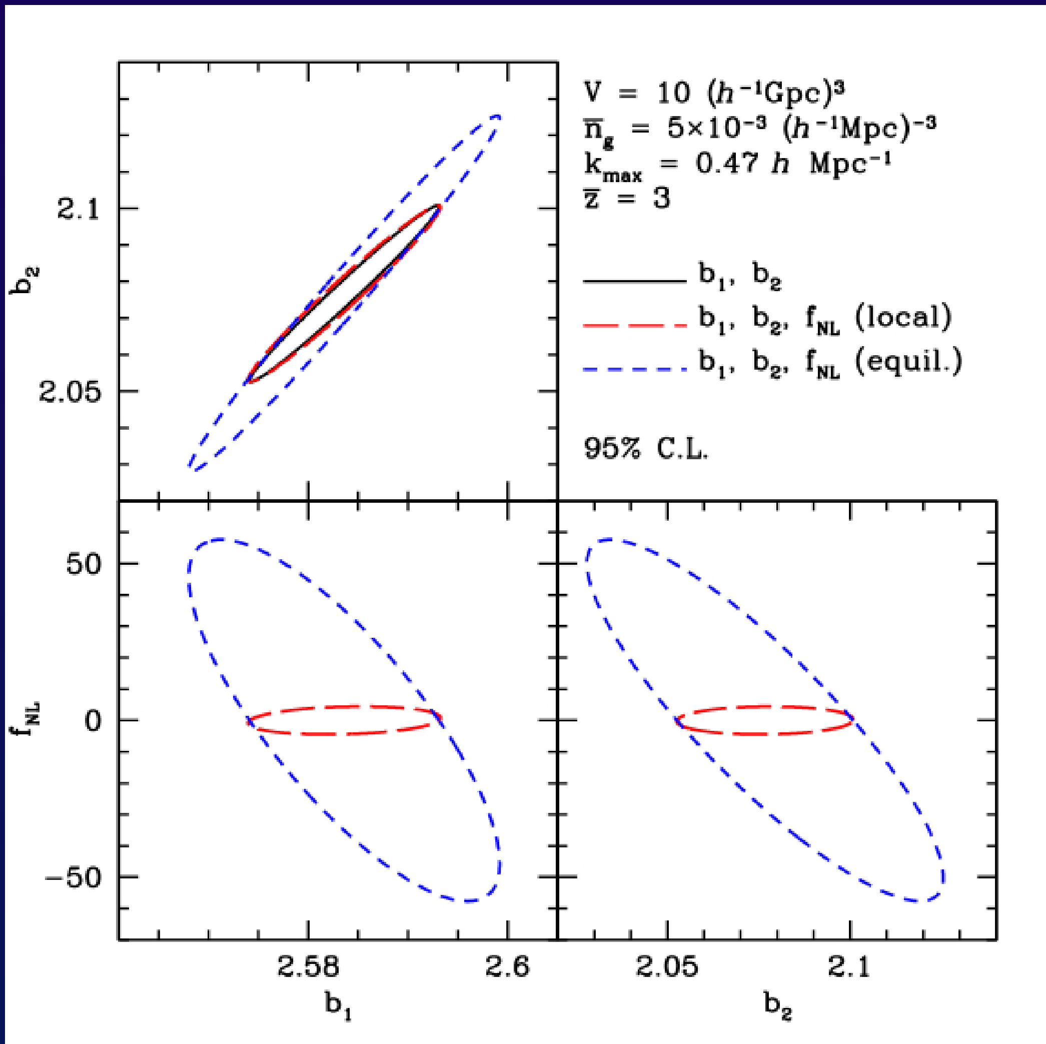
a *very* large number density $(n_g \geq 10^{-3}\, h^3\, \mathrm{Mpc}^{-3})$ is required at $z \geq 3$.

Degeneracies



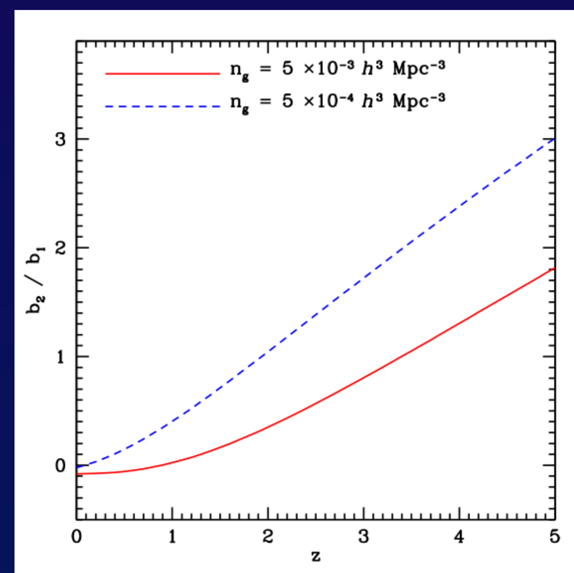
The equilateral model presents a large degeneracy with the bias parameters:

Degeneracies



The equilateral model presents a large degeneracy with the bias parameters:

However, an analysis in terms of the HOD lifts the degeneracy almost enterily by introducing a theoretical prior on the relation between linear and quadratic bias



$$b_i \simeq \frac{1}{n_g} \int_{M_{min}} dM n_h(M,z)\, b_i^h(M,z) \langle N \rangle_M ,$$

Present & future surveys: forecast

	V	n_g	z	$k_{\max}$	$\Delta b_1/b_1$	$\Delta b_2/b_2$	$\Delta f_{NL}^{\text{loc.}}$	$\Delta b_1/b_1$	$\Delta b_2/b_2$	$\Delta f_{NL}^{\text{eq.}}$
SDSS	0.3	30	0	0.09	26%	151%	260	38%	420%	1800
LRG	0.72	1	0.35	0.11	10%	37%	110	16%	76%	1000
APO-LSS	3.8	4	0.35	0.11	4%	33%	35	6.4%	76%	390
WFMOS1	1.6	5	0.7	0.14	4%	21%	41	6.6%	48%	430
	2.4	5	1.1	0.18	2%	8%	23	3.5%	17%	270
	combined				-	-	20	-	-	230
ADEPT	45	1	1.25	0.20	0.7%	1.8%	6.1	1.0%	3.2%	73
	55	1	1.75	0.26	0.5%	1.2%	4.5	0.7%	2.1%	53
	combined				-	-	3.6	-	-	43
WFMOS2	0.5	5	2.55	0.38	1.2%	4.8%	26	2.9%	8.7%	260
	0.5	5	3.05	0.48	1.6%	4.0%	22	2.4%	6.9%	210
	combined				-	-	17		-	160
HETDEX	0.68	5	2.25	0.34	1.7%	4.7%	24	2.7%	8.8%	240
	0.69	5	2.75	0.42	1.5%	3.8%	20	2.2%	6.7%	200
	0.67	5	3.25	0.53	1.3%	3.3%	18	2.0%	5.6%	180
	0.64	5	3.75	0.65	1.3%	3.2%	17	1.9%	5.1%	160
	combined				-	-	9.6	-	-	95

Marginalized, 1-σ uncertainties

An hypothetical, all-sky survey out to $z \sim 5$ should be able to provide

$$\Delta f_{NL}^{\text{loc.}} \simeq 0.2 \qquad \text{and} \qquad \Delta f_{NL}^{\text{eq.}} \simeq 2$$

Conclusions

- *Galaxy bispectrum and trispectrum in current surveys have as much signal-to-noise as the power spectrum when all measurable configurations down to mildly non-linear scales are taken into account.*
- *The information contained in the bispectrum is not limited to galaxy bias: a likelihood analysis of large-scale structure that combines power spectrum and bispectrum can improve our determination of cosmological parameters.*
- *Future high-z galaxy surveys can provide constraints on primordial non-Gaussianity comparable, or even better, than those from CMB observations.*

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Future work

- A more accurate model for galaxy biasing, possibly based on (renormalizable) perturbative techniques is required to fully exploit the information in higher-order correlation functions.
- The complementarity of Large-Scale Structure and CMB observations can become even more important as one allow for the possibility of a strongly scale-dependent primordial non-Gaussianity
- At high-z one might start to worry as well about weak lensing of higher-order correlation functions ...