Detection of gravitational lensing in the CMB

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Reference: Smith, Zahn, and Doré, 0705.3980
with key contributions from Mike Nolta
Gravitational lensing in the CMB

CMB photons are deflected by gravitational potentials between last scattering and observer. This remaps the CMB while preserving surface brightness:

\[ T(\hat{n}) \rightarrow T(\hat{n} + d(\hat{n})) \]

where \( d(\hat{n}) \) is a vector field giving the deflection angle along line of sight \( \hat{n} \).

Wayne Hu
Gravitational lensing in the CMB

To first order, deflection angles are a pure gradient field:

\[ d_a(\hat{n}) = \nabla_a \phi(\hat{n}) \]

where the lensing potential is given by the line-of-sight integral

\[ \phi(\hat{n}) = -2 \int_0^{\chi_*} d\chi \left( \frac{\chi_* - \chi}{\chi \chi_*} \right) \psi(\chi \hat{n}, \eta_0 - \chi) \]

RMS deflection: \( \sim 2.5 \text{ arcmin} \), coherent on degree scales (\( \ell \sim 100 \))

Scope of talk: We will present a 3.4\( \sigma \) detection from combining WMAP3 with radio galaxy counts from NVSS.
Gravity waves from inflation:
- In polarization, lensing is a contaminant unless removed

Neutrino mass:
- Complementary to neutrino oscillations ($\sum m_\nu$ vs $\Delta m^2_{\nu}$)
- e.g. Planck: $\sim 0.14$ eV from CMB lensing (Lesgourges et al 2005)

Counterpart to galaxy weak lensing:
- Probes same lensing potential as high-redshift galaxies, but completely different systematics
- **CMB**: Beam effects, point sources, SZ and other foregrounds
- **Galaxies**: PSF correction, intrinsic alignments, photo-z errors

Most robust measurement: cross-correlation between the two?
How can CMB lensing be detected in data?

**First idea:** Try to detect effect of lensing on power spectrum $C_{\ell}^{TT}$.

Effect is too small in WMAP: lensing can only be “detected” at $(1/3)\sigma$ directly from power spectrum.
CMB lens reconstruction

Idea: From observed CMB, reconstruct deflection angles (Hu 2001)
CMB lens reconstruction

Lensing weakly correlates CMB modes with $l \neq l'$:

$$\langle T(l) T(l')^* \rangle \propto \phi(l - l').$$

Reconstructed field $\hat{\phi}$ is quadratic in CMB temperature:

$$\hat{\phi}(\hat{n}) = \partial^a [\alpha(\hat{n}) \partial_a \beta(\hat{n})]$$

$$\alpha(\hat{n}) = \int \frac{d^2 l}{(2\pi)^2} \left( \frac{1}{C_{\ell TT} + N_{\ell TT}} \right) T(l) e^{il \cdot \hat{n}}$$

$$\beta(\hat{n}) = \int \frac{d^2 l}{(2\pi)^2} \left( \frac{C_{\ell TT}}{C_{\ell TT} + N_{\ell TT}} \right) T(l) e^{il \cdot \hat{n}}$$

Second idea for detecting CMB lensing: look for extra power in $\hat{\phi}$. Compute $C_{\ell}^{\phi \phi}$: quadratic in $\hat{\phi}$, or four-point in CMB.

WMAP3: statistical errors only give $1\sigma$.

In addition, systematics likely to be difficult...
CMB lens reconstruction: cross correlation

Third idea for detection: cross-correlate $\hat{\phi}$ to galaxy counts

⇒ Highly correlated, so “boosts” signal-to-noise

Systematics also tamer in cross-correlation

Compute $C^{\phi g}_\ell$: three-point estimator

First attempt: LRG’s from Sloan, $1\sigma$ result (Hirata et al 2004)

Our approach: Radio galaxies from NVSS.
1.4 GHz sky catalog, 50% complete at 2.5 mJy.

Mostly AGN-powered radio galaxies, quasars, nearby star-forming galaxies

Well-suited for cross-correlating to WMAP lensing potential:

- Nearly full sky coverage ($f_{\text{sky}} = 0.82$)
- Low shot noise ($N_{\text{gal}} \sim 1.8 \times 10^6$)
- High median redshift ($z \sim 0.9$)
Pipeline

Simulation (steps 1-4):
- Monte Carlo simulations used for calibration, assigning errors

Analysis (steps 5-8):
- Filter WMAP (Q-band, V-band, W-band) and NVSS datasets
- Lens reconstruction from WMAP
- Cross-correlate: estimate $C_{\ell}^{\phi g}$ in bands
Pipeline

Start
(1) \rightarrow \text{Gaussian fields } \{g_{\ell m}, \phi_{\ell m}, a_{\ell m}^{\text{unlensed}}\}

(2) \rightarrow \text{Lensed CMB } a_{\ell m}^{\text{lensed}}

\rightarrow \text{WMAP data}
(5) \rightarrow \text{Filtered CMB } \tilde{a}_{\ell m}

(6) \rightarrow \text{Reconstructed potential } \phi_{\ell m}

\rightarrow \text{Lensing estimator } \hat{C}_{\ell}^{\phi g}

CMB filtering (step 5):

Input: raw WMAP maps

Output: maximum likelihood map obtained by combining all channels
Pipeline

Gaussian fields \( \{ g_{\ell m}, \phi_{\ell m}, a_{\ell m}^{\text{unlensed}} \} \)

Lensed CMB \( a_{\ell m}^{\text{lensed}} \)

WMAP data

Filtered CMB \( \tilde{a}_{\ell m} \)

NVSS data

Filtered galaxy field \( \tilde{g}_{\ell m} \)

Reconstructed potential \( \phi_{\ell m} \)

Lensing estimator \( \hat{C}_{\ell}^{\phi g} \)

Lens reconstruction (step 6):

**Input:** Maximum likelihood CMB map

**Output:** Reconstructed lensing potential \( \hat{\phi} \) (shown bandlimited to \( 20 \leq \ell \leq 40 \):
Pipeline

Gaussian fields $\{g_{\ell m}, \phi_{\ell m}, a_{\ell m}^{\text{unlensed}}\}$

Lensed CMB $a_{\ell m}^{\text{lensed}}$

WMAP data

Filtered CMB $\tilde{a}_{\ell m}$

Reconstructed potential $\phi_{\ell m}$

Filtered galaxy field $\tilde{g}_{\ell m}$

Reconstructed potential $\tilde{\phi}_{\ell m}$

Lensing estimator $\hat{C}_\ell^\phi g$

NVSS filtering (step 7):

Input: NVSS source catalog

Output: Maximum likelihood galaxy map (shown bandlimited to $20 \leq \ell \leq 40$):
Pipeline

Gaussian fields \{g_{\ell m}, \phi_{\ell m}, a_{\ell m}^{\text{unlensed}}\} →

Lensed CMB \(a_{\ell m}^{\text{lensed}}\) →

WMAP data →

Filtered CMB \(\tilde{a}_{\ell m}\) →

Reconstructed potential \(\phi_{\ell m}\) →

Filtered galaxy field \(\tilde{g}_{\ell m}\) →

Lensing estimator \(\hat{C}_\ell^{\phi g}\) →

Cross-correlation (step 8):

Input: Lensing potential and galaxy fields (shown bandlimited to \(20 \leq \ell \leq 40\)):

Output: Cross correlation \(C_{\ell}^{\phi g}\), with estimator normalization and statistical errors computed by Monte Carlo

\(\ell^2 C_{\ell}^{\phi g} = (33.2 \pm 10.5) \times 10^{-7}\)

\((20 \leq \ell \leq 40, \text{stat.})\)
Main result (statistical errors only)

Values obtained by cross-correlating WMAP and NVSS datasets

Monte Carlo errors obtained by cross-correlating WMAP and NVSS simulations

Several-sigma (relative to simulations) correlation observed in data

How do we know that the correlation is lensing, rather than something else?

Rest of talk: Null tests, systematics.
Check: different ways of computing Monte Carlo errors

Our default procedure is to correlate pairs of sims, but could also:

- Correlate WMAP data with NVSS sims
- Correlate WMAP sims with NVSS data

Three-way consistency is an important check

Shows that result only depends on correctness of one of the simulation pipelines
Check: frequency dependence

Analyze each frequency channel in WMAP separately

Results consistent between frequencies

Because different frequencies are correlated, cannot combine three sets of error bars in a straightforward way.

Best overall result obtained from Q+V+W combined map as shown previously
Lensing potential is expected to be a pure gradient:

\[ d_a(\hat{n}) = \nabla_a \phi(\hat{n}) \]

but consider fictitious curl component instead:

\[ d_a(\hat{n}) = \epsilon_{ab} \nabla^b \psi(\hat{n}) \]

**Null test:** Should get \( C_\ell^{\psi g} = 0 \).

\( \chi^2 = 12.1/8 \): high at 1\( \sigma \), so \( C_\ell^{\psi g} \) null test passes.

Null test cannot monitor parity-invariant contaminants (e.g. point sources), analogous to \( C_\ell^{EB} = 0 \) in CMB polarization experiments.
Bispectrum perspective

Alternate approach to lensing estimator $C_{\ell}^{\phi g}$: optimal estimator for 3-point signal $b_{\ell_1 \ell_2 \ell_3}$ induced by gravitational lensing.

**Bispectrum:** depends on triple $\ell_1 \ell_2 \ell_3$ (power spectrum $C_{\ell}$ depends on single $\ell$).
NVSS systematics: bright sources

NVSS maps show “ringing” near bright sources

We treat this by masking \( \sim 2000 \) sources \( > 1 \) Jy

Source mask included in statistical errors

We include the mask in all results, but neither \( C_{\ell}^{gg} \) nor \( C_{\ell}^{\phi g} \) changes significantly.

NVSS raw map: \( 2^\circ \times 2^\circ \)
Consider NVSS galaxy power spectrum $C^{gg}_\ell$.

If analyzed straightforwardly, obvious systematic contamination at low $\ell$

**Known systematic effect:** equatorial striping (excess power for $\ell \lesssim 100$)

Projecting out $m = 0$ modes appears to remove contaminant (no evidence for higher values of $m$)
NVSS systematics: modeling uncertainty

NVSS redshift distribution is not known very well; we found that existing models, e.g. Gaussian (Pietrobon 2006)

\[ \frac{dN}{dz} \propto \exp \left( -\frac{(z - 1.1)^2}{2(0.8)^2} \right) \]

did not fit $C^g_g$ well.

However a small tweak, e.g. “lopsided Gaussian”:

\[ \frac{dN}{dz} \propto \begin{cases} 
\exp \left( -\frac{(z - 1.1)^2}{2(0.8)^2} \right) & (z < 1.1) \\
\exp \left( -\frac{(z - 1.1)^2}{2(0.3)^2} \right) & (z > 1.1) 
\end{cases} \]

results in a good fit. (Exception: $\ell \leq 10$.)
WMAP systematics: galactic foregrounds

Most of the foreground signal excluded by Kp0 mask: galactic plane, \(\sim 700\) resolved point sources.

**Dust:** use FDS template (Finkbeiner et al 1999)
Frequency dependence \(\propto \nu^2\)
V-band (60 GHz) RMS: 6.4 \(\mu K\).

**Free-free emission:** use \(H\alpha\) template (Finkbeiner 2003, Bennett et al 2003)
Frequency dependence \(\propto \nu^{-2.14}\)
V-band (60 GHz) RMS: 4.8 \(\mu K\).
WMAP systematics: galactic foregrounds

**Synchrotron:** templates available on degree scales (Haslam 408 MHz, WMAP K − Ka), but not on CMB scales ($\ell \sim 400$) used for lensing.

Assume systematic errors from synchrotron equal to dust + free-free!

<table>
<thead>
<tr>
<th>$(\ell_{\text{min}}, \ell_{\text{max}})$</th>
<th>Statistical</th>
<th>Dust</th>
<th>Galactic Free-free</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>(2, 20)</td>
<td>17.4 ± 22.4</td>
<td>±0.4</td>
<td>±1.4</td>
<td>±3.6</td>
</tr>
<tr>
<td>(20, 40)</td>
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<td>±1.4</td>
</tr>
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<td>(40, 60)</td>
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WMAP beams are asymmetric, but treated as isotropic in pipeline

(Q-band: 20% elliptical, V,W-band: 10-20 dB substructure)

- Include beam asymmetry in simulations, treat as source of systematic error.
- Multipole expansion of beam ($s = 0$ isotropic, $s = 1$ dipole, ...)
- Convolution with higher-$s$ multipoles includes sky-varying kernel which depends on scan strategy
After beam asymmetry, the only beam effect is uncertainty in the isotropic part

<table>
<thead>
<tr>
<th>$(\ell_{\text{min}}, \ell_{\text{max}})$</th>
<th>Statistical</th>
<th>Asymmetry</th>
<th>Beam Uncertainty</th>
<th>Total</th>
</tr>
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Point sources: approach

Only CMB point sources which are correlated to NVSS contribute

Too difficult to estimate point source contribution from models!

**Approach:** estimate level of point source contamination from data

Cross spectrum $C_{\ell}^{Tg}$ has wrong scaling; must estimate bispectrum

$$\Delta C_{\ell}^{Tg} \propto \sum_i S_i n_i \quad b_{\ell_1 \ell_2 \ell_3} \propto \sum_i S_i^2 n_i$$

Most general bispectrum considered: $b_{\ell_1 \ell_2 \ell_3} = F(\ell_3)$

- Allows arbitrary point source clustering on degree scales
- Assumes clustering negligible on CMB scales ($\ell \sim 400$)
- Nonlinear evolution neglected
Point sources: estimator

Optimal estimator for point source bispectrum:
- “Quadratic reconstruction” $s(\hat{n})$ for point source power in CMB
- Cross-correlate to NVSS: $C_{\ell}^{sg}$.

No evidence for point sources seen in data: $\chi^2 = 11.7/12$.

**Allows tight systematic errors:** any point source contribution must be hidden beneath the detection threshold.
Consider ensemble of simulations with varying point source levels.

Restrict to simulations with same observed point source level as data.

Point source contribution to lensing:

\[ \Delta \hat{C}_\ell^{\phi g} = (-0.5 \pm 1.7) \times 10^{-7} \]

Treat shift as part of systematic error: \( \pm 2.2 \times 10^{-7} \).

**One final detail:** Sunyaev-Zeldovich effect can be treated as part of point source contribution for WMAP (SZ clusters not resolved by WMAP beam).
Point sources: bottom line

Point sources are largest source of systematic error:

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<th>$(\ell_{\text{min}}, \ell_{\text{max}})$</th>
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Combine statistical errors with systematic errors considered previously:

- WMAP beam effects
- Galactic CMB foregrounds
- Point sources + SZ

To assess total statistical significance: fit to one large bandpower in multiple of fiducial $C_{\ell}^{\phi g}$.

Result: $1.15 \pm 0.34$, i.e. a $3.4\sigma$ detection, consistent with the fiducial model.
Milestone: 3.4σ detection, not enough for precision cosmology but in agreement with the predicted level.

Future prospects: unlikely to exceed 5σ in next few years; different story after Planck/SPT/ACT (e.g. Hu 2001: ~ 60σ from Planck alone).

Many systematic checks: “sims vs data”, frequency dependence, curl null test, WMAP beam effects, point sources + SZ

Systematics largely unexplored outside WMAP/NVSS datasets: point source + SZ contamination seems to be the biggest problem (in particular, beam effects are small) but this may not apply to upcoming higher-resolution surveys.