Measuring Primordial Non-Gaussianity using CMB T & E data

Benjamin Wandelt
Amit Yadav
University of Illinois at Urbana-Champaign

Life beyond the Gaussian, KITP, June 8, 2007
Outline

- **Looking for non-Gaussianity**
  - Generic primordial non-Gaussianity/anisotropy can be probed using reconstructed primordial curvature perturbation
  - Complementarity of T and E data

- **Fast estimator for** $f_{NL}$ **using temperature and polarization**
  - Generalizes Komatsu, Spergel and Wandelt 2005

- **Towards the Planck data**
  - Dealing with real data—inhomogeneous noise, sky cut
Tomographic reconstruction of inflationary scalar curvature perturbations from CMB

We construct filters that invert linear radiative transport. Generates a single scalar that contains all the information from T&E. Anyone intending to test primordial non-Gaussianity and anisotropy in T&E data should do so using curvature perturbations obtained with our filters.

Yadav and Wandelt 2006
Reconstructed Primordial Perturbations

\[
\Phi_{\ell m} = O_l a_{\ell m}
\]

SW limit

\[
\frac{\delta \phi}{\delta T} = -\frac{1}{3} \frac{\delta T}{T}
\]

Reconstructed Primordial perturbations with T alone

Wiener filter \( O \)

\[
\beta^{i}_{\ell}(r) = \frac{2b^{i}_{\ell}}{\pi} \int k^2 dk P_{\phi}(k) g^{i}_{\ell}(k) j_{\ell}(kr)
\]
Temperature and polarization are complementary

Out of phase

Yadav, and Wandelt, PRD (2005)
Yadav, and Wandelt, PRD (2005)
Yadav, and Wandelt, PRD (2005)
Reconstructed perturbations at different radii

Curvature fluctuations

Yadav, and Wandelt, PRD (2005)
Detecting of Non-Gaussianity specific models

\[ \Phi(x) = \Phi_G(x) + f_{NL} \Phi_G^2(x) \]

Characterizes the amplitude of non-Gaussianity

**Non-Gaussianity from Inflation**

- \( f_{NL} \approx 0.05 \) canonical inflation (single field, couple of derivatives)  \( \text{(Maldacena 2003, Acquaviva et al. 2003)} \)
- \( f_{NL} \approx 0.1-100 \) higher order derivatives
  - DBI inflation \( \text{(Alishahiha, Silverstein and Tong 2004)} \)
  - UV cutoff \( \text{(Creminelli and Cosmol, 2003)} \)
- \( f_{NL} > 10 \) curvaton models \( \text{(Lyth, Ungarelli and Wands, 2003)} \)
- \( f_{NL} \approx 100 \) ghost inflation \( \text{(Arkani-Hamed et al., Cosmol, 2004)} \)

Salopek & Bond 1990
Komatsu & Spergel 2001
Current Status

Non-Gaussianity from Inflation

\[ f_{\text{NL}} \sim 0.05 \] canonical inflation (single field, couple of derivatives)  
(Maldacena 2003, Acquaviva et al 2003)

\[ f_{\text{NL}} \sim 0.1-100 \] higher order derivatives  
DBI inflation (Alishahiha, Silverstein and Tong 2004)

\[ f_{\text{NL}} > 10 \] curvaton models  
(Lyth, Ungarelli and Wands, 2003)

\[ f_{\text{NL}} \sim 100 \] ghost inflation  
(Arkani-Hamed et al., Cosmol, 2004)

\[ \Delta f_{\text{NL}} \sim 70 \]

We are far from \( \Delta f_{\text{NL}} \sim 1 \) but can already start putting constraints on some models like DBI inflation, ghost inflation etc.
$f_{NL} = 0$

Temperature ($f_{NL} = 0$)

Liguori, Yadav, Hansen, Komatsu, Matarrese, and Wandelt (in preparation)
$f_{NL} = 10^1$

Temperature ($f_{NL} = 10$)

Liguori, Yadav, Hansen, Komatsu, Matarrese, and Wandelt (in preparation)
$f_{NL} = 10^2$

Temperature ($f_{NL} = 10^2$)

Liguori, Yadav, Hansen, Komatsu, Matarrese, and Wandelt (in preparation)
$f_{NL} = 10^3$

Liguori, Yadav, Hansen, Komatsu, Matarrese, and Wandelt (in preparation)
$f_{NL} = 10^4$

Temperature ($f_{NL} = 10^4$)

Liguori, Yadav, Hansen, Komatsu, Matarrese, and Wandelt (in preparation)
The challenge of optimal $f_{NL}$ estimation

- The optimal brute force bispectrum estimator for $f_{NL}$ from polarized CMB maps (Babich and Zaldarriaga 2004) requires $O(l^5)$ computations – not feasible for Planck!
- We prove that for homogeneous noise this estimator is equivalent to a fast estimator that scales as $O(l^3)$ – a factor of millions for Planck.
- Studies of higher order correlation functions (4 pt etc) should be similarly possible.

Komatsu, Spergel, Wandelt 2005

Fast, bispectrum based estimator of local $f_{\text{NL}}$}

**Cubic Statistic:**

$$\hat{S}_{\text{prim}} = \frac{1}{f_{\text{sky}}} \int r^2 dr \int d^2 \hat{n} B(\hat{n}, r) B(\hat{n}, r) A(\hat{n}, r)$$

$B(\hat{n}, r) \equiv \sum_{ip} \sum_{lm} (C^{-1})^{ip} a_{\ell m}^{ip} \beta_{\ell}^{p}(r) Y_{\ell m}(\hat{n})$

$B(r)$ is a map of reconstructed primordial perturbations

$A(\hat{n}, r) \equiv \sum_{ip} \sum_{lm} (C^{-1})^{ip} a_{\ell m}^{ip} \alpha_{\ell}^{p}(r) Y_{\ell m}(\hat{n})$.

$A(r)$ picks out relevant configurations of the bispectrum

Above statistics combine all configurations of bispectrum such that it most sensitive to “local” primordial non-Gaussianity i.e $f_{\text{NL}}$. 

Komatsu, Spergel and Wandelt 2005
Minimum detectable non-Gaussianity as we go to smaller scales

For an ideal CMB experiment and using both temperature and polarization we can get down to $\Delta f_{NL} \sim 1$

For Planck the Cramer Rao limit is $\Delta f_{NL} \sim 3$.

Sky cut adds a challenge

- The cut couples $a_{lm}$ at low $l$, but our estimator is optimal for full sky coverage.
- For T this effect only degrades the efficiency of the estimator slightly.
- For T+P, some low l polarization modes are contaminated by power from other scales. We found that this can be removed by just chopping off a few low l modes.

<table>
<thead>
<tr>
<th>reionization</th>
<th>$f_{sky}$</th>
<th>$l_{min}$</th>
<th>Std. dev. without liner term</th>
</tr>
</thead>
<tbody>
<tr>
<td>no</td>
<td>100</td>
<td>2</td>
<td>5.7</td>
</tr>
<tr>
<td>no</td>
<td>90</td>
<td>2</td>
<td>54.8</td>
</tr>
<tr>
<td>no</td>
<td>90</td>
<td>10</td>
<td>24.5</td>
</tr>
<tr>
<td>no</td>
<td>90</td>
<td>20</td>
<td>13.7</td>
</tr>
<tr>
<td>yes</td>
<td>100</td>
<td>2</td>
<td>6.8</td>
</tr>
<tr>
<td>yes</td>
<td>90</td>
<td>2</td>
<td>54.9</td>
</tr>
<tr>
<td>yes</td>
<td>90</td>
<td>10</td>
<td>48.5</td>
</tr>
<tr>
<td>yes</td>
<td>90</td>
<td>20</td>
<td>15.4</td>
</tr>
<tr>
<td>yes</td>
<td>90</td>
<td>25</td>
<td>13.5</td>
</tr>
</tbody>
</table>
Tests with Gaussian Monte Carlo simulations

Dealing with real data

- The estimators we wrote down are optimal only for uniform sky coverage and noise distribution.
- For non-uniform noise inclusion of a linear term reduces the variance of the estimator (Creminelli et al. 2005)
- Very simple to generalize this to include polarization.

\[
\hat{S}_{\text{linear}}^{\text{prim}} = \frac{-3}{f_{\text{sky}}} \int r^2 dr \int d^2 \hat{n} \left\{ B(\hat{n}, r) S_{AB}(\hat{n}, r) + S_{BB}(\hat{n}, r) A(\hat{n}, r) \right\}
\]

\[
B(\hat{n}, r) \equiv \sum_{ip} \sum_{lm} (C^{-1})^{ip} a_{\ell m}^i \beta_{\ell}^p (r) Y_{\ell m}(\hat{n})
\]

\[
A(\hat{n}, r) \equiv \sum_{ip} \sum_{lm} (C^{-1})^{ip} a_{\ell m}^i \alpha_{\ell}^p (r) Y_{\ell m}(\hat{n})
\]
Inhomogeneous noise

- Mask = infinite noise in galactic plane

\[
S_{AB}(\hat{n}, r) = \sum_{ipqr} \sum_{\ell_1 m_1 \ell_2 m_2} \beta_i^r(r)(C^{-1})^{ip}_{\ell_1} Y_{\ell_1 m_1}(\hat{n}) \alpha_j^q(r)(C^{-1})^{jq}_{\ell_2} Y_{\ell_2 m_2}(\hat{n}) \langle a_{\ell_1 m_1}^p a_{\ell_2 m_2}^q \rangle
\]

\[
S_{BB}(\hat{n}, r) = \sum_{ipqr} \sum_{\ell_1 m_1 \ell_2 m_2} \beta_i^r(r)(C^{-1})^{ip}_{\ell_1} Y_{\ell_1 m_1}(\hat{n}) \beta_j^q(r)(C^{-1})^{jq}_{\ell_2} Y_{\ell_2 m_2}(\hat{n}) \langle a_{\ell_1 m_1}^p a_{\ell_2 m_2}^q \rangle
\]

- Compute linear weight maps using Monte Carlo simulations.
Estimation with inhomogenous noise

1) Non-Gaussian Simulations ($f_{NL} = 100$, $l_{max} = 500$, $f_{sky} = 90$):

- without linear term:
  - mean $f_{NL} = 117.5$
  - stddev = 74.7
- with linear term:
  - mean $f_{NL} = 116.61$
  - stddev = 43.7

Gaussian Simulations ($l_{max} = 500$, $f_{sky} = 90$):

- without linear term
  - stddev = 54.8
- with linear term included:
  - stddev = 29.2

Linear term partially removes effects of sky cut
SUMMARY

- Detection of non-Gaussianity will give complementary information about the early universe (class of inflationary models)

- Current status: $\Delta f_{NL} \sim 70$ (using CMB T )

- T and E polarization are complimentary

- With CMB T and E polarization: $\Delta f_{NL} \sim 3$ (in $\sim 5$ yrs!)
  - Fast estimators exists!
  - Estimators tested against Gaussian and non-Gaussian simulations with and without inhomogeneous noise

- Extensions to more general $f_{NL}$ straightforward
...and the search for non-Gaussianity continues

The End
\[ \beta^i_\ell(r) = \frac{2b^i_\ell}{\pi} \int k^2 dk P_\phi(k) g^i_\ell(k) j_\ell(kr) \]

\[ \alpha^i_\ell(r) = \frac{2b^i_\ell}{\pi} \int k^2 dk g^i_\ell(k) j_\ell(kr). \]