Measuring Primordial Non-Gaussianity using CMB T & E data

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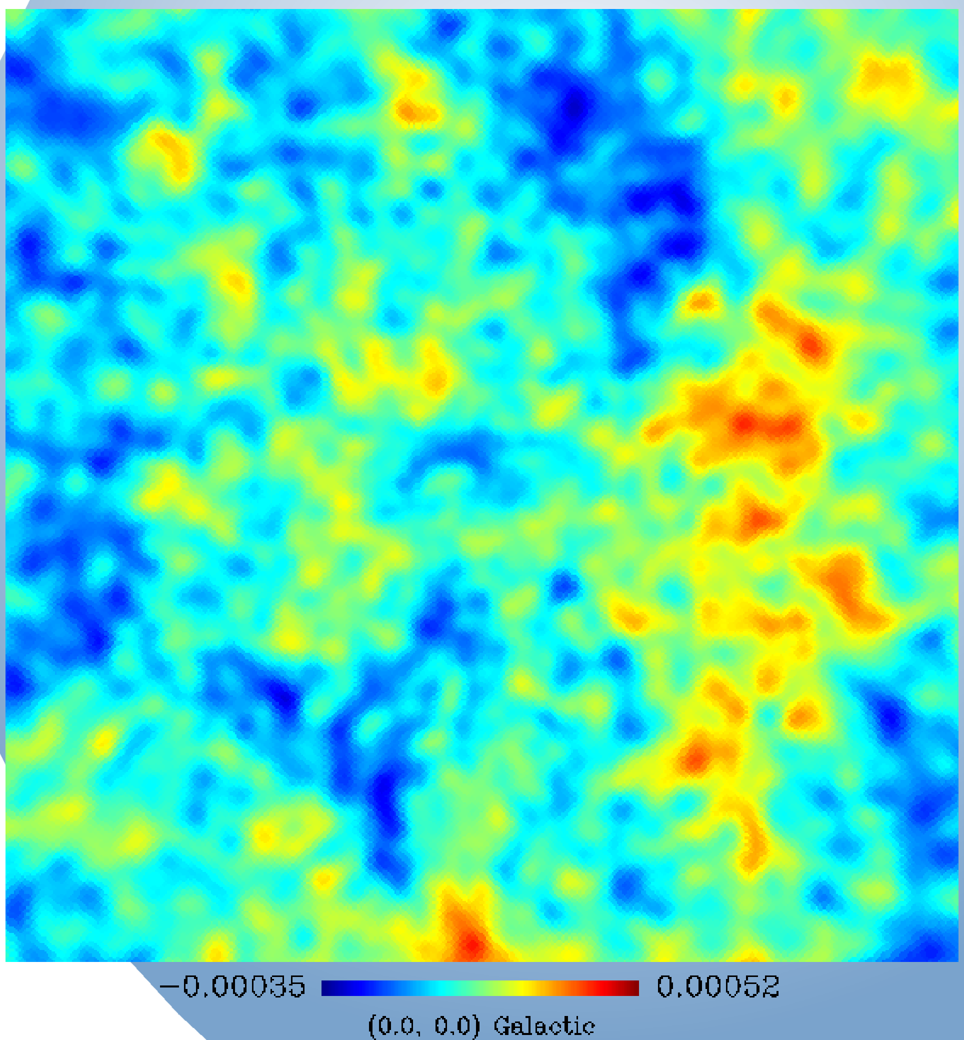
Life beyond the Gaussian, KITP, June 8, 2007

Outline

- Looking for non-Gaussianity
  - Generic primordial non-Gaussianity/anisotropy can be probed using reconstructed primordial curvature perturbation
  - Complementarity of T and E data
- Fast estimator for f_{NL} using temperature and polarization
  - Generalizes Komatsu, Spergel and Wandelt 2005
- Towards the Planck data
  - Dealing with real data—inhomogeneous noise, sky cut

Tomographic reconstruction of inflationary scalar curvature perturbations from CMB

Curvature fluctuations



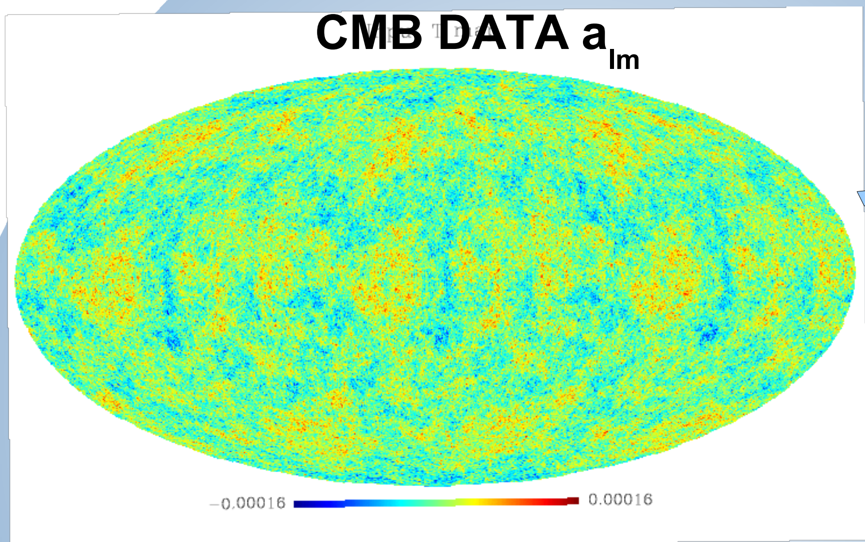
We construct filters that invert linear radiative transport.

Generates a single scalar that contains all the information from T&E.

Anyone intending to test primordial non-Gaussianity and anisotropy in T&E data should do so using curvature perturbations obtained with our filters.

Yadav and Wandelt 2006

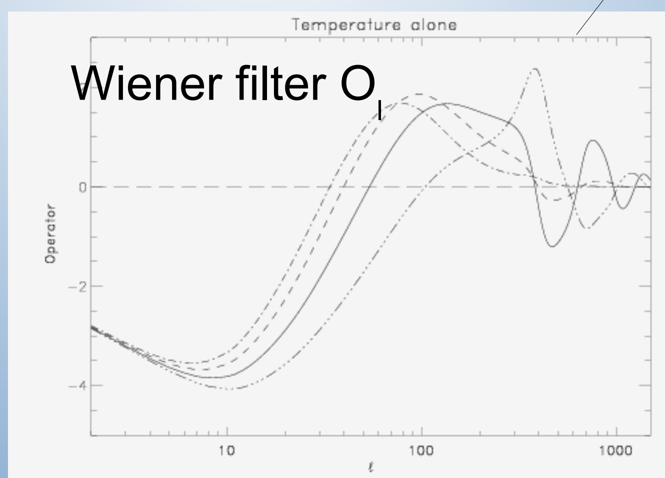
Reconstructed Primordial Perturbations

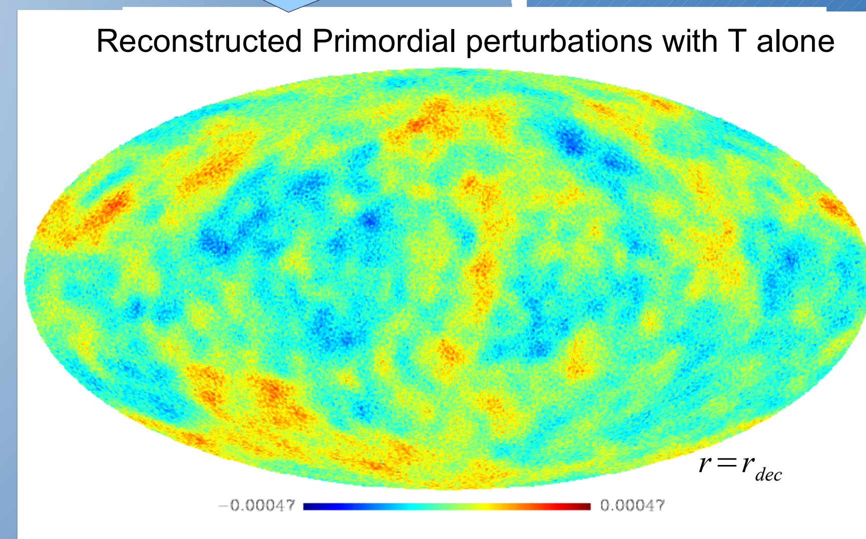


$\Phi_{lm} = O_l \, a_{lm}$

SW limit

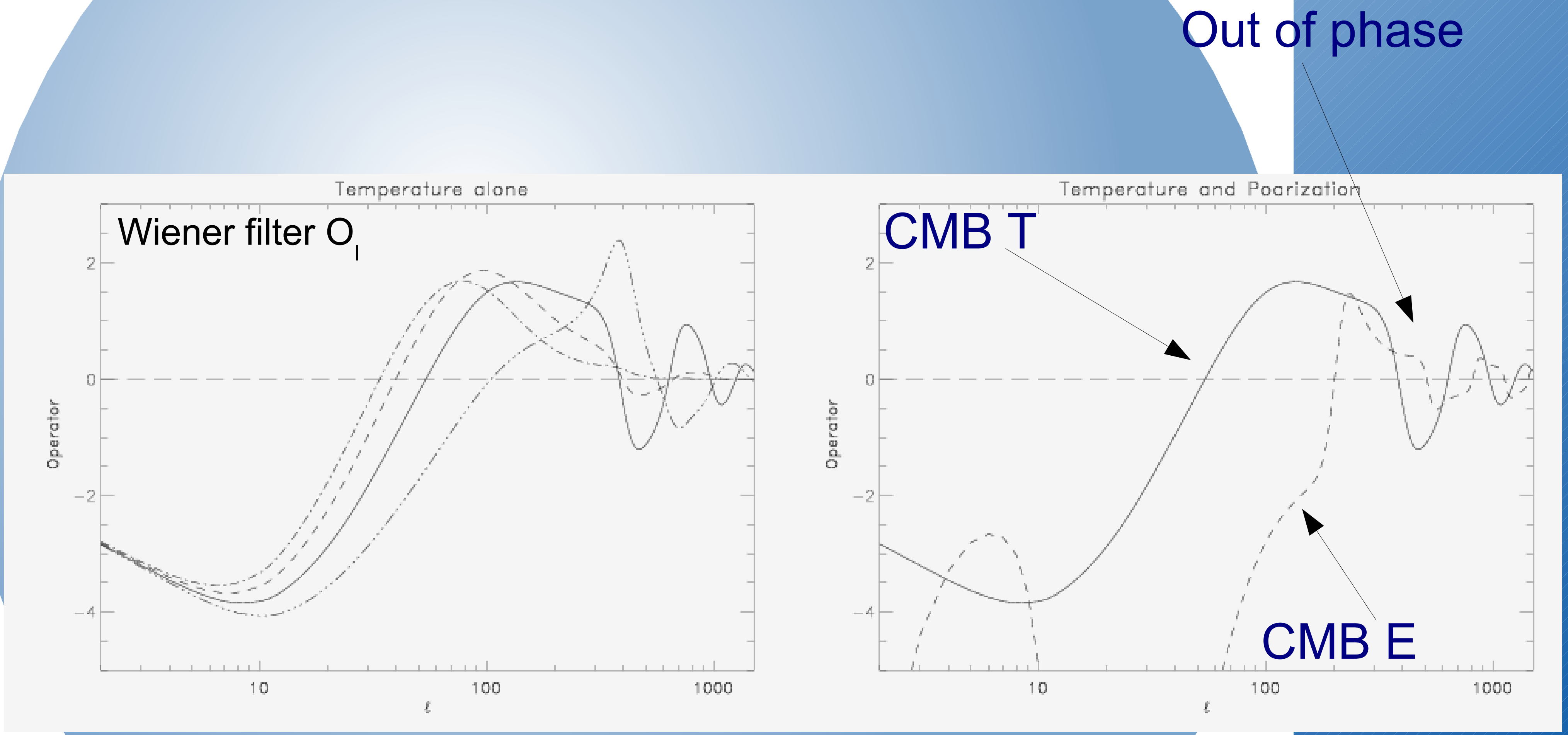
$$\frac{\delta \phi}{\phi} = \frac{-1}{3} \frac{\delta T}{T}$$



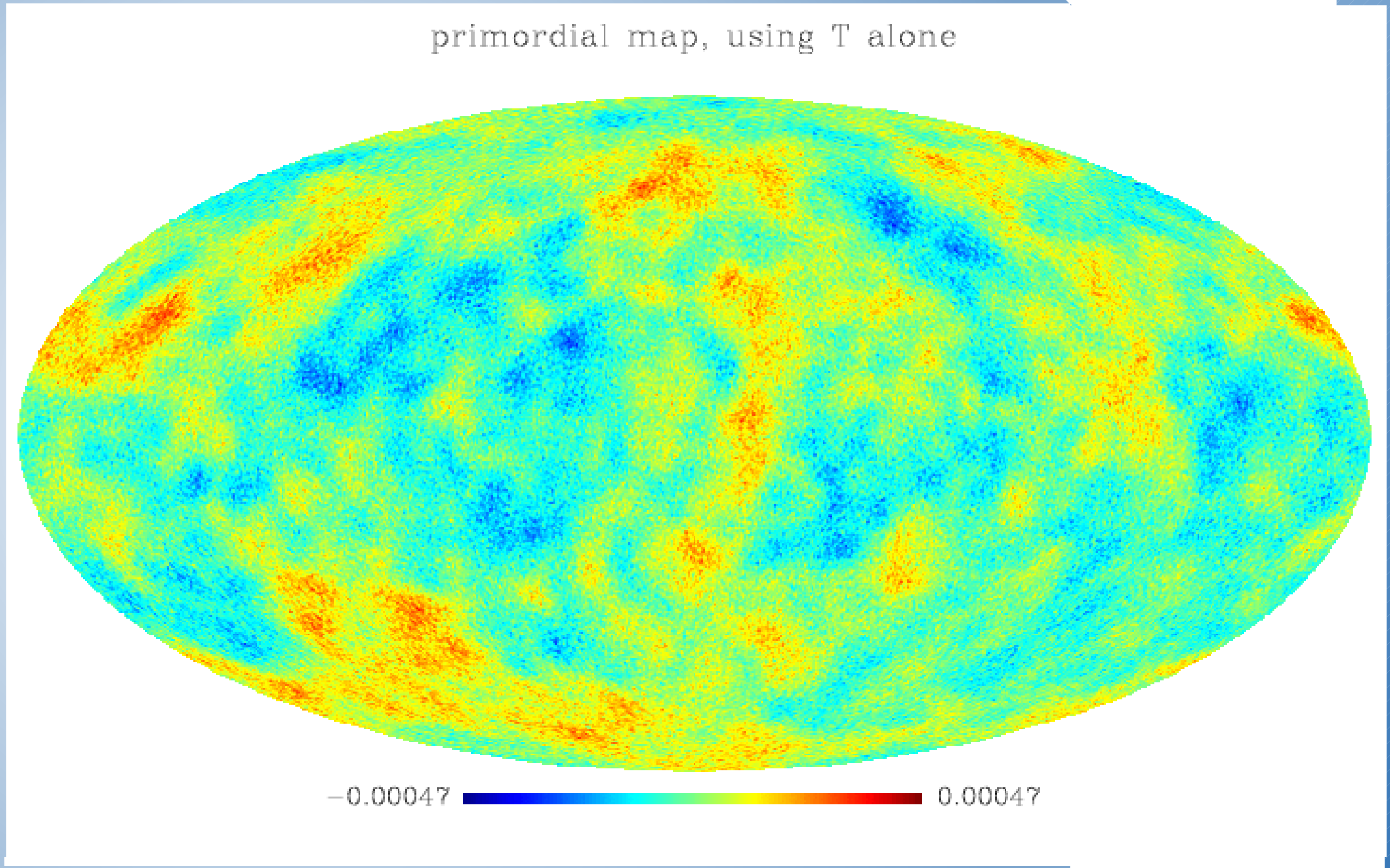


$$\beta^i_\ell(r) = \frac{2b^i_\ell}{\pi} \int k^2 dk P_\phi(k) g^i_\ell(k) \, j_\ell(kr),$$

Temperature and polarization are complementary



Yadav, and Wandelt, PRD (2005)



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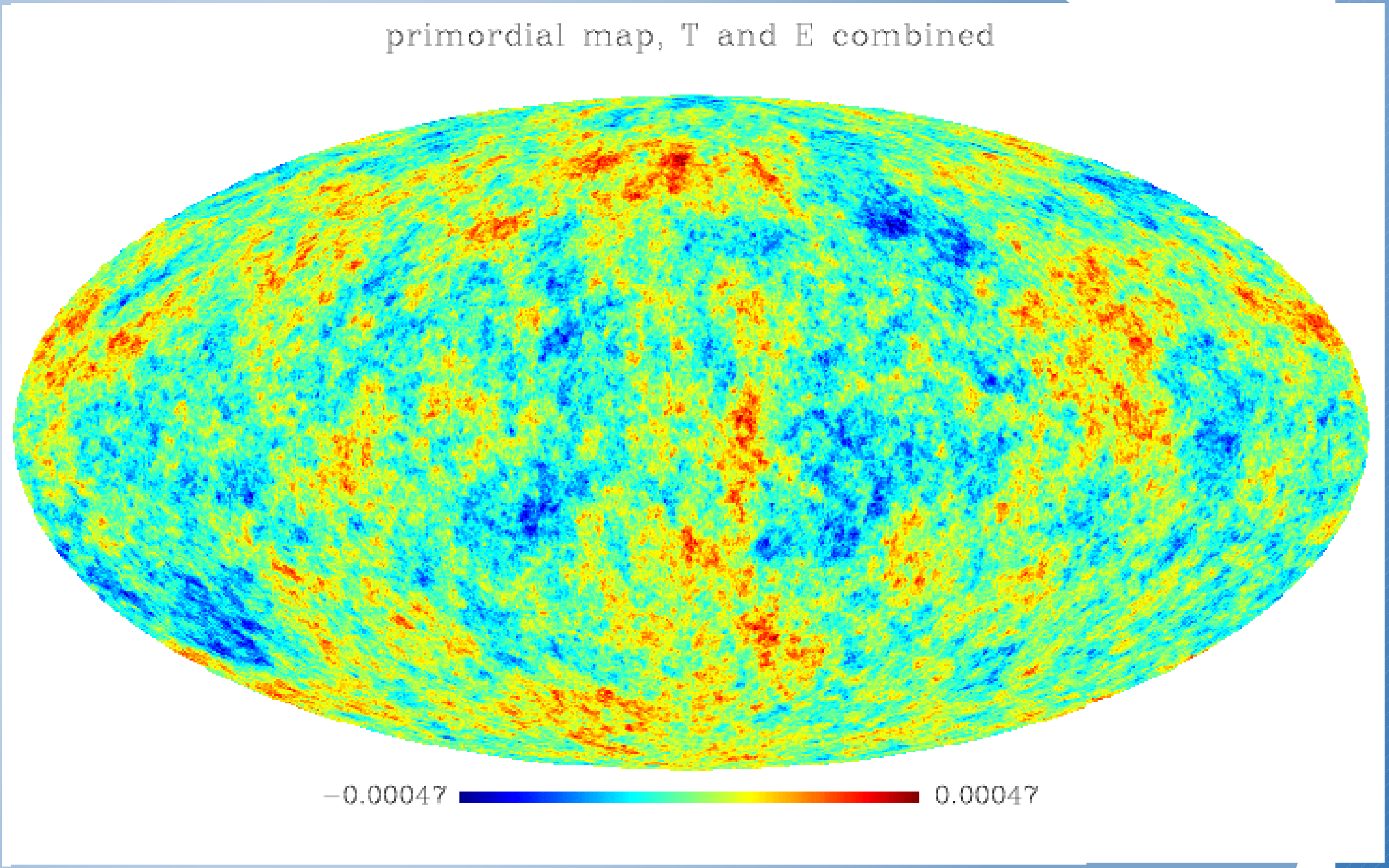
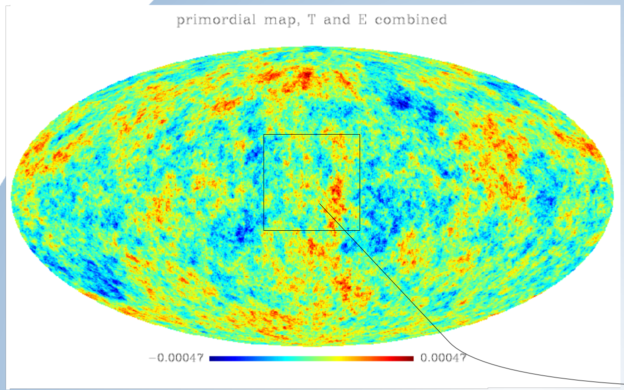
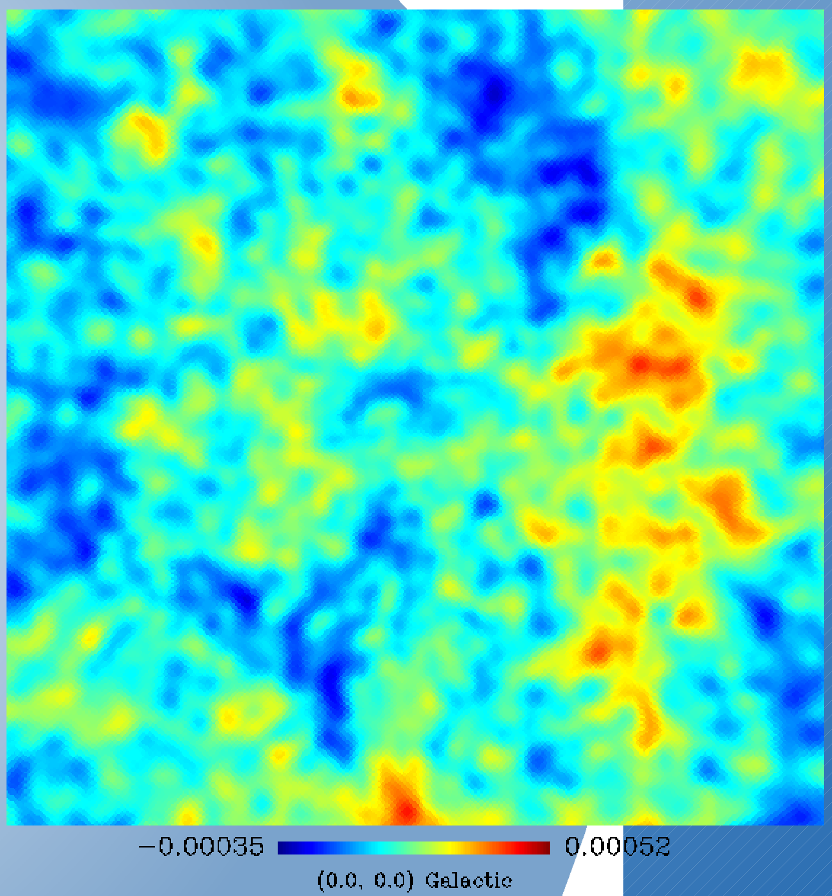


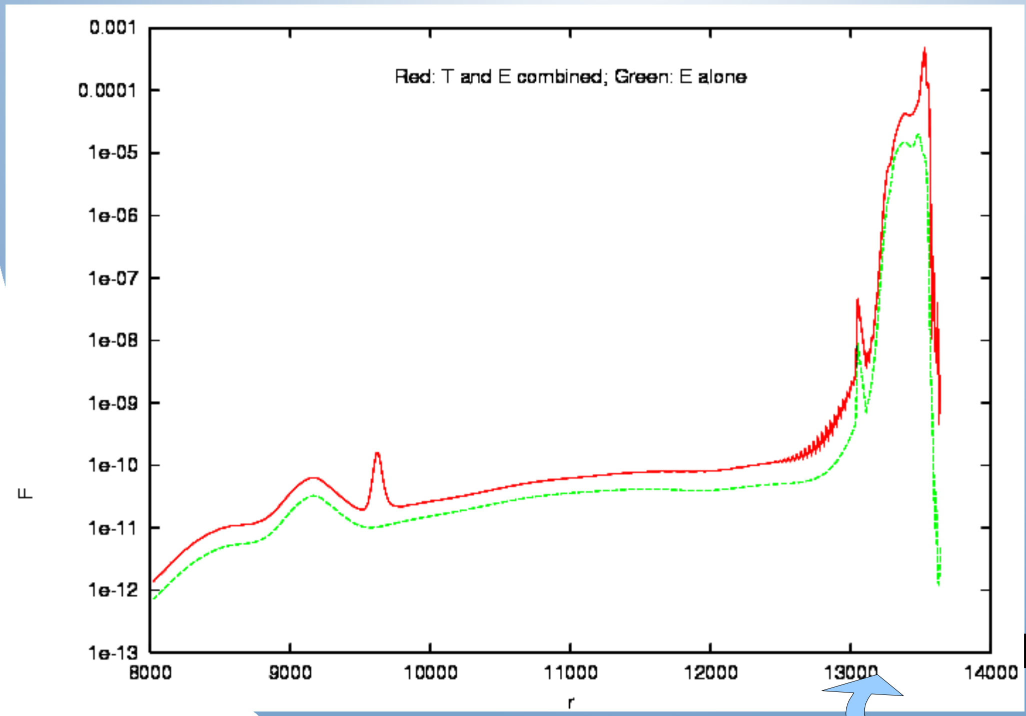
Yadav, and Wandelt, PRD (2005)

Reconstructed perturbations at different radii



Curvature fluctuations





Decoupling

Yadav, and Wandelt, PRD (2005)

Detecting of Non-Gaussianity specific models

$$\Phi(x)=\Phi_G(x)+f_{NL}\Phi_G^2(x)$$

Salopek & Bond 1990
Komatsu & Spergel 2001

Characterizes the amplitude of non-Gaussianity

Non-Gaussianity from Inflation

f_{NL} ~ 0.05 canonical inflation (single field, couple of derivatives) (Maldacena 2003, Acquaviva etal 2003)

f_{NL} ~ 0.1--100 higher order derivatives

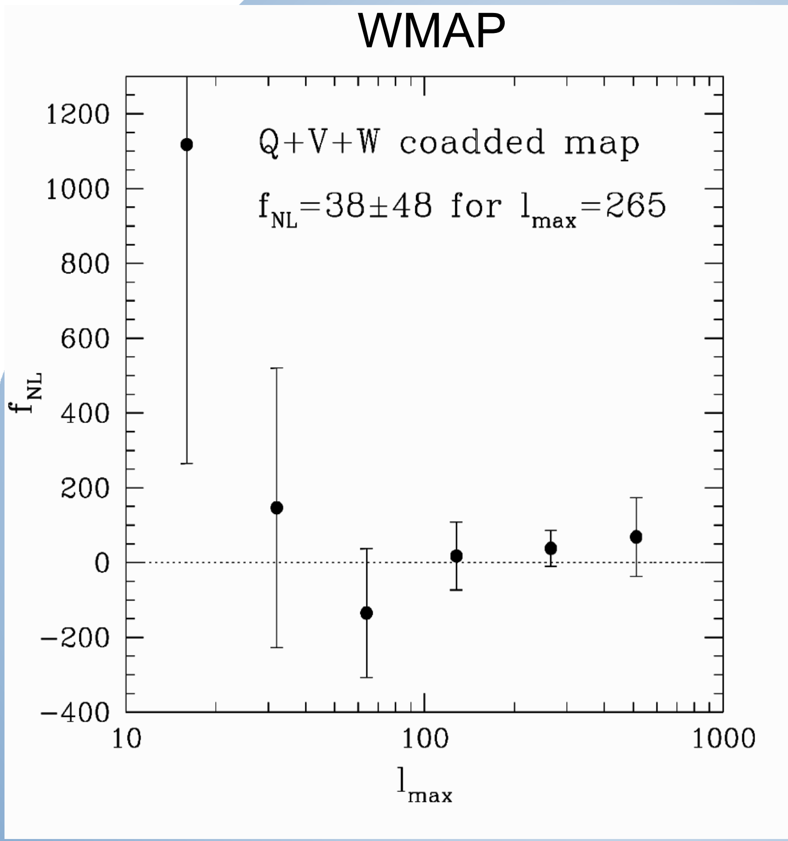
DBI inflation (Alishahiha, Silverstein and Tong 2004)

UV cutoff (Creminelli and Cosmol, 2003)

f_{NL} >10 curvaton models (Lyth, Ungarelli and Wands, 2003)

f_{NL} ~100 ghost inflation (Arkani-Hamed et al., Cosmol, 2004)

Current Status



$-58 < f_{NL} < 137$ (95%) **WMAP 1yr**

$-54 < f_{NL} < 114$ (95%) **WMAP 3yr**

$-36 < f_{NL} < 100$ (95%) **Creminelli et. al. 2006 using WMAP 3yr data**

$\Delta f_{NL} \sim 70$

Non-Gaussianity from Inflation

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DBI inflation (Alishahiha, Silverstein and Tong 2004)

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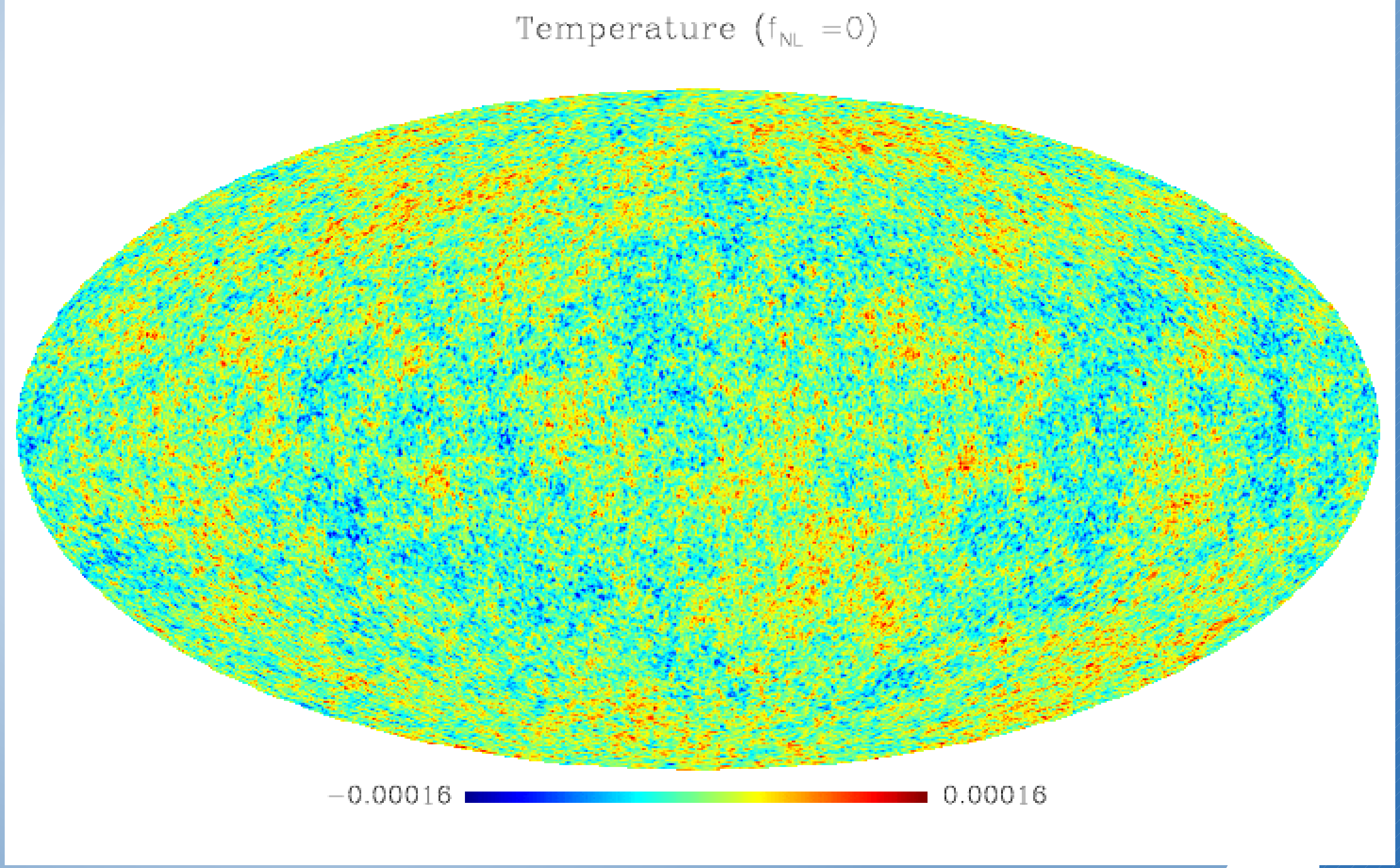
$f_{NL} > 10$ curvaton models (Lyth, Ungarelli and Wands, 2003)

$f_{NL} \sim 100$ ghost inflation (Arkani-Hamed et al., Cosmol, 2004)

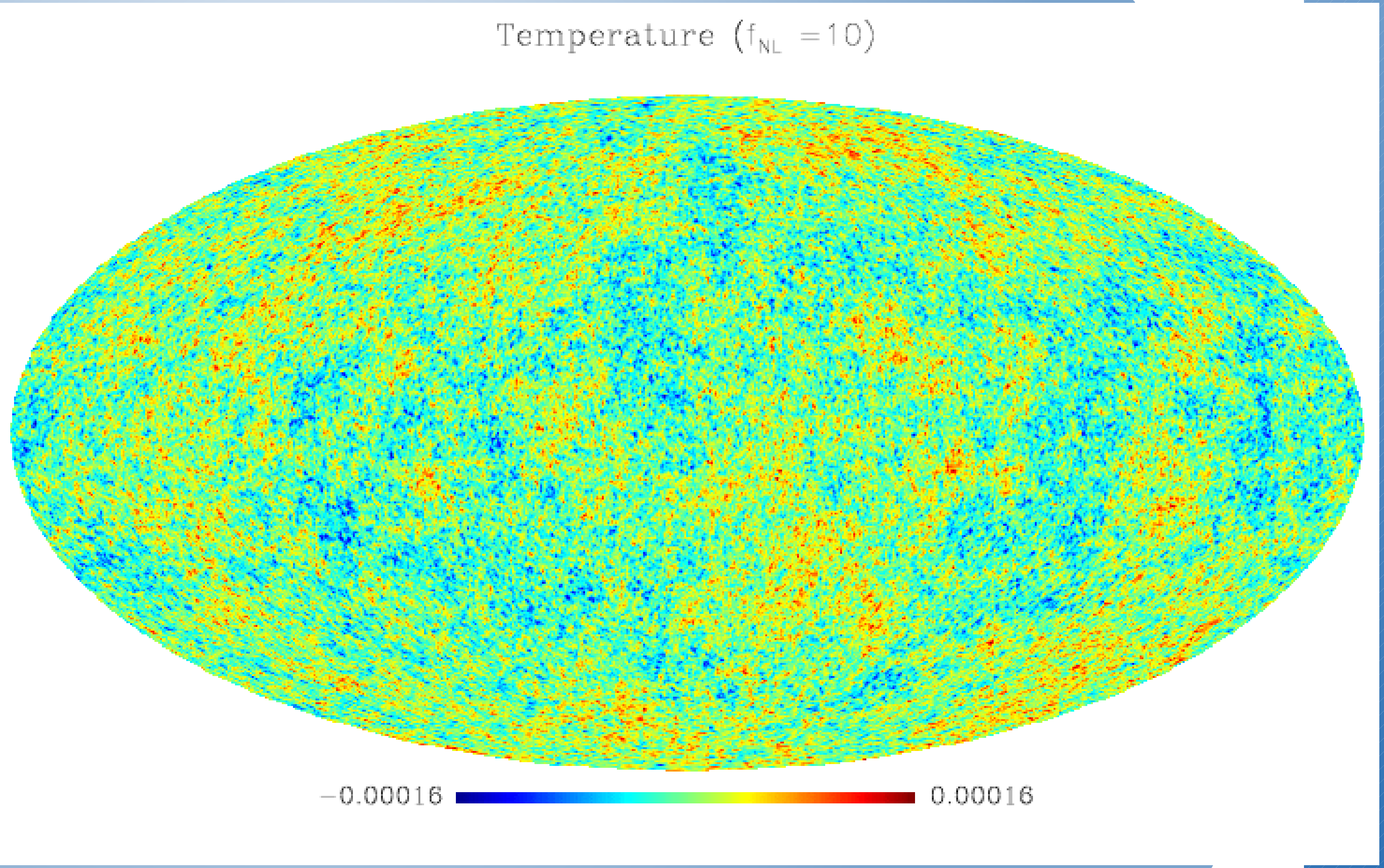
We are far from $\Delta f_{NL} \sim 1$ but can already start putting constraints on some models like DBI inflation, ghost inflation etc.

$$f_{NL}=0$$



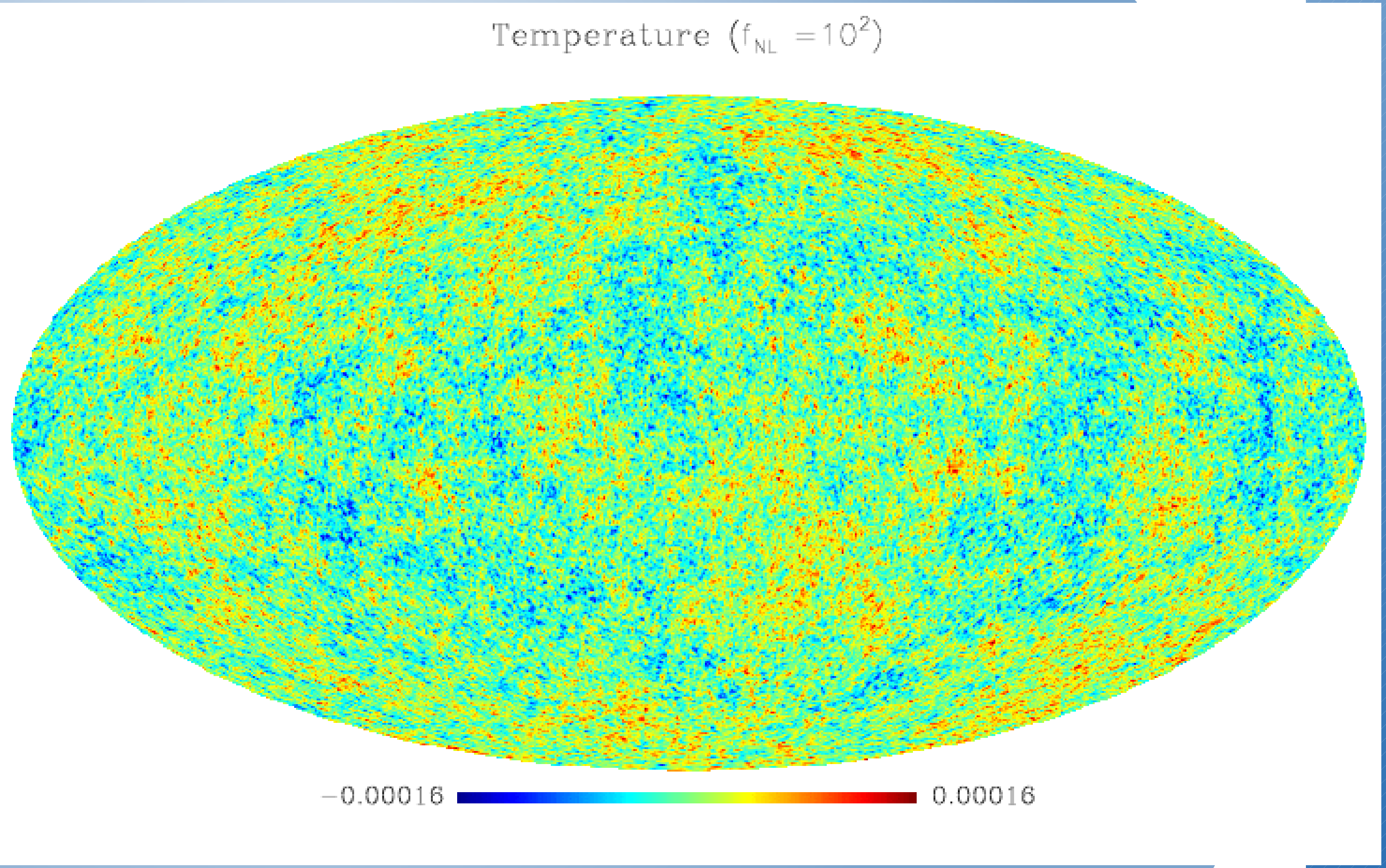
Liguori, Yadav, Hansen, Komatsu, Matarrese, and Wandelt (in preparation)

$$f_{NL} = 10^1$$



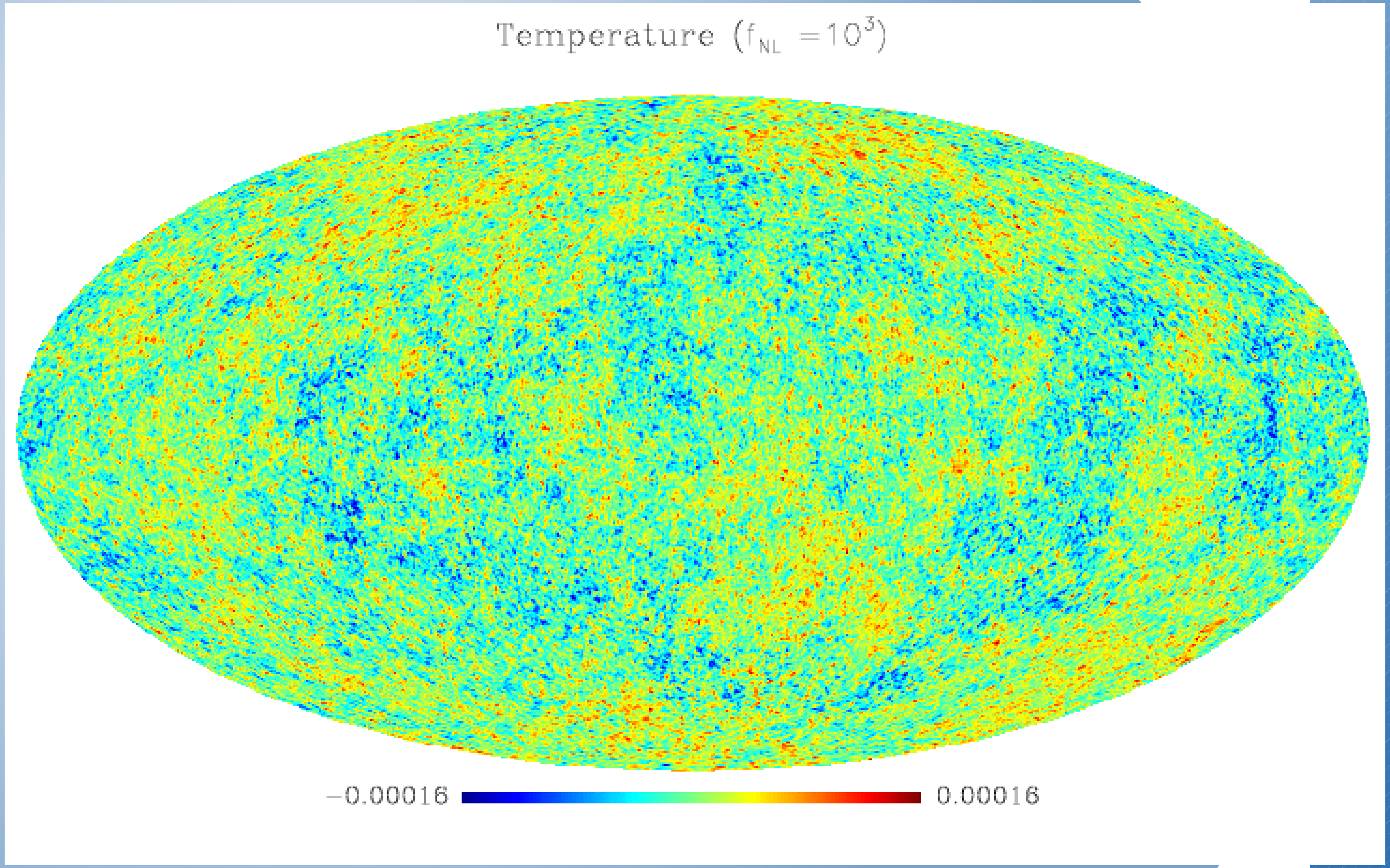
Liguori, Yadav, Hansen, Komatsu, Matarrese, and Wandelt (in preparation)

$$f_{NL} = 10^2$$



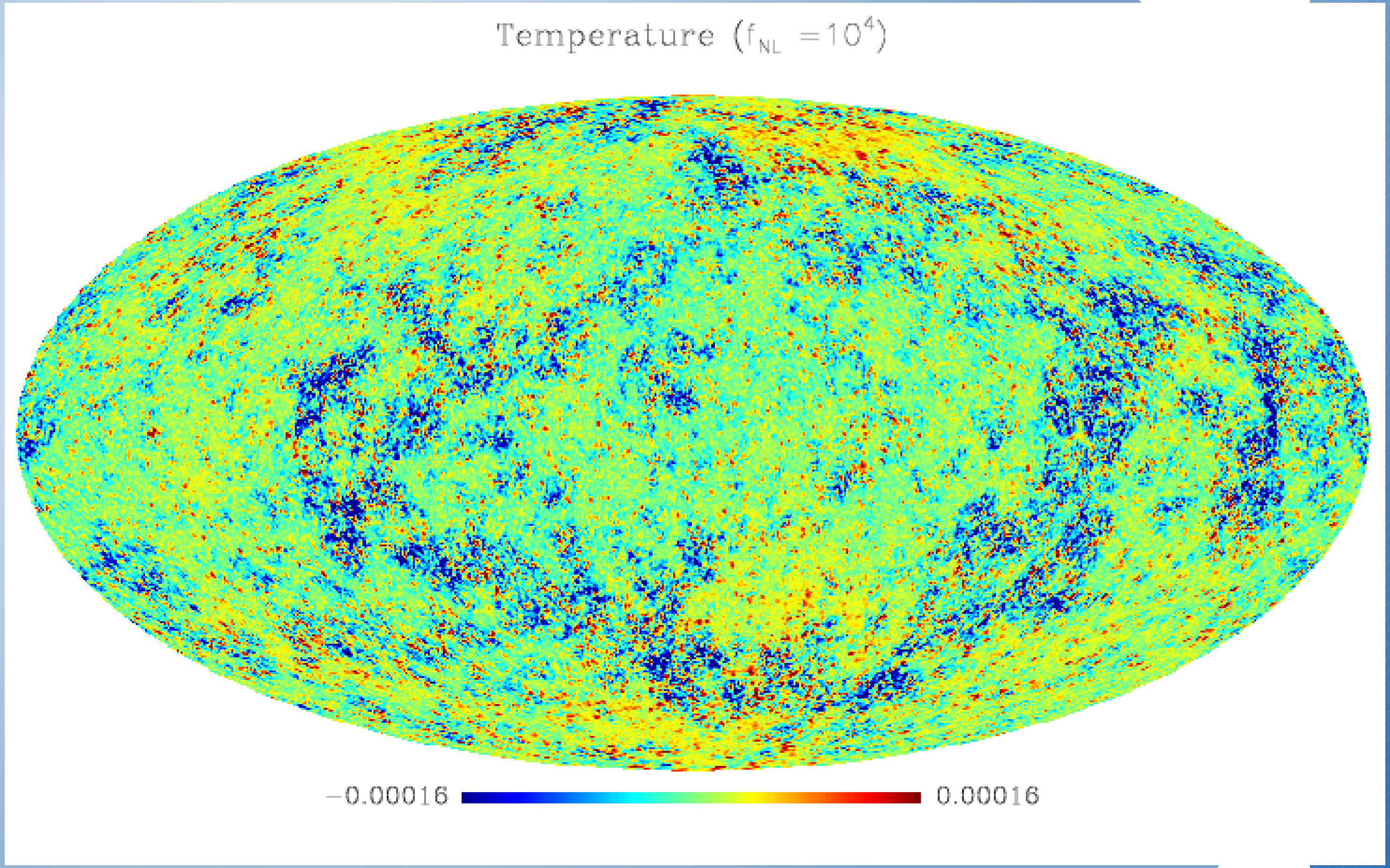
Liguori, Yadav, Hansen, Komatsu, Matarrese, and Wandelt (in preparation)

$$f_{NL} = 10^3$$



Liguori, Yadav, Hansen, Komatsu, Matarrese, and Wandelt (in preparation)

$$f_{NL} = 10^4$$



Liguori, Yadav, Hansen, Komatsu, Matarrese, and Wandelt (in preparation)

The challenge of optimal fNL estimation

- The optimal brute force bispectrum estimator for fNL from polarized CMB maps (Babich and Zaldarriaga 2004) requires $O(l^5)$ computations – not feasible for Planck!
- We prove that for homogeneous noise this estimator is equivalent to a fast estimator that scales as $O(l^3)$ – a factor of millions for Planck.
- Studies of higher order correlation functions (4 pt etc) should be similarly possible.

Komatsu, Spergel, Wandelt 2005

Yadav, Komatsu, Wandelt astro-ph/0701921, ApJ in press

Fast, bispectrum based estimator of local f_{NL}

Cubic Statistic:

$$\hat{S}_{prim} = \frac{1}{f_{sky}} \int r^2 dr \int d^2\hat{n} B(\hat{n}, r) B(\hat{n}, r) A(\hat{n}, r)$$

Komatsu, Spergel and Wandelt 2005

$$B(\hat{n}, r) \equiv \sum_{ip} \sum_{lm} (C^{-1})^{ip} a^i_{\ell m} \beta^p_\ell(r) Y_{\ell m}(\hat{n})$$

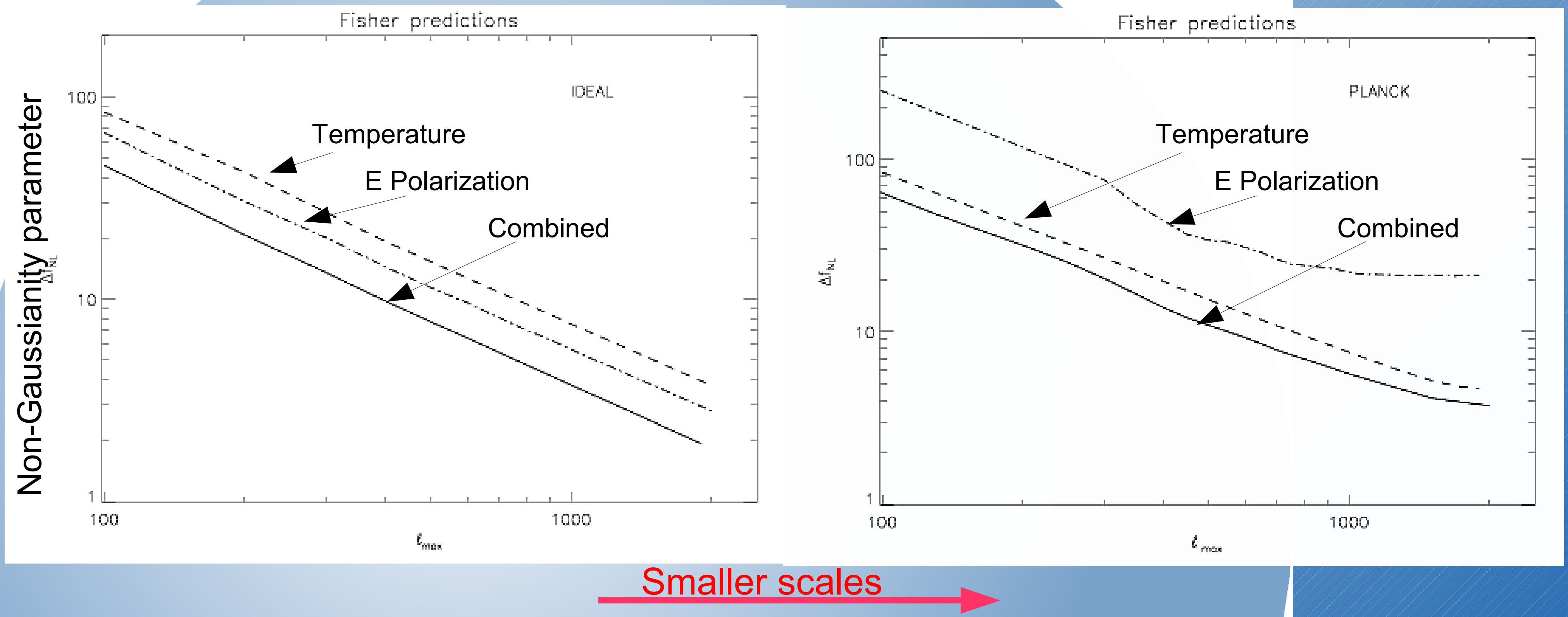
B(r) is a map of reconstructed primordial perturbations

$$A(\hat{n}, r) \equiv \sum_{ip} \sum_{lm} (C^{-1})^{ip} a^i_{\ell m} \alpha^p_\ell(r) Y_{\ell m}(\hat{n}).$$

A(r) picks out relevant configurations of the bispectrum

Above statistics combine combine all configurations of bispectrum such that it most sensitive to "local" primordial non-Gaussianity i.e f_{NL}`

Minimum detectable non-Gaussianity as we go to smaller scales



For and Ideal CMB experiment and using both temperature and polarization we can get down to $\Delta f_{NL} \sim 1$

For Planck the Cramer Rao limit is $\Delta f_{NL} \sim 3$.

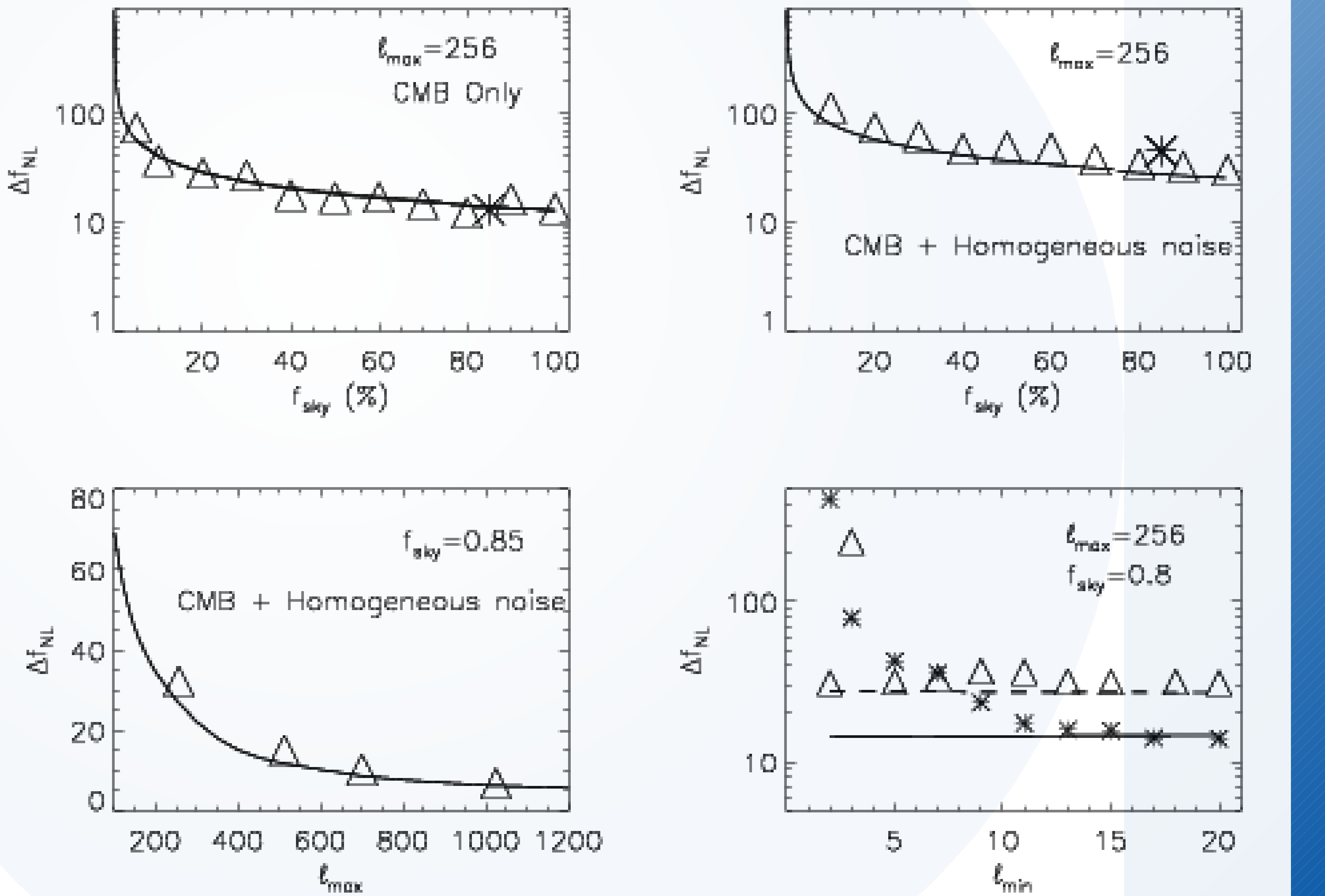
Yadav, Komatsu and Wandelt, astro-ph/0701921, ApJ (in press)

Sky cut adds a challenge

- The cut couples a_{lm} at low l, but our estimator is optimal for full sky coverage.
- For T this effect only degrades the efficiency of the estimator slightly.
- For T+P, some low l polarization modes are contaminated by power from other scales. We found that this can be removed by just chopping off a few low l modes.

reionization	f_{sky}	l_{min}	Std.dev. without liner term
no	100	2	5.7
no	90	2	54.8
no	90	10	24.5
no	90	20	13.7
yes	100	2	6.8
yes	90	2	54.9
yes	90	10	48.5
yes	90	20	15.4
yes	90	25	13.5

Tests with Gaussian Monte Carlo simulations



Yadav, Komatsu, and Wandelt, ApJ (in press)

Dealing with real data

- The estimators we wrote down are optimal only for uniform sky coverage and noise distribution.
- For non-uniform noise inclusion of a linear term reduces the variance of the estimator (Creminelli et al. 2005)
- Very simple to generalize this to include polarization.

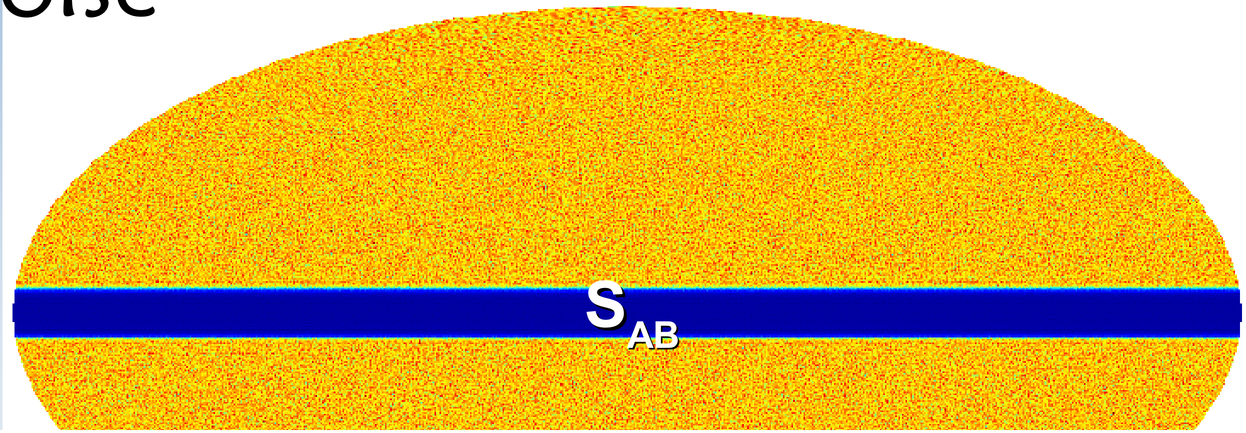
$$\hat{S}_{prim}^{linear} = \frac{-3}{f_{sky}} \int r^2 dr \int d^2\hat{n} \left\{ B(\hat{n},r) S_{AB}(\hat{n},r) + S_{BB}(\hat{n},r) A(\hat{n},r) \right\}$$

$$B(\hat{n},r) \equiv \sum_{ip} \sum_{lm} (C^{-1})^{ip} a^i_{\ell m} \beta^p_\ell(r) Y_{\ell m}(\hat{n})$$

$$A(\hat{n},r) \equiv \sum_{ip} \sum_{lm} (C^{-1})^{ip} a^i_{\ell m} \alpha^p_\ell(r) Y_{\ell m}(\hat{n})$$

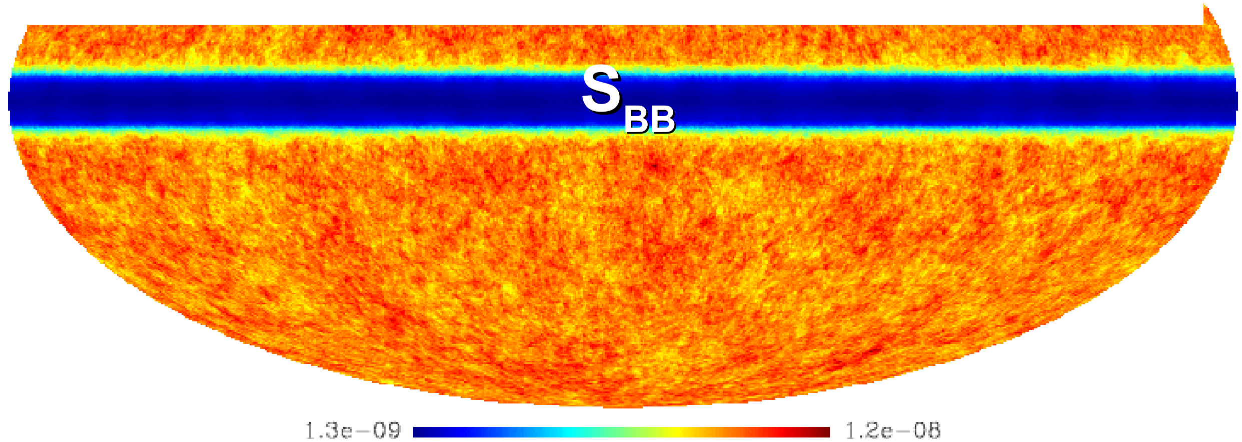
Inhomogeneous noise

- Mask = infinite noise in galactic plane



$$S_{AB}(\hat{n},r) \equiv \sum_{ipqr} \sum_{\ell_1 m_1 \ell_2 m_2} \beta^i_{\ell_1}(r)(C^{-1})^{ip})_{\ell_1} Y_{\ell_1 m_1}(\hat{n}) \alpha^j_{\ell_2}(r)(C^{-1})^{jq}_{\ell_2} Y_{\ell_2 m_2}(\hat{n}) \langle a^p_{\ell_1 m_1} a^q_{\ell_2 m_2} \rangle$$

$$S_{BB}(\hat{n},r) \equiv \sum_{ipqr} \sum_{\ell_1 m_1 \ell_2 m_2} \beta^i_{\ell_1}(r)(C^{-1})^{ip})_{\ell_1} Y_{\ell_1 m_1}(\hat{n}) \beta^j_{\ell_2}(r)(C^{-1})^{jq}_{\ell_2} Y_{\ell_2 m_2}(\hat{n}) \langle a^p_{\ell_1 m_1} a^q_{\ell_2 m_2} \rangle$$



- Compute linear weight maps using Monte Carlo simulations.

f_{NL} Estimation with inhomogenous noise

- 1) Non-Gaussian Simulations (f_{NL}=100, l_{max}=500, f_{sky}=90):
  - without linear term:
    - mean f_{NL}= 117.5
    - stdev= 74.7
  - with linear term:
    - mean f_{NL}=116.61
    - stdev=43.7

- Gaussian Simulations(l_{max}=500, f_{sky}=90):
  - without linear term
    - stdev= 54.8
  - with linear term included:
    - stdev= 29.2

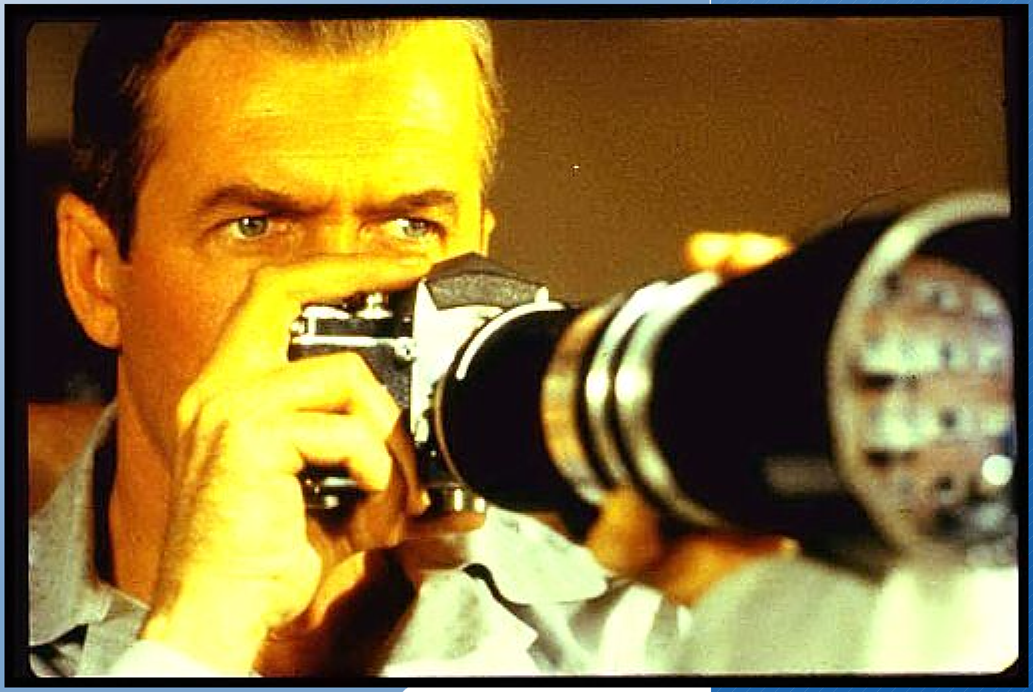
Linear term partially removes effects of sky cu

SUMMARY

- Detection of non-Gaussianity will give complementary information about the early universe (class of inflationary models)
- Current status: $\Delta f_{NL} \sim 70$ (using CMB T)
- T and E polarization are complimentary
- With CMB T and E polarization: $\Delta f_{NL} \sim 3$ (in ~5 yrs!)
  - Fast estimators exists!
  - Estimators tested against Gaussian and non-Gaussian simulations with and without inhomogeneous noise
- Extensions to more general f_{NL} straightforward

...and the search for non-Gaussianity continues

The End



$$\beta_\ell^i(r) = \frac{2b_\ell^i}{\pi} \int k^2 dk P_\phi(k) g_\ell^i(k)\, j_\ell(kr),$$

$$\alpha_\ell^i(r) = \frac{2b_\ell^i}{\pi} \int k^2 dk\, g_\ell^i(k)\, j_\ell(kr),$$