Inferring substructure in strong lenses using simulation-based inference

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Based on work in progress with Johann Brehmer, Joeri Hermans, Gilles Louppe and Kyle Cranmer

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Gravitational lensing: effect of substructure

Presence of substructure down to small scales is one of the key predictions of CDM

Substructure causes percent-level shifts in strongly lensed image

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Lovell et al [1104.2929]

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Siddharth Mishra-Sharma (NYU) | LSST Dark Matter Workshop 2019
Substructure likelihood

\[
p(x \mid \theta) = \int dz \, p(x, z \mid \theta)
\]

Parameters of interest
- Subhalo population parameters: \( \theta = \{f_{\text{sub}}, \beta\} \)

Latent variables
- \( z = \{\vec{m}_{\text{sub}}, \vec{r}_{\text{sub}}\} \)

Huge latent space — full likelihood is intractable!
Inferring substructure through collective effects

Power spectrum decomposition

Unlensed source \( m_{AB} = 24 \) HST WFC3 F555W

Cyr-Racine et al [1806.07897]

See also
Rivero et al [1707.04590]
Rivero et al [1809.00004]
Brennan et al [1808.03501]
Hezaveh et al [1403.2720]

Trans-dimensional methods

\[ P_{\text{sub}}(k) \] \([\text{kpc}^2]\)

Daylan et al [1706.06111]

See also
Brewer et al [1508.00662]

Summary statistics

HST data

Birrer et al [1702.00009]
Goal

Future surveys like LSST, *Euclid* expected to deliver large samples of galaxy-galaxy strong lenses

\( \mathcal{O}(10,000) \)

Collett et al [1507.02657]
Future surveys like LSST, Euclid expected to deliver large samples of galaxy-galaxy strong lenses $(\mathcal{O}(10,000))$

Collett et al [1507.02657]

Propose method to infer high-level substructure properties that is

✓ Fast

✓ Capture maximum information from the data

✓ Scalable to a large sample of lenses

✓ Can deal with a large number of nuisance/latent parameters
Likelihood-free inference: opening the black box

“Traditional” LFI treats the simulator as a generative black box: parameters in, samples out.

No one cares how the sausage is made.
Likelihood-free inference: opening the black box

“Traditional” LFI treats the simulator as a generative black box: parameters in, samples out.

No one cares how the sausage is made.

But in real-life problems, we have access to the simulator code and some understanding of the microscopic processes.

We can extract more simulation from the simulator and use it to improve inference.
Overview

1. Simulation

Parameters $\theta$ → Simulator

Latent $z$ → Observables $x$ → $r(x, z \mid \theta)$ → Augmented data

2. Machine Learning

$\arg\min_g L[g] \rightarrow \hat{r}(x \mid \theta)$

3. Inference

Limit setting with standard hypothesis tests

Extract additional information from simulator

Use this information to train estimator for likelihood ratio

Brehmer et al [1805.00013]
Brehmer et al [1805.00020]
Stoye et al [1808.00973]
Simulation

1. Simulation

- Parameters $\theta$
  - $\hat{r}(x|\theta)$
  - Augmented data

2. Machine Learning

- Latent $z$
  - $x$ (Observables)

3. Inference

- $\arg\min_g L[g]$
- Likelihood ratio estimate
- $\theta_j$
- $\theta_i$
Simulation

1. Simulation

Parameters $\theta$

Simulator

Latent $z$

Observables $x$

Calculating the joint likelihood ratio

For each simulated event, we can calculate the joint likelihood ratio which depends on the specific evolution of the simulation:

$$r(x, z | \theta_0, \theta_1) \equiv \frac{p(x, z | \theta_0)}{p(x, z | \theta_1)} = \frac{p(x | z)}{p(x | z)} \frac{p(z | \theta_0)}{p(z | \theta_1)}$$

Brehmer et al [1805.00013]
Brehmer et al [1805.00020]
Stoye et al [1808.00973]
Machine learning

1. Simulation

- Parameters $\theta$
- Simulator
- Latent $z$
- Observables $x$

2. Machine Learning

- $r(x, z | \theta)$
- Augmented data
- $\arg\min_g L[g]$
- Likelihood ratio estimate $\hat{r}(x | \theta)$

3. Inference

Brehmer et al [1805.00013]
Brehmer et al [1805.00020]
Stoye et al [1808.00973]
Inference

1. Simulation

2. Machine Learning

3. Inference

Parameters $\theta$ → Observables $x$ → Augmented data $r(x, z | \theta)$ → Likelihood ratio estimate $\hat{r}(x | \theta)$

$\text{Simulator}$

Latent $z$
Inference

IX. **On the Problem of the most Efficient Tests of Statistical Hypotheses.**

By J. NEYMAN, Nencki Institute, Soc. Sci. Lit. Varsoviensis, and Lecturer at the Central College of Agriculture, Warsaw, and E. S. PEARSON, Department of Applied Statistics, University College, London.

*(Communicated by K. Pearson, F.R.S.)*

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Application to substructure in strong lenses

Parameters of interest
Subhalo population parameters
\( \theta = \{ f_{\text{sub}}, \beta \} \)

Observables
Lensed image

Joint likelihood ratio
\[
\mathcal{L}(x, z | \theta_0, \theta_1) = \frac{p(m_{\text{sub}}, r_{\text{sub}} | f_{\text{sub},0}, \beta_0)}{p(m_{\text{sub}}, r_{\text{sub}} | f_{\text{sub},1}, \beta_1)}
\]

Likelihood ratio estimate
\[
\hat{\mathcal{L}}(x | \theta)
\]
Application to substructure in strong lenses

After training the parameterized LR estimator...

\[ \hat{r}(x|\boldsymbol{\theta}) = \arg \min_g L[g] \]

… can infer parameters of interest for given lensed image!
Proof of principle

Use simulated ensemble of galaxy-galaxy lenses observable by *Euclid*

Collett et al [1507.02657]

1. Train likelihood ratio estimator with $f_{\text{sub}} \sim [0, 0.2], \beta \sim [-2.5, -1.5]$

2. Test on simulated data with $f_{\text{sub}} = 0.05, \beta = -1.9$
Inferred likelihood ratios

\[ f_{\text{sub}} = 0.05, \ \beta = -1.9 \]
Inferred likelihood ratios

Analysis of Individual images

\[ f_{\text{sub}} = 0.05, \beta = -1.9 \]
Inferred likelihood ratios

\[ f_{\text{sub}} = 0.05, \beta = -1.9 \]

**Analysis of Individual images**

**Combined analysis (20 images)**
Inferred likelihood ratios

\[ f_{\text{sub}} = 0.05, \beta = -1.9 \]

Profiles at \( \beta = -1.9 \)
Bayesian interpretation

\begin{align*}
\text{Uniform prior on } \{f_{\text{sub}}, \beta\} \\
\text{Gaussian prior for } \beta \sim \mathcal{N}(-1.9, 0.1)
\end{align*}

\[f_{\text{sub}} = 0.05, \beta = -1.9\]
Summary

Estimating the likelihood ratio with machine learning

*Powerful simulation-based estimators of the likelihood ratio provide a principled way to perform inference using additional information extracted from the simulator*

Brehmer et al [1805.00013], Brehmer et al [1805.00020]

Inferring substructure in strong lenses

*Fast, efficient, scalable way to analyze a population of galaxy-galaxy strong lenses to infer underlying substructure properties*