Outline

- Microwave Yield:
  - Method I;
  - Method II;

- Background:
  - General discussion;
  - Estimate for CROME;

- EAS Simulation
Experimental Setup for Reference Signal
SLAC T471/E165

• radiator to create showers;
• NO conversion to photons;
• number of $e^- \text{ per bunch (at 28.5 GeV)}$ $N_{e^-}^{beam} \sim 1.2 \times 10^7$;
• number of $e^-$ at $X_{max}$ (from PDG 30 GeV showers) $N_{e^-}^{max} \sim 90 e^-$;
• observed shower length $\Delta l = 0.65 \text{ m}$;
• distance from shower axis $r = 0.4 \text{ m}$;
• antenna effective area $A_{eff} = \pi r^2 = 0.05 m^2$;
• measurement bandwidth 1.5 – 6 GHz.
Measurements

The signal $S_{\text{meas}}[\text{W/m}^2]$ measured at distance $d$ is:

$$S_{\text{meas}} = S_0 \ e^{-t/\tau}; \quad (1)$$

where $S_0 = 1 \times 10^{-6} \ \text{W/m}^2$ is the signal at peak and $\tau = 7 \ \text{ns}$ is the decay constant of the signal.

The measured energy per unit of area $\mathcal{E}_{\text{meas}}[\text{J/m}^2]$ is:

$$\mathcal{E}_{\text{meas}} = \int_{0}^{+\infty} S_0 \ e^{-t/\tau} \ dt = S_0 \ \tau; \quad (2)$$
Absolute Signal: Method I

Being the shower in the experiment at distance $r = 0.4\, \text{m}$ and assuming isotropic emission, the radiated energy at the source $E$ is:

$$E = E_{\text{meas}} A_{\text{eff}} \frac{4\pi}{\langle \Delta \Omega_{\text{ref}} \rangle}; \quad (3)$$

where $\langle \Delta \Omega_{\text{ref}} \rangle = 2\pi (1 - \cos \langle \theta_{\text{ref}} \rangle)$ is the mean solid angle of the receiver.

Assuming that the Bremsstrahlung radiated energy per unit of frequency $S$ is proportional to the deposited energy and the "Microwave Yield" $Y_{\text{MW}}$:

$$E_{\text{MW}} = Y_{\text{MW}} \frac{dE}{dX} \Delta X = Y_{\text{MW}} E_{\text{dep}}^{\text{tot}}; \quad (4)$$
Absolute Signal: Method I

Total energy deposited $E_{\text{dep}}^{\text{tot}}$ in the chamber can be estimated as:

$$E_{\text{dep}}^{\text{tot}} = N_{e^-}^{\text{beam}} N_{e^-}^{\text{max}} \frac{dE_{e^-}}{dX} \rho_{\text{ground}} \Delta l;$$

where $\frac{dE_{e^-}}{dX}$ is the energy deposited by one single $e^-$ per unit of depth and $\rho_{\text{ground}}$ is the Air density at ground, $\Delta l$ the length of the chamber.

Microwave Yield

The yield $Y_{MW}[Hz^{-1}]$ can be calculated as:

$$Y_{MW} = \frac{E_{MW}}{\Delta \nu_{\text{ref}} E_{\text{dep}}^{\text{tot}}} = \frac{4\pi A_{\text{eff}} S_0 \tau}{\Delta \nu_{\text{ref}} \langle \Delta \Omega_{\text{ref}} \rangle E_{\text{dep}}^{\text{tot}}} = 1.5 \times 10^{-19} Hz^{-1};$$

where $\Delta \nu_{\text{ref}} = 4.5$ GHz is the bandwidth in which the signal is measure in Gorham.

Lower Limit: this value for $Y_{MW}$ is a lower limit. If the energy deposited in the chamber is lower than the estimated value, the $Y_{MW}$ has to be higher to observe the same signal in the antenna.
Absolute Signal: Method II

In an equivalent way we can use Monte Carlo to reproduce the shape of the signal measured by Gorham. The scale in the total signal value will give the *Microwave Yield* $Y_{MW}$.

Monte Carlo
The Monte Carlo includes:

- travel time through the chamber;
- propagation from emission point to antenna;
- time delay for the starting of emission ($\sim 3\text{ns}$);
- exponential decay of the emitted signal with time ($\sim e^{-t/\tau}$ with $\tau = 7 \text{ns}$).
- background noise fluctuates gaussian around the mean value;
Absolute Signal: Method II

The total power emitted $S_{MW}$ is:

$$S_{MW} \propto \frac{\Delta \nu_{ref}}{4\pi} \int \frac{dE}{dX} \Delta \Omega(X) dX;$$  \hspace{1cm} (7)

where $dE/dX$ is the energy deposited in a depth $dX$ of the chamber along the shower axis. The proportionality with the signal is given by $Y_{MW}$ through Microwave Yield

In this case the Microwave Yield can be calculated as:

$$Y_{MW} = \frac{4\pi S_0 \tau A_{eff}}{\Delta \nu_{ref} \sum_i E_{dep}^i \Delta X_i \Delta \Omega_i};$$  \hspace{1cm} (8)

In this case the approximation of using a mean solid angle is not used. The value of $Y_{MW}$ obtained with this method is discussed in the next slides.
Simulated of Gorham Signal

Signal vs Time

- Simulated signal is in very good agreement with Gorham measurement;
- The background slightly differs from measurement (Noise calculation in next slides).
Determination of $Y_{MW}$

Gorham measurement has been simulated several times, fluctuating some parameter, and the yield has been calculated:

- number of electrons in the beam;
- number of electrons at shower maximum
- Energy deposited along shower axis.

From the distribution of the yield we get the value:

$$Y_{MW} = (1.17 \pm 0.21) \times 10^{-19} \text{Hz}^{-1};$$
Background

Definition

Background can be calculated as:

\[ N_{MW} = k_B \Delta \nu \ T_{sys}; \]  

(9)

where:

\[ T_{sys} = T_{rec} + T_{sky} \ A_{eff} \ \Delta \Omega / \lambda^2 \]  

(10)

\( k_B \) is the Boltzmann constant, \( T_{sys} \) is the temperature of the measuring system, \( T_{sky} \approx 4 \ K \) for zenith observation, \( \lambda \) is the wavelength at the center of the considered bandwidth \( \Delta \nu \), \( A_{eff} \) is the antenna effective area and \( \Delta \Omega = 2\pi \ (1 - \cos(\theta_A/2)) \) is the solid angle of the antenna. 
\( \theta_A = 70^\circ \ \lambda/D \) is the opening angle of the antenna and \( D \) the diameter.

Example:

Assuming \( D = 0.9 \ m \ (\theta_A \approx 6^\circ) \), \( T_{sys} \approx 100 \ K \) in the C Band (3.6 - 4.2 GHz):

\[ N_{MW} = 6.64 \times 10^{-13} \ W \ (\sim -90 \ \text{dBm}); \]
Sky Brightness Temperature

Sky Brightness Temperature depends on:

- CMB
- syncrotron emission from galactic plane;
- atmosphere

Details in: www.skatelescope.org
Noise Estimation for CROME setup

Calibration from measured Noise

- $T_{\text{sky}} \approx 4 \, \text{K}$ is negligible
- $T_{\text{rec}}$ can be estimated directly from data

- Conversion between measured Voltage and dBm measured;
- Noise trace in dBm ($\sim \log_{10}(\text{power}[\text{mW}])$) is gaussian;
Noise Estimation for CROME setup

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Data vs Simulated noise

Data Noise

Simulated Noise

Noise from data has a correlation from bin to bin.
Data vs Simulated noise FFT

Data Noise

Simulated Noise

Noise has to be parameterized in frequency domain.
EAS Simulation

- Gaisser Hillas profiles (iron or protons);
- Number of particles is converted into $E_{\text{dep}}$;
- $E_{\text{dep}}$ is converted in microwave signal:

$$MW_{\text{sign}} = E_{\text{dep}} \ Y_{MW} \ \frac{\Delta \nu_{\text{rec}}}{\Delta t_{\text{rec}}} \ \Delta \Omega \ \varepsilon_{\text{antenna}}; \quad (11)$$

- $E_{\text{dep}}$ is distributed according to Gora function (LDF);
- time propagation of the signal at the detector:

$$t_{\text{tot}} = t_{\text{emission}} + t_{\text{rec}} + t_{\text{decay}}; \quad (12)$$

- detector geometry taken into account;
- NO attenuation of the signal due to atmosphere (negligible);
Simulated Shower
Backup Slide I - Atmospheric Attenuation

Graph showing the absorption coefficient $\kappa_a$ [dB km$^{-1}$] vs frequency [GHz] for different gases: $H_2O$, $O_2$, and total.

Absorption Coefficient $\kappa_a$ [dB km$^{-1}$] vs Frequency [GHz]


Backup Slide II - Solid Angle Factor

\[ d_{rec} = |\vec{x}_{emission} - \vec{x}_{rec}|; \]  \hspace{2cm} (13)

\[ \gamma = \arctan \left( \frac{D_{antenna}}{2 \cdot d_{rec}} \right); \]  \hspace{2cm} (14)

\[ \Delta \Omega = \frac{1 - \cos(\gamma)}{2}; \]  \hspace{2cm} (15)
Data vs Simulated noise (dBm)

Data Noise

![Data Noise Graph]

Simulated Noise

![Simulated Noise Graph]