A REVIEW OF UHE NEUTRINO DETECTION USING THE ASKARYAN EFFECT

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March 1, 2016

UHEAP2016 @ The Kavli Institute for Cosmological Physics
I. The Continuing Story of GZK neutrinos and Radio

II. *Regathering* our knowledge of the Askaryan effect
   B. The basic effect, some definitions and the LPM Effect

III. Towards a complete analytic formalism
   A. Our implementation of RB, agreement with ZHS
   B. Accounting for the LPM effect

IV. Numerical Work
   A. Constructing the shower, given numerical constraints
   B. Deriving the general form factor provided by Geant4

V. Results in the Fourier domain, LPM and form factor together

VI. Newest work: analytic equations in the time-domain
   A. Graphs of asymmetric vector potential
REGATHERING OF KNOWLEDGE
The basic effect is coherent radiation from a negative charge excess:

\[ \eta = \left( \frac{a}{\Delta z_{coh}} \right)^2 \]  \hspace{1cm} (1)

Fraunhofer regime: 
\( E(\omega) \) has spherical symmetry (\( \propto 1/R \))

Fresnel regime: \( E(\omega) \) cylindrical symmetry (\( \propto 1/\sqrt{\rho} \))

Feynman’s formula:
\[ E_{rad} \propto \text{sgn}(1 - n\hat{u} \cdot \vec{\beta}) \ddot{\theta} \]
Feynman’s Formula - Imagine charge passing right to left.
Some definitions and remarks.

*Longitudinal:* refers to the $z'$ coordinate, or shower axis.

*Lateral:* refers to the $\rho'$ coordinate ($z^2 + \rho^2 = R^2$, $\rho/R = \sin \theta$).

$\theta$, the viewing angle

$k$, wavevector in the medium

*Shower width:* $a$ (m)

$\propto \sqrt{3/2 \ln(E)}$

*Excess charged particles:* $n_{max} \propto E/\sqrt{\ln(E)}$

**Energy-scaling:** Product of $n_{max}a \propto E$ (area under Gaussian to first order)
Papers and other references.


**Greisen parameterization** (Prog. in Cosmic Ray Physics, 1956, ch. 1) E&M shower model. Leads to Rossi B approximation etc. Solution for lateral charge evolution (see below).
\[ \vec{J} = \vec{v} n(z') f(z' - ct', \vec{\rho}') \quad (2) \]

The main result from RB:

\[ R\tilde{E}(\omega) = 2.52 \times 10^{-7} \frac{a}{m} \frac{n_{\text{max}}}{1000 \text{ GHz}} \frac{\nu}{F(\vec{q})} \psi \tilde{E}(\theta, \eta) \quad (3) \]

\[ \psi = -i \exp(ikR) \sin \theta \quad (4) \]

\[ \tilde{E}(\theta = \theta_c, \eta) = \tilde{e}_\theta (1 - i\eta)^{-1/2} \quad (5) \]

\[ \vec{q} = (\omega/c, k\vec{\rho}/R) \quad (6) \]

Rossi showed that the Greissen solution for \( n(z') \) with depth \( a \) can be approximated as a gaussian with width \( a \).

The linear \( \omega \) dependence comes from acceleration factor in Lienard-Wiechert fields.
Coherence zones, and other useful approximations. The Fraunhofer approximation leads to an insight:

\[ R = |x - x'| \gg \rho \]  
\[ R = |x - x'| \gg \lambda \]  
\[ i|k||x - x'| \approx ikR - ik \cdot \rho(\tau) \]  

Beginning with the Lienard-Wiechert retarded potentials for decelerating charge, and focusing only on the radiation term, one can show \( y = \pi \nu \delta t(1 - n \beta \cos \theta) \):

\[ E(\omega, x) \propto i\omega \frac{e^{ikR} \sin y}{R \frac{y}{y}} \]  

Similar to single-slit diffraction with length \( \approx a \).
A more subtle approximation, keeping another order...

\[ |\mathbf{x} - \mathbf{x}'| = \sqrt{(z - z')^2 + (\rho - \rho')^2} \tag{11} \]

\[ |\mathbf{x} - \mathbf{x}'| \approx R(z') - \frac{\rho \cdot \rho'}{R} + \left( \frac{\rho'^2}{R} \right) \tag{12} \]

\[ R(z') = \sqrt{(z - z')^2 + \rho^2} \tag{13} \]

Scale of the instantaneous charge excess is small compared to the longitudinal shower development. Keep the first two terms, drop the third. Integrals decouple into the form factor, and the Fresnel-Fraunhofer integrals.
The 3D Fourier transform of the charge distribution, $f$, the normalized charge excess distribution. (Dropping bold font for vectors).

$$\int d^3x'f(x') = 1 \quad (14)$$

$$F(q) = \int d^3x' \exp(-iq \cdot x')f(x') \quad (15)$$

$$q = \left( \frac{\omega}{c}, \frac{k}{R} \rho' \right) \quad (16)$$

The structure of the Askaryan electric field is derived in RB, parameterized in ZHS, and fit in the time domain by ARVZ. In addition to the LPM effect, the main thrust of this work is to analytically derive $F(q)$, and match to Monte Carlo simulations from Geant4.
Simple incorporation: draw the $a$-parameter from the EM curve below, rather than Greisen.

![Graph showing shower length vs. neutrino/electron energy]
ASKARYAN FIELDS
ANALYTIC FORMS OF ASKARYAN FIELDS

Viewing Angle: 0 degrees

Frequency [MHz]

E-Field (1 km) [V/m/MHz]

ZHS
Ralston and Buny

1.0e-08 1.0e-07 1.0e-06 1.0e-05 1.0e-04 1.0e-03
ANALYTIC FORMS OF ASKARYAN FIELDS

Viewing Angle: 2 degrees

E-Field (1 km) [V/m/MHz]

Frequency [MHz]
ANALYTIC FORMS OF ASKARYAN FIELDS

Viewing Angle: 4 degrees

E-Field (1 km) [V/m/MHz]

Frequency [MHz]
$E_\nu = 10^{17}$ eV, $\rho_0 = 10 \text{ m}^{-1}$, w/LPM, @ 1000 m
Viewing Angle: 0 degrees

E-Field (1 km) [V/m/MHz]

Frequency [MHz]
ACCOUNTING FOR THE LPM EFFECT - SCALING

Viewing Angle: 2 degrees

**E-Field (1 km) [V/m/MHz]**

- **ZHS** (Black line)
- **RB (no LPM)** (Light grey line)
- **RB (with LPM)** (Dark grey line)

**Frequency [MHz]**

- 10e-08 to 10e-03
- 10e+00 to 10e+04
Viewing Angle: 3 degrees

E-Field (1 km) [V/m/MHz]

Frequency [MHz]
ACCOUNTING FOR THE LPM EFFECT - SCALING

Viewing Angle: 4 degrees

E-Field (1 km) [V/m/MHz]

Frequency [MHz]
EM Showers, $E_\nu \approx 10^{16}$ eV, RB Model

- LPM On
- LPM Off
EM Showers, $E_{\nu} \approx 10^{17}$ eV, RB Model

$\sigma_\theta$ (deg)

Freq$^{-1}$ (GHz$^{-1}$)

LPM On

LPM Off
EM Showers, $E_\nu \approx 10^{18}$ eV, RB Model

$\sigma_\theta$ (deg) vs. $\text{Freq}^{-1}$ (GHz$^{-1}$)

- LPM On
- LPM Off
NUMERICAL WORK
Limitations:

Few hundred jobs at once.

Charged RUs from finite account, 1 RU = 10 CPU-hours

Memory use < 8 GB, (MC thresholds)

Strategy: Implement pre-shower sub-shower strategy, with Geant4.

Utilizes back-fill (each sub-shower is 10 cpu-minutes).

Obeys memory constraints

Cost: few hundred RUs, courtesy of Dr. Amy Connolly, @ OSU

Goal: $F(\omega, \theta)$ using Geant4 pre-showers and sub-showers.
GEANT4 SIMULATIONS
Fractional negative charge excess, $\Delta q$. (MC threshold-dependent)
All tracks in 10 ps window @ 5 ns after primary interaction:
All tracks in 10 ps window @ 10 ns after primary interaction:
All tracks in 10 ps window @ 15 ns after primary interaction:
GEANT4 SIMULATIONS - $Z'$-FORM FACTOR DEPENDENCE?

All tracks in 10 ps window @ 20 ns after primary interaction:

![Graph showing electron tracks in $z$-axis](image-url)
All tracks in 10 ps window @ 25 ns after primary interaction:
GEANT4 SIMULATIONS - Z'-FORM FACTOR DEPENDENCE?

All tracks in 10 ps window @ 30 ns after primary interaction:
The "instantaneous" form factor in z’ is so small, it doesn’t limit the Askaryan radiation...

Unless the time-scale that matters is actually the Nyquist frequency of the RF detectors (1 GHz or 1 ns).

If that were true, then the z-shape could matter (long tail, flat top is limited by time-window).

One can show that the phase shift due to any z-dependence in form factors goes like

\[
\phi/\Delta\theta \approx 2\pi n \left\{ \frac{\nu \Delta t}{\Delta \theta} - \frac{R}{\lambda} \sin \theta_c \right\}
\] (17)
GEANT4 SIMULATIONS - $\rho'$-FORM FACTOR DEPENDENCE

OSC Results
Greisen, 5 rad. lengths
Greisen, 10 rad. lengths
Greisen, 20 rad. lengths

Charged Particle Density (cm$^{-2}$)

$\rho$ (Moliere units = 10.4, g cm$^{-2}$)
Necessary to explain why decelerating charge doesn’t radiate up to optical frequencies: \( E(k) \approx k \).

\[
F_{ZHS}(k) = \frac{1}{1 + \left(\frac{k}{k_0}\right)^2} = \frac{k^2}{k_0^2 + k^2}
\]  

What does the corresponding charge distribution (inverse Fourier transform) resemble? Must treat the poles carefully.

\[
f(\rho') = \frac{k_0^2}{2\pi} \int_{-\infty}^{\infty} \frac{e^{ik\rho'}}{(k + ik_0)(k - ik_0)} dk, k \in \mathbb{C}
\]

\[
f(\rho')/k_0^2 = \frac{1}{2\pi i} \oint \frac{ie^{ik\rho'}(k + ik_0)^{-1}}{(k - ik_0)}
\]
ZHS FORM FACTOR

\[
\begin{align*}
  f(\rho')/k_0^2 &= \frac{1}{2\pi i} \int ie^{ik\rho'}(k + ik_0)^{-1} \frac{1}{(k - ik_0)} \\
  f(\rho') &= k_0^2 \left( ie^{ik\rho'}(k + ik_0)^{-1} \right)_{k=ik_0} \\
  f(\rho') &= \frac{k_0}{2} e^{-k_0\rho'} 
\end{align*}
\]  

Exponential (interesting), normalized to $\frac{1}{2}$. Using the oppositely oriented contour, we get a different distribution:

\[
\begin{align*}
  f(\rho') &= -k_0^2 \left( ie^{ik\rho'}(k - ik_0)^{-1} \right)_{k=-ik_0} = \frac{k_0}{2} e^{k_0\rho'} 
\end{align*}
\]

We must choose the contour that follows Jordan’s lemma - (see below).
What about a form factor like:

\[ f(\rho') = k_0 \exp(-k_0 z'), \rho' > 0 \]  

(25)

Fourier transform gives the form factor:

\[
F_{\text{JCH}}(k) = \int_{-\infty}^{\infty} dz' e^{-ik\rho'} k_0 e^{-k_0 \rho'} = \frac{k_0}{k_0 - ik}
\]  

(26)

Only one pole, at \( k = -ik_0 \), and still cuts off the high-frequency spectrum. So in the 1D case, just remove a pole, or, take the contour that converges.
Taking the inverse Fourier transform of $F_{JCH}$ requires closing the contour around the one pole, and using Cauchy’s formula.

$$f(\rho')/k_0 = \frac{1}{2\pi} \int_{-\infty}^{\infty} \frac{e^{ik\rho'}}{k_0 - ik} dk$$  \hspace{1cm} (27)

$$f(\rho')/k_0 = \frac{1}{2\pi i} \oint \frac{ie^{ik\rho'}}{k_0 - ik} dk = \frac{1}{2\pi i} \oint \frac{e^{ik\rho'}}{k - ik_0} dk$$  \hspace{1cm} (28)

$$f(\rho') = k_0 e^{-k_0 \rho'}, \ \rho' > 0$$  \hspace{1cm} (29)

Notice that $\Re(F_{JCH}) = F_{ZHS}$, and $|F_{JCH}| = \sqrt{F_{ZHS}}$ (for same $k_0$). This means that my form factor also cuts off the spectrum at high frequencies, and reduces to ZHS if we ignore imaginary $E$ before taking the magnitude. Interesting that $\arg(F_{JCH}) \approx k/k_0$, so $k_0$ should be large, to avoid adding extraneous phases.
\( E_\nu = 10^{17} \text{ eV}, \text{ RB EM shower (1 km), no LPM} \)

Form factor \( \rightarrow f(z) = b \exp(bz), z<0 \)

\( b^{-1} \approx 1 \text{ cm} \)
3D CASE - $\rho'$-FORM FACTOR DEPENDENCE

For the general, 3D case, I propose a $\rho'$-dependence as follows:

$$f(x') = f_0 \delta(z') \exp(-\sqrt{2\pi} \rho_0 \rho'), \quad \int dz' d^2 \rho' f(x') = 1, \quad f_0 = \rho_0^2$$  \hspace{1cm} (30)

$$F(q) = \int_{-\pi}^{\pi} \int_0^\infty \int_0^\infty dz' \rho' d\rho' d\phi' e^{-i q \cdot x'} f(x')$$  \hspace{1cm} (31)

$$\gamma = k \sin \theta \ (m^{-1})$$  \hspace{1cm} (32)

$$\sigma = \frac{\gamma}{\sqrt{2\pi} \rho_0}$$  \hspace{1cm} (33)

So $\sigma$ is the ratio of the lateral projection of the wave-vector and the lateral charge extent. Perform $z'$-integration and substitute:

$$F(q) = \rho_0^2 \int_0^\infty \rho' d\rho' \int_{-\pi}^{\pi} d\phi' \exp\{-i \gamma \cos \phi + \gamma/\sigma \rho'\}$$  \hspace{1cm} (34)
ρ'-FORM FACTOR DEPENDENCE

Shift $\phi \rightarrow \phi - \pi/2$ (cylindrical symmetry), and perform $\phi$-integration:

$$F(q) = \rho_0^2 \int_0^\infty \rho' d\rho' \int_{-\pi}^\pi d\phi' \exp\{-i\gamma \cos \phi + \gamma/\sigma \rho'\}$$  \hspace{1cm} (35)

$$F(q) = \rho_0^2 \int_0^\infty \rho' d\rho' \exp\left\{-\frac{\gamma}{\sigma} \rho'\right\} \int_{-\pi}^\pi d\phi' \exp\{-i\gamma \rho' \sin \phi\}$$  \hspace{1cm} (36)

$$F(q) = 2\pi \rho_0^2 \int_0^\infty d\rho' \rho' \exp\left\{-\frac{\gamma}{\sigma} \rho'\right\} J_0(\gamma \rho')$$  \hspace{1cm} (37)

$$F(q) = \sigma^{-2} \int_0^\infty du' u' \exp\{-u'/\sigma\} J_0(u')$$  \hspace{1cm} (38)

Table of integrals...and finally:

$$F(k, \theta) = \frac{1}{(1 + \sigma^2)^{3/2}} = \left(1 + \left(\frac{k}{\rho_0}\right)^2 \left(\frac{\sin \theta}{2\pi}\right)^2\right)^{-3/2}$$  \hspace{1cm} (39)
COMBINED RESULTS
Shower energy: $10^{17}$ eV.
$10^{16}$ eV, $(2\pi)^{1/2} \rho_0 = 250 \text{ m}^{-1}, \text{ LPM, 1 km}$
$10^{17}$ eV, $(2\pi)^{1/2} \rho_0 = 250$ m$^{-1}$, LPM, 1 km
$10^{18}$ eV, $(2\pi)^{1/2} \rho_0 = 250$ m$^{-1}$, LPM, 1 km
$10^{16}$ eV, $(2\pi)^{1/2} \rho_0 = 25 \text{ m}^{-1}$, LPM, 1 km
COMPLETE FIELD - $\theta$ POLARIZATION

$10^{17}$ eV, $(2\pi)^{1/2}$ $\rho_0 = 25$ m$^{-1}$, LPM, 1 km

Viewing Angle (deg)

Time (ns)

52
53
54
55
56
57
58
59
60

-4 -2 0 2 4

-1
-0.5
0
0.5
1

-4 -2 0 2 4
$10^{18}$ eV, $(2\pi)^{1/2} \rho_0 = 25$ m$^{-1}$, LPM, 1 km
$10^{16}$ eV, $(2\pi)^{1/2} \rho_0 = 2.5 \text{ m}^{-1}$, LPM, 1 km
$10^{17}$ eV, $(2\pi)^{1/2} \rho_0 = 2.5 \text{ m}^{-1}$, LPM, 1 km
$10^{18}$ eV, $(2\pi)^{1/2} \rho_0 = 2.5 \text{ m}^{-1}$, LPM, 1 km
$10^{18}$ eV, $(2\pi)^{1/2} \rho_0 = 25.0 \text{ m}^{-1}$, LPM, 1 km
NEW RESULTS - PURELY ANALYTIC
TIME-DOMAIN FIELDS
At the Cerenkov cone, with ideal form factor \( F(\omega, \theta) = 1 \), the E-field takes a convenient form:

\[
RE(\omega, \theta_C) = -\frac{i\omega E_0 \sin \theta_C e^{i\omega R/c}}{(1 - i\eta)^{1/2}} \hat{e}_\theta \quad [\text{V/Hz}]
\]  

(40)

\( E_0 \): Energy scaling-normalization (goes as \( n_{\text{max}} a \), or the area under the Gaussian \( n(z') \), i.e. total charged particles).

\( \eta = k(a^2 \sin^2 \theta)/R \), same parameter from RB, \( \eta = \omega/\omega_C \).

\( \hat{e}_\theta \) is the spherical unit vector (field is linearly polarized orthogonal to viewing direction).

\[
RE(t_r, \theta_C) \approx \frac{i\omega_C E_0 \sin \theta_C}{\pi} \hat{e}_\theta \frac{d}{dt_r} \int_C d\omega \frac{e^{-it_r \omega}}{\omega + 2i\omega_C}
\]  

(41)
JORDAN’S LEMMA, INVERSE FOURIER TRANSFORM

Phase/π, $t=-2.0$ ns

Phase/π, $t=2.0$ ns

$log_{10}(\text{Norm})$, $t=-2.0$ ns

$log_{10}(\text{Norm})$, $t=2.0$ ns
Under the Lorentz gauge condition for Maxwell’s equations, in the absence of any static potentials, the negative derivative of the vector potential yields the electric field: 

\[-\partial A/\partial t = E.\]

\[\begin{align*}
R\mathbf{E}(t_r, \theta_C) &\approx 4E_0 \sin(\theta_C) \omega_C^2 \hat{e}_\theta \\
&= \begin{cases} 
\exp(2\omega_C t_r) & t_r \leq 0 \\
-\exp(-2\omega_C t_r) & t_r > 0
\end{cases} [V] 
\end{align*}\]  

\[\begin{align*}
R\mathbf{A}(t_r, \theta_C) &\approx -2E_0 \omega_C \sin \theta_C \hat{e}_\theta \\
&= \begin{cases} 
\exp(\omega_C t_r) & t_r \leq 0 \\
\exp(-\omega_C t_r) & t_r > 0
\end{cases} [V \cdot \mathbf{s}]
\end{align*}\]
The coherence limiting frequency

\[ \log_{10}(\nu_C \text{ [GHz]}), \quad \theta_C = 55.8 \text{ deg} \]

\[ \nu_C = \frac{cR}{2\pi a^2 \sin^2 \theta_C} \quad (44) \]
Let $\sigma = \omega/\omega_{CF}$.

$$RE(\omega, \theta_C) = -\frac{i\omega E_0 \sin \theta_C e^{i\omega R/c}}{(1 - i\omega/\omega_C)^{1/2}(1 + (\omega/\omega_{CF})^2)^{3/2}} \hat{e}_\theta \quad \text{[V/Hz]} \quad (45)$$

In the limit $\sigma < 1$, and $\eta < 1$, $\omega_0 = 2/3 \omega_{CF}$.

$$RE(t_r, \theta_C) \approx \frac{2i}{3\pi} \hat{e}_\theta \frac{d}{dt_r} \int d\omega \frac{E_0 \sin \theta_C \omega_{CF}^2 \omega_C e^{-it_r\omega}}{(\omega + 2i\omega_C)(\omega + i\omega_0)(\omega - i\omega_0)} \quad \text{[V]} \quad (46)$$

**Key figure of merit:** ratio of form factor limiting frequency, and the coherence limiting frequency:

$$\epsilon' = (\sqrt{2\pi} \rho_0 \rho) \left(\frac{a}{R}\right)^2 \quad (47)$$
$\log_{10}(\omega_{CF}/\omega_{C}), \ R = 500 \ m$

\[ \epsilon' = (\sqrt{2\pi} \rho_0 \rho) \left(\frac{a}{R}\right)^2 \]  

(48)
FINAL RESULTS (VECTOR POTENTIAL)

\[ R|A(t_r, \theta_C)|, \ (2\pi)^{1/2} \rho_0 = 1.0 \text{ m}^{-1} \]

\[ a=(1,2,...,10) \text{ m} \]

\[ R|A(t_r, \theta_C)|, \ (2\pi)^{1/2} \rho_0 = 5.0 \text{ m}^{-1} \]

\[ a=(1,2,...,10) \text{ m} \]

\[ R|A(t_r, \theta_C)|, \ (2\pi)^{1/2} \rho_0 = 10.0 \text{ m}^{-1} \]

\[ a=(1,2,...,10) \text{ m} \]
Our model reproduces the shape, width, and asymmetry, of the semi-analytical approach. (Taylor expansion of $\omega_C$ exponential gets the form of ARVZ $A(t, \theta_C)$.)
CONCLUSIONS
I. Developed a fully analytic RB-model of Askaryan radiation
   A. Accounts for LPM effect, and $F(\omega, \theta)$.
   B. $F(\omega, \theta)$ was derived with the help of the Greisen parameterization, Geant4, and the OSC

II. Results in the Fourier domain, LPM and form factor together

III. Newest work: **analytic equations in the time-domain**

IV. This work will be published in a forthcoming paper, and posted on arXiv by April 23

*The code is on github:*

```
git clone https://github.com/918particle/AraSim2 AraSim2
```
CONCLUSIONS - DEVELOPERS NEEDED FOR ARASIM2

Spectrum: neutrino energy spectrum model with ability to insert multiple models.

Event Generator: Neutrino theta, phi, energy, from Spectrum, in a fractional coordinate system.

Askaryan: charge profile is computed based on hadronic/electromagnetic parameterization.

Antropogenic: collection of anthropogenic noise events.

Event: the full set of information to be compared to data in analysis.

Detector: collection of stations.

Output collection: similar to record class in Arasim 1.0, a storage container for all modules' optional outputs.

Signal: the complete electric field, with phase and magnitude, and voltage waveforms.

Geophysics: Ice absorption, scattering, index, density, temperature, flow, birefringence, surface roughness, ocean surface roughness.

Front End: Properties of the data acquisition that depend on frequency, including gain, and group delay, and voltage linearity.

Trigger Requirements: Number of channels required to satisfy a given threshold, a threshold, allowing for scam.

Measurement: number of channels, response from Trigger Requirements, station location in fractional coordinates, live-time.

LPM effect handler: our sub-peak sneering strategy (parameters). Background: showing a thermal noise sample from Thermal background.

Build System Ideas: Use BuildBot from BuildIot.com as an open source build system that interfaces with Git versioning, and looking into Boost C++ as a tool to build frames and also convenient features.